# The Timing and Magnitude of Exchange Rate Overshooting

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**Abstract**

Empirical evidence suggests that a monetary shock induces the exchange rate to overshoot its long-run level. The estimated magnitude and timing of the overshooting, however, varies across studies. This paper generates delayed overshooting in a New Keynesian model of a small open economy by incorporating incomplete information about the true nature of the monetary shock. The framework allows for a sensitivity analysis of the overshooting result to underlying structural parameters. It is shown that policy objectives and measures of the economy's sensitivity to exchange rate dynamic affect the timing and magnitude of the overshooting in a predictable manner, suggesting a possible rationale for the cross-study variation of the delayed overshooting phenomenon.

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# 1 Introduction

Dornbusch's (1976) seminal work on expectations and exchange rate dynamics has had a lasting impact on theoretical modeling in international macroeconomics. The most striking prediction of his model is the overshooting result. That is, when prices are sticky and the nominal exchange rate adjusts without friction to equalize foreign and domestic expected returns, an unanticipated permanent change in the money supply causes the exchange rate to *immediately* overshoot its long-run level and subsequently revert to its new equilibrium rate.

Unfortunately, despite the model's simplicity and intuitive appeal, empirical evidence seems to provide only mixed support for the overshooting prediction. Indeed, early studies using estimated vector-autoregressive models (VARs) typically indicate that the exchange rate response to a monetary shock is not immediate but occurs with a lag. For example, Eichenbaum and Evans (1995) find that the peak of the nominal and real overshooting occurs between 24 to 40 months after the initial shock. Other studies such as Grilli and Roubini (1996) find similar results. This empirical puzzle has popularly been referred to as the delayed overshooting phenomenon. In recent years, however, some observers have questioned the robustness of these results. In particular, the exact timing of the overshooting has come under significant scrutiny. For instance, Faust and Rogers (2003) show that the delayed overshooting result can be quite elusive and extremely sensitive to the underlying assumptions necessary to identify the initial monetary shock. For some specifications they find that the peak of the overshooting can occur as early as within one month or as late as 30 months after the shock. Additionally, Kim and Roubini (2000) find that the timing of the delayed overshooting differ from country to country, indicating the importance of country specific factors.[1]

The purpose of this paper is to take a theoretical approach in order to shed some light on possible determinants of the magnitude and timing of the overshooting result. Since delayed overshooting implies that the nominal exchange rate fails, at least ex-post, to equalize foreign and domestic expected returns (i.e., the uncovered interest parity condition (UIP) does not hold), our framework must address this failure. Theoretically, the breakdown of the UIP could be due to either a time-varying risk-premium or persistent expectational errors. However, since empirical evidence seems to indicate that monetary shocks only account for a small share of the variation in output, it is likely that fluctuations in the risk premium are also relatively small.[2] Hence,

[1] Other studies, such as Cushman and Zha (1997) and Bjornland (2005) find little support of delayed overshooting.

[2] Smets and Wouters (2003) finds that monetary policy shocks contributed very little to euro area output variations in the 1980s and 1990s and that most of euro area output variations can be attributed to various supply and demand shocks.

for monetary shocks to generate UIP deviations, these deviations are more likely to be due to expectational errors. Following this rationale, our theoretical set-up assumes that agents are rational but that they do not have enough information regarding the true degree of persistence of the monetary shock. As a result, they are forced to learn by observing realized outcomes and form optimal forecasts of the future path of the policy instrument. This learning process induces persistent expectational errors which give rise to ex-post UIP deviations and delayed overshooting.

A similar approach is taken by Gourinchas and Tornell (2004). In their set-up agents cannot fully distinguish between persistent and temporary shifts in the forward premium (i.e., the differential between domestic and foreign interest rates). As in our model, they show that such an assumption leads to UIP deviations and delayed overshooting of the nominal exchange rate. Our model differs from Gourinchas and Tornell (2004) in two important ways. First, we explicitly model the underlying structure of the economy through a standard dynamic stochastic general equilibrium model. This allows us to examine *both* nominal and real exchange rate dynamics. It also permits us to see how sensitive the overshooting result is to structural characteristics of the economy.[3] Second, in our model monetary policy is specified by a generic interest rate rule and not by an exogenously assumed stochastic process as in Gourinchas and Tornell (2004). This means that exchange rate movements, as well as domestic inflation and output, are allowed to contemporaneously feedback into interest rate dynamics through the central banks reaction function. By endogenizing monetary policy, we can examine the sensitivity of the overshooting result to various policy specification.

The model provides several interesting results with respect to the magnitude and timing of the exchange rate overshooting result.

1. *Overshooting and learning:* (i) Incomplete information combined with learning produces a delayed exchange rate response to a monetary shock. The more persistent the shock, the more delayed and the greater is the magnitude of the overshooting. (ii) The peak of the nominal exchange rate generally occurs later than that of the real exchange rate.

2. *Overshooting and policy objectives:* (i) The higher the degree of policy inertia, the more delayed and the smaller is the magnitude of the overshooting. (ii) The more concerned the central bank is about output fluctuation (relative to inflation fluctuation), the greater is the magnitude of the overshooting.

[3] Pierdzioch (2005) also analyzes nominal and real exchange rate overshooting by specifing a dynamic general equilibrium model. In contrast to our model, however, persistence in exchange rate expectations is exogenously assumed through a "rule of thumb" behavior. That is, some portion of market participants behave irrationally.

3. *Overshooting and exchange rate exposure:* The more open the economy and the greater the exchange rate pass-through, the smaller is the magnitude and the more delayed is the exchange rate overshooting.

The paper is structured as follows. Section 2 highlights the importance of future expected monetary policy on exchange rate dynamics. Section 3 presents the canonical representation of Monacelli's (2005) model of a small open economy. Section 4, describes monetary policy, the nature of the policy shock, and the optimal filtering problem facing price setters. Section 5, presents and discusses the sensitivity of the overshooting result due to partial information and learning for different policy parameters and degrees of exchange rate exposure. Finally, section 6 concludes.

## 2 Exchange Rate Determination and Monetary Policy

Under the assumption of full capital mobility and perfect substitutability between domestic and foreign assets, a no arbitrage assumption imply that the expected rates of returns of the two assets (measured in the same currency) must equalize. This *uncovered* interest rate parity condition can be written as:

$$i_t = i_t^* + E_t^p \Delta e_{t+1} \tag{1}$$

where $i_t$ and $i_t^*$ are the domestic and foreign nominal interest rate, respectively. The nominal exchange rate is denoted by $e_t$, and $E_t^p$ refers to the public's expectations at time $t$ with respect to the information set $\Omega_t^p$. Realizing that (1) is a difference equation we can solve for the current exchange rate and iterate the expression forward to derive the following expression:

$$e_t = \bar{e}_t - E_t^p \sum_{i=0}^{\infty} \left( i_{t+i} - i_{t+i}^* \right) \tag{2}$$

where $\bar{e}_t = E_t^p e_{t+\infty}$ i.e., the long-run expected nominal exchange rate. Thus, given the foreign interest rate, the current exchange rate is determined by (i) the future expected sum of short-term nominal interest rates and (ii) its expected long-run level. Since the monetary authorities control the short-term interest rate, equation (2) clearly shows that perceived future monetary policy actions have a significant impact on exchange rate dynamics. Hence, if the monetary authorities are not transparent enough with respect to future intended policy actions (i.e., $\Omega_t^p \subset \Omega_t$, where $\Omega_t$ refers to the complete information set), or if their announcements are not perceived as credible, the uncovered interest rate parity does not have to hold ex-post. For example, if a persistent decrease in the nominal interest rate is initially mis-percieved by the public as a temporary expansionary action, the initial exchange rate response is likely to be muted. However, as private agents learn about the true nature of the policy shift, they revise the expected future path of the interest rate. Depending on the speed of the learning process and the persistence of the shock, the exchange rate response may be significantly restrained and delayed.

The same rationale holds for the real exchange rate. Substituting the definition for the real exchange rate (i.e. $q_t = e_t + p_t^* - p_t$ , where $p_t$ and $p_t^*$ denote the domestic and foreign consumer price index (CPI), respectively) into (1), we can derive the following expression:

$$q_t = \overline{q}_t + E_t^p \sum_{i=0}^{\infty} \left\{ \left( i_{t+i}^* - \pi_{t+i}^* \right) - \left( i_{t+i} - \pi_{t+i} \right) \right\} \tag{3}$$

where $\overline{q}_t = E_t^p q_{t+\infty}$ is the long-run expected real exchange rate. Equation (3) states that the real exchange rate is determined by the future expected path of the real interest rate. As long as monetary policy can systematically affect the real interest rate rate (via the nominal interest rate), imperfect information about the true nature of monetary shocks can play an important role in determining real as well as nominal exchange rate dynamics.

Finally, notice that if the purchasing power parity holds in the long-run, the long-run real exchange rate should be constant and independent of the nature of the monetary shock. The long-run nominal exchange rate, on the other hand, depends on the impact that the monetary shock has on the domestic consumer price index (relative to the foreign consumer price index). That is, the current nominal exchange rate is not only determined by the expected future path of the nominal interest rate, but also the expected long-run price level. This observation turns out to be important when comparing real and nominal exchange rate dynamics under incomplete information.[4]

# 3 A Canonical Representation of a Small Open Economy

This section presents the structure of a small open economy. More precisely, we employ the model specified by Monacelli (2005). Since the derivation of the model is available in the previous reference the following sections only describe the reduced form equations. To simplify matters we assume that the foreign (or world) price level, interest rates and output level are time-invariant and unaffected by the domestic economy.

[4] Indeed, Gourinchas and Tornell (2004) assumes that the learning behavior of the private sector does not affect the long-run exchange rate. In our model, on the other hand, we allow for the partial information structure to also affect the long-run price level and thus the long-run nominal exchange rate.

### 3.1 Price Levels, Exchange Rates and Imperfect Pass-Through

The log-linerized Consumer Price Index (CPI) in the small open economy can be written as:

$$p_t = (1-\alpha)\, p_{H,t} + \alpha p_{F,t} \tag{4}$$

where $p_{H,t}$ denotes the price index of domestically produced goods, and $p_{F,t}$ denotes the domestic currency price index of imports. The parameter $\alpha$ represents the share of domestic consumption allocated to foreign produced goods. It follows from (4) that the CPI inflation rate can be expressed as:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{5}$$

where $s_t$ is the terms of trade defined as $p_{F,t} - p_{H,t}$ . Thus, the more open the economy, as measured by $\alpha$, the greater is the impact of domestic prices of imported goods on inflation. Note that if the *law of one price* (l.o.p) holds then it must be true that $p_{F,t} = e_t + p_t^*$. To allow for deviation from the l.o.p and incorporate imperfect pass-through, Monacelli (2005) defines a new variable $\psi_t$:

$$\psi_t = (e_t + p_t^*) - p_{F,t} \tag{6}$$

Thus, $\psi_t$ (labeled as the *law-of-one-price gap,* l.o.p gap), measures the wedge between the world prices of foreign goods and the domestic prices of imported goods set by local importers. The real exchange rate can then be expressed as:

$$q_t = \psi_t + (1-\alpha)\, s_t \tag{7}$$

Hence, the l.o.p gap separates the terms of trade and the real exchange rate. Under complete pass-through, given the domestic price level, a nominal depreciation causes an increase in the terms of trade and a rise in CPI inflation. Under incomplete pass-through, on the other hand, the nominal depreciation will have less of a direct effect on prices of imported goods and therefore less of an effect on CPI-inflation. Instead, the real exchange rate will absorb more of the nominal depreciation.

### 3.2 The Supply Side

There are two groups of price setters in the economy: (i) the domestic producers and (ii) the local importers. All price setters face downward sloping demand and are constrained by their infrequent ability to adjust retail prices as the economic environment changes. The optimal pricing behavior of the importers is thus primarily influenced by current and expected future dynamics of the l.o.p gap, $\psi_t$. For example, if the domestic currency is expected to depreciate (i.e., l.o.p gap is expected to increase given the prices of imported goods), importers are likely to raise prices today since they may not have the opportunity to raise them in the near future. The supply curve for imported goods can then be derived as:

$$\pi_{F,t} = \beta E_t^p \pi_{F,t+1} + \lambda_F \psi_t \tag{8}$$

where the parameter $\lambda_F = \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}$ and where $\theta_F$ governs the degree of exchange rate pass-through. If the probability to revise retail prices on imported goods is low then $\theta_F$ is close to one and $\lambda_F$ consequently relatively small.

Similarly, domestic producers set their prices in anticipation of current and expected future dynamics of marginal cost. The marginal cost, in turn, is proportionally affected by domestic output through its effect on real wages.[5] However, under incomplete pass-through, the l.o.p gap has an additional affect on the marginal cost above and beyond the domestic output level. To see this, suppose a change in the nominal exchange rate is not passed through to domestic import prices (i.e., l.o.p gap increases), then the real exchange rate absorbs the change completely (see equation 7). The real depreciation causes a decrease in the domestic expected real interest rate (relative to the foreign expected real interest rate) increasing current consumption which puts upward pressure on the real wages. As a result, the aggregate supply curve for domestic goods takes the following form:

$$\pi_{H,t} = \beta E_t^p \pi_{H,t+1} + \kappa_x x_t + \kappa_\psi \psi_t \tag{9}$$

where $x_t$ denotes the domestic output gap (i.e., the difference between domestic output, $y_t$, and potential output, $\overline{y}$). The parameter $\kappa_x$ and $\kappa_\psi$ are non-negative constants and are negatively related to the stickiness of retail prices of domestic and imported goods.[6]

Combining (8), (9), and (4) we can now write the CPI-based aggregate supply curve as:

$$\pi_t = \beta E_t^p \pi_{t+1} + \kappa_x^c x_t + \kappa_\psi^c \psi_t \tag{10}$$

where $\kappa_x^c = (1-\alpha)\kappa_x > 0$ and $\kappa_\psi^c = (1-\alpha)\kappa_\psi + \alpha\lambda_F > 0$. Thus, the slope of the aggregate supply curve, $\kappa_x^c$ , is negatively related to the openness of the economy. That is, the larger the

[5] This proportionality factor differ between an open and closed economy. In an open economy, an increase in output does not only have a direct impact on real wages but also causes a real depreciation (a decrease in the price of domestic goods relative to foreign goods) which increase the marginal cost further through its effect on the product wage for any given real wage.

[6] In fact, $\kappa_x = \lambda_H \left(\varphi + \frac{\sigma}{\omega_\alpha}\right)$ and $\kappa_\psi = \lambda_H \left(1 - \frac{\omega_\psi}{\omega_\alpha}\right)$. The stickyness of domestic goods prices is determined by $\lambda_H = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}$ where $\theta_H$ is the Calvo parameter. The parameters $\varphi$ and $\sigma$ denotes the inverse of the intertemporal elasticities of labor supply and consumption, respectively. Finally, $\omega_\alpha = 1 + \alpha(2-\alpha)(\sigma\eta - 1)$ and $\omega_\psi = 1 + \alpha(\sigma\eta - 1)$. Thus in the case where $\sigma\eta = 1$, the direct impact of the l.o.p gap on domestic prices dissapears.

relative weight is on prices of imported goods in the CPI, the less of an effect domestic output movements have on the overall price level.

### 3.3 The Demand Side

The demand side of the domestic economy under incomplete pass-through can be described by the following IS-equation:

$$x_t = E_t^p x_{t+1} - \frac{\omega_\alpha}{\sigma}\left(i_t - E_t^p \pi_{H,t+1}\right) + \Gamma_y E_t^p \Delta\psi_{t+1} \tag{11}$$

The inverse of the intertemporal elasticity of substitution is represented by $\sigma$, and the parameter $\omega_\alpha = 1 + \alpha(2-\alpha)(\sigma\eta - 1)$, where $\eta$ is the elasticity of substitution between home and foreign goods. Finally, the slope coefficient in front of the l.o.p gap can be derived as $\Gamma_y = \frac{\alpha(1-\alpha)(\sigma\eta-1)}{\sigma}$. Thus, the degree of openness affects the output gap's sensitivity to movements in both the interest rate and l.o.p gap (as long as $\sigma\eta \neq 1$). This is because a change in the interest rate affects the exchange rate which in turn leads to an expenditure switching effect. Notice that under incomplete pass-through and $\sigma\eta > 1$, the expenditure switching effect is mitigated by movements in the l.o.p gap.

## 4 Monetary Policy, Partial Information and Optimal Filtering

Finally, to close the model we need to specify the behavior of the monetary authorities. The central bank is assumed to implement policy according to the following rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\pi_t^* + \rho_\pi\left(\pi_t - \pi_t^*\right) + \rho_y x_t\right] \tag{12}$$

The policy rule indicates that the central bank adjusts the nominal interest rate so that the ex-post real interest rate rises when CPI-inflation increases above its target, $\pi_t^*$, or if output rises above its potential level. Moreover, policy is assumed to display an intrinsic degree of inertia as represented by the smoothing parameter $\rho_i$. Thus, the greater the smoothing process, the smaller is the contemporaneous impact of policy feedback on the interest rate.

Equation (12) also allows for a time-invariant inflation target. In fact, in our simulations we model the monetary shock as an innovation to the inflation target.[7] Similar to Erceg and Levin

[7] The assumption of a time-varying inflation target have been used widely in papers that try to estimate DSGE models using Bayesian methods (see for instance Smets and Wouters (2003), Adolfson et al (2005). In addition, recent papers within the Affine Term Structure Modelling literature have also assumed a time-varying inflation target in the interest rate rule (see Rudebusch and Wu (2003) and Lildholdt, Peacock and Panigirtzoglou, 2006)

(2003), we allow the inflation target, $\pi_t^*$, to be subject to persistent and temporary shocks. That is,

$$(\pi_t^* - \bar{\pi}) = (\pi_{p,t} - \bar{\pi}) + \chi_t \tag{13}$$

$$(\pi_{p,t} - \bar{\pi}) = \rho_p (\pi_{p,t-1} - \bar{\pi}) + v_t \tag{14}$$

where $0 < \rho_p < 1$ and $\bar{\pi}$ is the long-run inflation target. The stochastic variables $\pi_{p,t}$ represent the persistent shocks to the target, while $\chi_t$ and $v_t$ are independent normally distributed random variable with variances $\sigma_\chi^2$ and $\sigma_v^2$, respectively.

Under full information the public observes both the persistent and the temporary components of the target when forming expectation. However, under partial information the public cannot perfectly determine whether a shift in the current target is going to be persistent or simply transitory. In the latter case the public is faced with an optimal filtering problem in order to estimate the two components. Using the Kalman filter the optimal estimate of the persistent component can be written as:

$$E_t^p (\pi_{p,t} - \bar{\pi}) = \rho_p E_{t-1}^p (\pi_{p,t-1} - \bar{\pi}) + \frac{K}{\rho_p} \left(\pi_t^* - E_{t-1}^p \pi_t^*\right) \tag{15}$$

The parameter $K$ represents the weight or importance that agents assign to new information (i.e., the realized forecast error of the inflation target) in updating their previous estimate. The weight $K$, the so-called constant gain coefficient, is positively related to the ratio of $\sigma_v^2/\sigma_\chi^2$ (the variance of persistent shocks relative to variance of temporary shocks). That is, the more important permanent shocks are to the dynamics of the inflation target (relative to temporary shocks), the more information can be gained from the realized forecast errors.

# 5 Monetary Shocks and Exchange Rate Overshooting

Since there are no closed form solutions we proceed by simulating the model and derive impulse responses of relevant variables. The parameterization used in the benchmark simulation is given in Table 1. The value of the discount factor $\beta$ indicates an interest rate of 1 percent in the steady state. We follow other work in this literature, e.g. Devereux and Engel (2003), and set $\sigma$, the relative risk aversion parameter, greater than one, $\sigma = 2$. The elasticity of substitution between home and foreign goods $\eta$ is set equal to 1.5 as found by Hooper and Marquez (1995, Table 4.1). The import share $\alpha$ in the steady state is assumed to be 40% for the small open economy. Average import shares in the post Bretton Woods era range from 10% in Japan to almost 30% in Canada and 31% in the UK. Following Rotemberg and Woodford (1997) the inverse of Frisch elasticity of labor supply $\varphi$ is set equal to 0.47. The price adjustment parameter of domestic and foreign goods producers is set such that the average time between price adjustment for a firm is one year. This implies $\theta_H = \theta_F = 0.75$. The parameters for the monetary policy rule are those estimated for the UK in Lubik and Schorfheide (2003) except we assume that the domestic central bank does not target nominal exchange rates. Furthermore, without loss of generality, the inflation target $\bar{\pi}$ equals 0. Since we are not calibrating our model to any particular economy, we have adopted a pragmatic approach in terms of specifying the learning parameters. Under incomplete information, our constant gain Kalman scalar, $K$, is set equals 0.06.[8] Our learning process implies that agents have incorporated half of any permanent shock to the inflation target into their expectations within two years. We set the Kalman scalar, $K$ to 1 when we analyze the model under full information.

Figure 1-6 and tables 2-7 summarize the results. Each figure contains nine sub-figures:(a) output gap, (b) nominal interest rate, (c) UIP deviations (ex-post), (d) real exchange rate, (e) nominal exchange rate, (f) l.o.p gap, (g) CPI inflation, (h) domestic inflation, and (i) inflation target. Table 2-7, show the relationship between the particular parameter of interest and the timing and magnitude of the exchange rate overshooting.

## 5.1 Overshooting and Learning

Figure 1 displays the impulse response functions of a one percent positive shock to the persistent component of the inflation target under full information (dashed) and partial information (solid line), respectively.

First, let us consider the case of full information. To achieve the new higher inflation target the central bank must expand the economy by lowering the real interest rate. The private agents know that the shock is persistent and that it therefore has a prolonged expansionary effect. Consequently, inflation expectations increases significantly and causes a drop in the real interest rate. As a result, the central bank does not have to adjust the nominal interest rate by much to achieve the desired inflationary effect. The reduction in the nominal and real interest rate causes the real and nominal exchange rate to immediately depreciate and overshoot their long-run levels. Since the purchasing power parity holds in the long-run, the real exchange rate returns to its initial level while the nominal exchange rate converges to a higher level due to the permanent affect that the shock has on the domestic price level.

Under incomplete information private agents cannot be sure whether the shock to the inflation

[8] Not surprisingly, there is little empirical guidence for the value of K. Using US data, Erceg and Levin (2003) estimate the ratio of the variance of persistent shocks relative to variance of temporary shocks $\sigma_v^2/\sigma_\chi^2$ and finds a Kalman gain of 0.13. A lower value for $K$ seems approporiate for our analysis since U.S Federal Reserve is percived as relative transparent and enjoyes a fair amount of credibility.

target is temporary or persistent. Consequently, they underestimate the persistency of the policy shock. As a result inflation expectations are slow to adjust. The central bank must therefore be more aggressive and reduce the nominal interest rate by more compared to the scenario with complete information.

As agents learn about the nature of the shock, they adjust their expectations. Similar to Gourinchas and Tornell (2003), this leads to persistent ex-post UIP deviations and a delayed overshooting of the exchange rate. Table 2 shows that the real exchange rate peaks in the second period while the nominal exchange rate peaks in the third period. The latter is due to the fact that agents not only have to revise the expected path of the interest rate, but also the long-run domestic price level which determines the long-run nominal exchange rate. The initial underestimation of the persistency of the monetary shock causes a gradual upward revision of the expected long-run domestic price level and thus of the nominal exchange rate. This causes an additional delay in the overshooting dynamics.

Furthermore, since inflation expectations are sticky under imperfect information, the cost of achieving the new inflation target (in terms of a higher output gap) is greater than under complete information. The inflation response to the monetary shock is therefore muted. As a result, the cumulative effect on the price level is smaller under incomplete information and learning and leads to a lower long-run nominal exchange rate level. Thus, in contrast to Gourinchas and Tornell (2003), learning does not only cause a delayed nominal exchange rate overshooting, but it also influence the long-run nominal exchange rate level.

Figure 2 and table 3 show how the results under incomplete information changes with different persistence levels of the shock. The lower the persistence level of the shock, the smaller is the expectational errors and the faster the economy returns to its steady state. Not surprisingly, this implies that the UIP deviations are less severe and less persistent. The delayed overshooting is therefore less pronounced. The magnitude of the real exchange rate overshooting is positively related to the persistence level of the shock while the opposite is true for the nominal exchange rate. This occurs because a less persistent shock has a smaller effect on the long-run price level and therefore on the long-run nominal exchange rate.

## 5.2 Overshooting and Monetary Policy Objectives

This section analyzes how changes in policy parameters affect the magnitude and timing of the overshooting result derived in the previous section. In each of the following simulations we assume that agents have incomplete information regarding the inflation target. Thus, the speed at which agents learn about the nature of the shock is identical across simulations but the parameter specifications differ. Given the specification of the policy rule and the structure of the monetary shock, we focus on two aspects of policy: (i) the degree of policy inertia as represented by the interest rate smoothing parameter, and (ii) the relative importance that the central bank assigns too achieving the new inflation target as supposed to stabilizing real activity.

Figure 3 and table 4 show the impact of different degrees of interest rate smoothing. Note that the benchmark case discussed in figure 1 and 2 corresponds to an interest smoothing parameter of 0.65. A higher interest rate smoothing parameter implies that a shock to the inflation target has a lower contemporaneous feedback effect (via output and inflation) on policy. That is, the monetary authority cares more about smoothing the interest rate than the effect that the shock has on current output and inflation dynamics. Consequently, the perceived cost of achieving the new inflation target is smaller resulting in a more persistent and more inflationary shock. Price setters realize this and adjust their inflation expectation upward. This makes it possible for the central bank to achieve its desired inflationary effect without reducing the nominal interest rate by much. At the same time, policy inertia prolongs the impact of the shock and makes expectational errors more pronounced. As a result, the real and nominal exchange rate overshooting is further delayed. Table 4 shows that when $\rho_i = 0.9$ the real exchange rate peaks in the third period while the nominal exchange rate does not peak until the seventh period. On the other hand, when $\rho_i = 0.5$ they both peak in the second period. The increased differential between the timing of the peak of the nominal and real exchange rate overshooting is due to the fact that a higher degree of interest rate smoothing leads to larger revisions of the expected long-run price level and thus in the expected long-run nominal exchange rate. Finally, the muted response of the interest rate under a high degree of policy inertia results in a low magnitude of the overshooting for both the real and nominal exchange rate.

Figure 4 and table 5 show the results for different values of the relative weights on the output gap. Given the shock to the inflation target, the more the monetary authorities care about the output gap, vis-a-vis inflation, the less aggressively they will pursue the new target. Although price setters do not know the true persistence of the shock, they realize that the real effect of the shock should be muted because of the central bank's aversion to real output gap fluctuation. Inflation expectations, which depend on the stream of future expected output gaps, are therefore lower. The less aggressive policy response to the nominal shock and the consequent lower inflation expectations reduces the shock's impact on expected future real interest rates. As a result, the magnitude of the real exchange rate overshooting is lower. As table 5 shows, the opposite is true for the nominal exchange rate. The differential behavior between the real and nominal exchange rate is due effect on the long-run nominal exchange rate. The more concerned the central bank is about output gap fluctuation, the smaller the cumulative effect is on the long-run price level. The new equilibrium nominal exchange rate is therefore lower and the magnitude of the overshooting greater.

## 5.3 Overshooting and Exchange Rate Exposure

There are many underlying structural parameters that may be of interest when discussing the sensitivity of the overshooting result. This section focuses on two obvious parameters of interest for a small open economy: (i) the degree of openness as measured by the share of domestic consumption allocated to foreign produced goods,i.e., $\alpha$, and (ii) the degree of exchange rate path-through as measured by $\theta_F$. Again, the speed at which agents learn about the nature of the shock is identical across simulations but the parameter specifications differ.

Figure 5 shows the impulse response functions with respect to three different values of $\alpha$. There are three effects due to an increase in the degree of openness. First, the direct impact of exchange rate movements on CPI-inflation is stronger since the price of imported goods has a higher weight in the CPI. Second, the expenditure switching effect is also stronger, making domestic demand more interest rate sensitive. Third, the slope of the Phillips curve is flatter (i.e., inflation is less output gap sensitive). The two first effects suggests that the leverage policy has on inflation should increase with the degree of openness while the last effect suggests the opposite. Under complete pass-through the net effect of these forces is an increase the leverage effect of policy on inflation.[9]

As Figure 5 shows, the interest rate reduction necessary to achieve the new inflation target is less pronounced in a more open economy. This must be the case since the real interest rate required to achieve the new policy objective is lower due to the increase in the policy leverage on inflation. It follows that the muted interest rate response leads to a decrease in the magnitude of the overshooting result for both the nominal and real exchange rate. Additionally, since the inflation response is greater in a more open economy, the cumulative effect on the price level is larger and the long-run nominal exchange rate higher. Of course, a higher new long-run nominal exchange rate combined with learning gives rise to a more delayed overshooting of the nominal relative to the real exchange rate, as previously discussed. Table 6 confirms both of the magnitude and timing prediction for different degrees of openness.

Figure 6 and table 7 show the impact of a monetary shock under different degrees of exchange rate pass-through. A greater degree of pass-through has similar effects as an increase in the openness of the economy. That is, because of the direct impact of the exchange rate on foreign

[9] Under incomplete pass-through, both the direct effect of exchange rate movements on inflation and the expenditure switching effect are muted and weekens the leverage effect of policy on inflation. However, to focus on the degree of openness we simplify the framework by assuming complete pass-through.

prices, monetary policy has a greater leverage effect on inflation. The real and nominal interest rate response to the shock is thus muted causing a decrease in the magnitude of the overshooting result.[10]. Also, the greater inflationary impact of the shock under a higher degree of pass-through increases the long-run nominal exchange rate which in conjecture with learning causes a more pronounced delayed overshooting in the nominal exchange rate.

# 6 Conclusion

The empirical evidence of the delayed overshooting phenomenon can at best be described as mixed. Timing and magnitude of the overshooting varies significantly across studies. One potential source for the delayed overshooting result is that agents do not have complete information about the true nature of the monetary shock. In this paper we incorporate learning into a standard New Keynesian model of a small open economy in order to analyze the magnitude and timing of the overshooting result and its sensitivity to underlying parameters. Impulse response dynamics indicate that policy objectives (i.e., policy inertia and importance of real activity relative to inflation) and measures of the economy's exchange rate exposure (i.e., the degree of openness and exchange rate pass-through) do affect the overshooting result in predictable manners. This suggests that controlling for these parameters across countries and time (e.g., policy transparency, credibility, policy inertia, openness and exchange rate pass-through) could potentially shed some light on why the delayed overshooting result appears so elusive.

[10]Notice that, unlike the degree of openness, the degree of pass-through does not directly affect the interest sensitivity of domestic demand (see the IS-equation (11)). Thus, the output dynamics do not differ much for different degrees of pass-through. The degree of pass-through does however affect the l.o.p deviation and thus the expenditure switch effect. Thus the output response is slightly more pronounces under a higher degree of pass-through.

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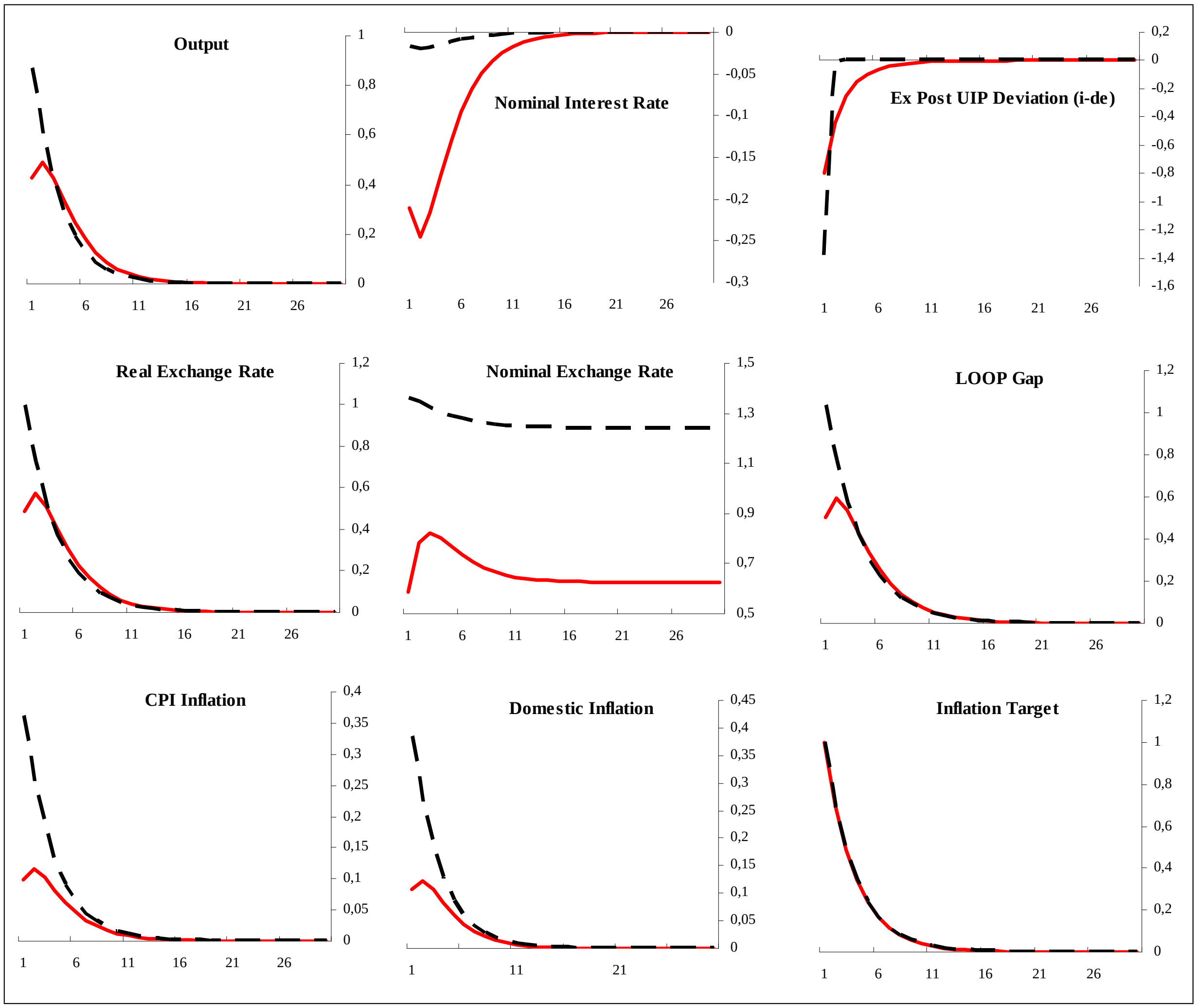


Figure 1: Full - vs. incomplete information. The solid line corresponds to incomplete - and the dashed line to full information.

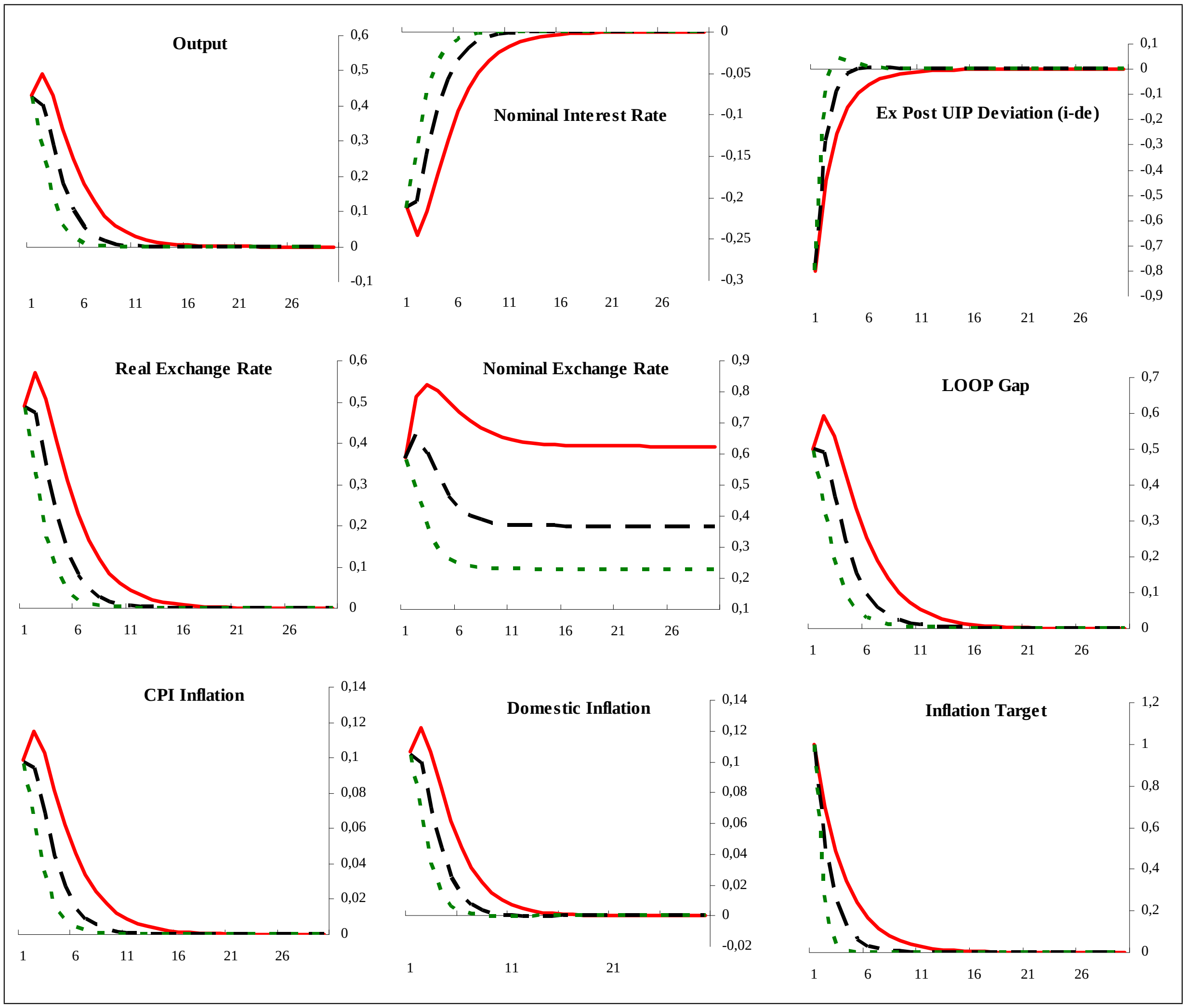


Figure 2: Different degrees of persistency of the shock. The solid line corresponds to a shock persistence of 0.70, the dashed line to 0.50 and the dotted line to 0.20, respectively.

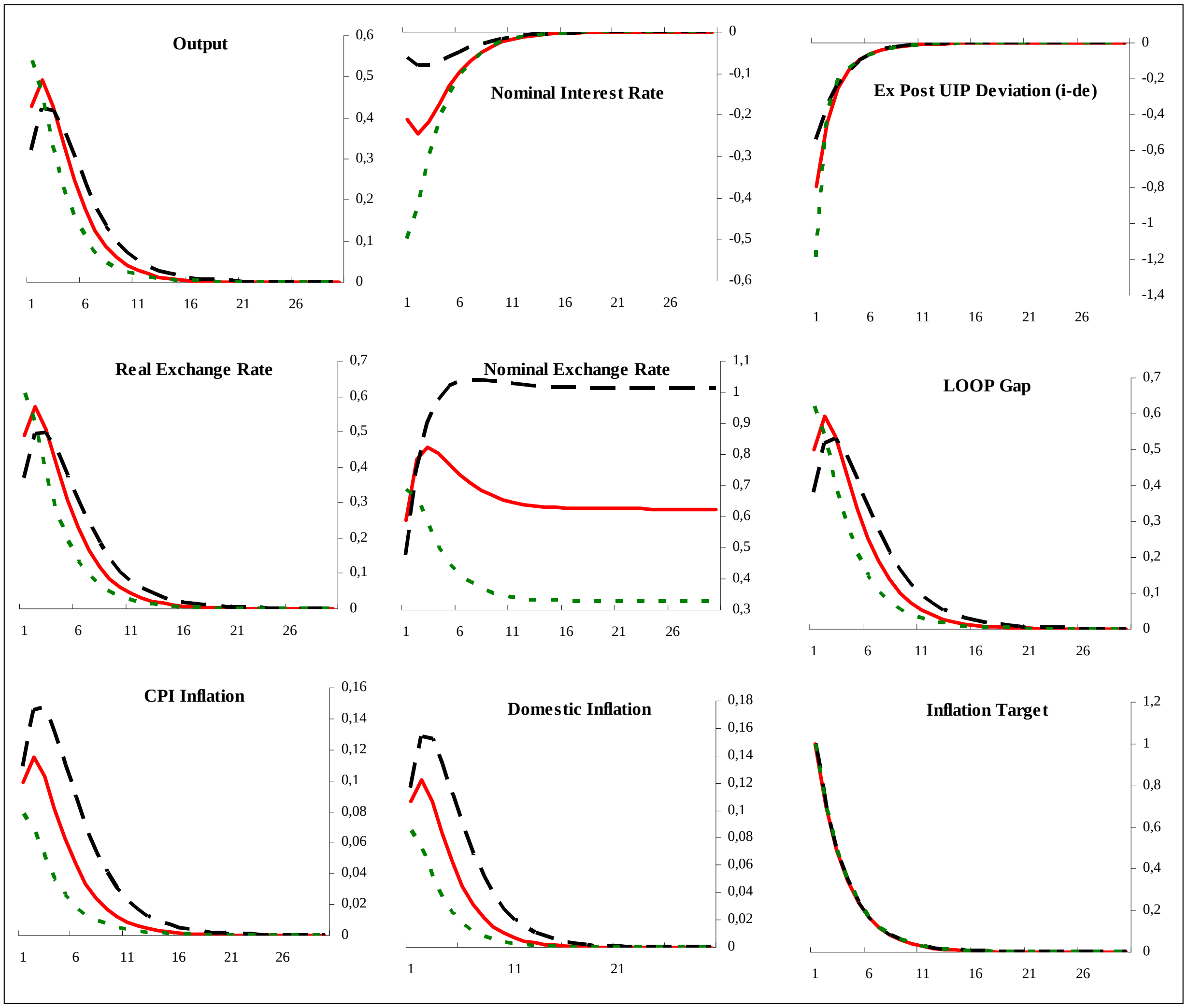


Figure 3: Different degrees of interest rate smoothing. The solid line corresponds to an interest rate smoothing of 0.65, the dashed line to 0.90 and the dotted line to 0.20, respectively.

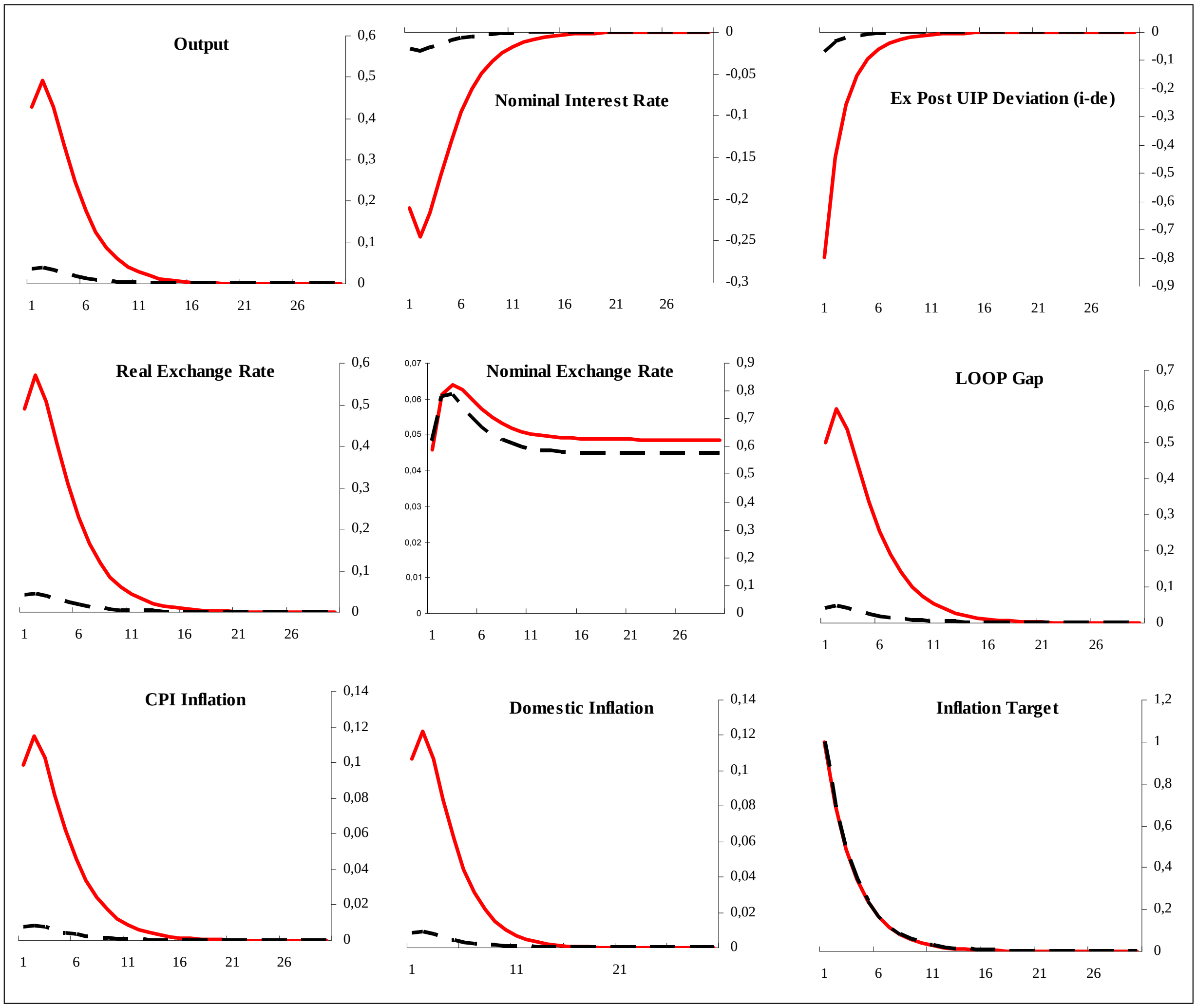


Figure 4: Different values of the relative weights on the output gap (Partial information). The solid line corresponds to a weight on the output gap of 0.15 and an inflation weight of 1.85 while the dotted line shows a weight on the output gap of 0.9 and on inflation of 1.10.

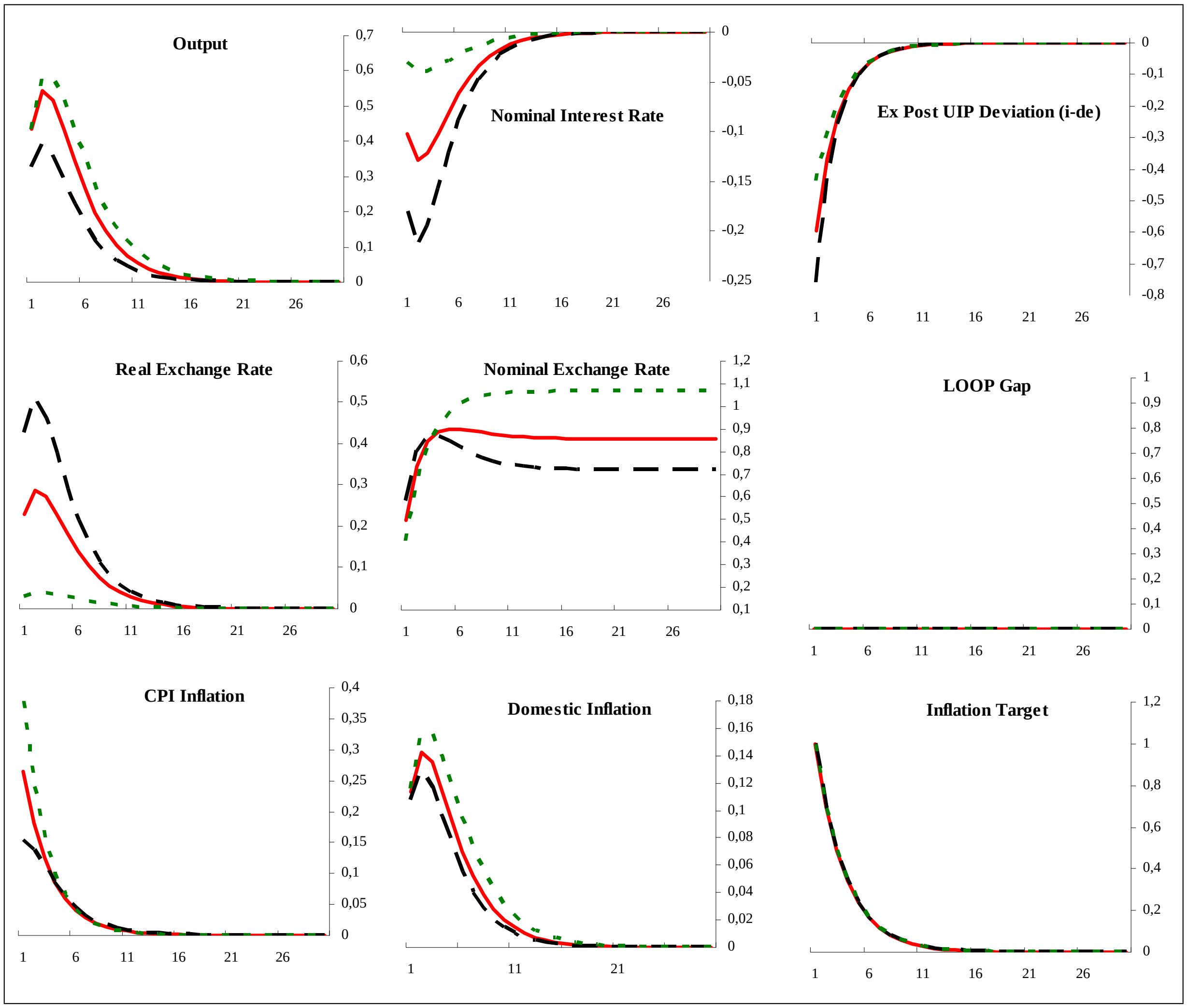


Figure 5: Different degrees of openness under full exchange rate pass through (Partial information). The solid line corresponds to 40%, the dashed line to 10% and the dotted line to 90% of foreign goods in the overall consumption basket.

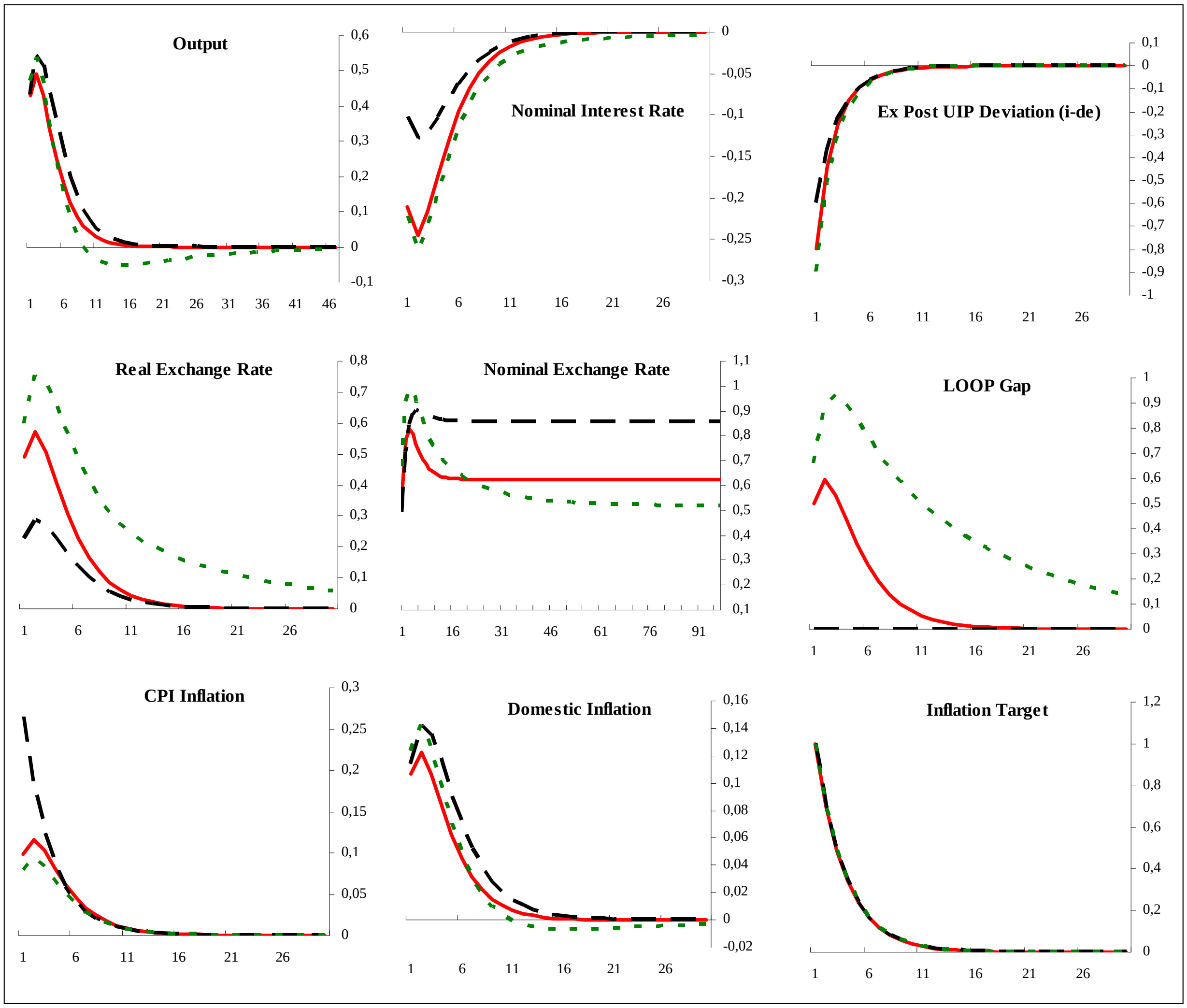


Figure 6: Different degrees of exchange rate pass through (Partial information). The solid line corresponds to intermediate -, the dashed line to full - and the dotted line to low exchange rate pass through.

| Table 1 | | |
|---|---|---|
| Parameter | Description | Value |
| $\beta$ | Discount Factor | 0.99 |
| $\sigma$ | Parameter of relative Risk Aversion | 2 |
| $\eta$ | Intratemporal Elasticity of Substitution | 1.5 |
| $\varphi$ | Inverse of Frisch Elasticity of Labor Supply | 0.47 |
| $\alpha$ | Share of Foreign Goods in CPI Basket | 0.4 |
| $\theta_H$ | Domestic Output Price Calvo Stickiness | 0.75 |
| $\theta_F$ | Domestic Import Price Calvo Stickiness | 0.75 |
| $\bar{\pi}$ | Inflation Target | 0 |
| $\rho_i$ | Interest Smoothing Coefficient | 0.65 |
| $\rho_\pi$ | Inflation Targeting Coefficient | 1.85 |
| $\rho_y$ | Output Gap Targeting Coefficient | 0.15 |
| $K$ | Kalman scalar (Full Information) | 1 |
| $K$ | Kalman scalar (Partial Information) | 0.06 |

Notes: The structural parameters of the model.

| Table 2 | | | | |
|---|---|---|---|---|
| | Timing (peak) of Overshooting (in Period) | | Magnitude of Overshooting | |
| | Real exchange rate | Nominal exchange rate | Real Exchange Rate | Nominal exchange rate |
| Full Info. | initial period | initial period | 0.999 | 1.097 |
| Partial Info. | 2 | 3 | 0.571 | 1.319 |

Notes: The case for full - and partial information, respectively. For the nominal exchange rate the magnitude of overshooting is expressed by the peak relative to its long run level while for the real exchange rate it refers to the absolute peak.

| Table 3 | | | | |
|---|---|---|---|---|
| | Timing (peak) of Overshooting (in Period) | | Magnitude of Overshooting | |
| $\rho_p$ | Real exchange rate | Nominal exchange rate | Real exchange rate | Nominal exchange rate |
| 0.7 | 2 | 3 | 0.571 | 1.319 |
| 0.5 | initial period | 2 | 0.487 | 1.805 |
| 0.3 | initial period | initial period | 0.486 | 2.232 |
| 0.2 | initial period | initial period | 0.486 | 2.550 |
| 0.1 | initial period | initial period | 0.485 | 2.868 |

Notes: Different degrees of persistence of the inflation target shock, $\pi_{p,t}$. For the nominal exchange rate the magnitude of overshooting is expressed by the peak relative to its long run level while for the real exchange rate it refers to the absolute peak.

| Table 4 | | | | |
|---|---|---|---|---|
| | Timing (peak) of Overshooting (in Period) | | Magnitude of Overshooting | |
| $\rho_i$ | Real exchange rate | Nominal exchange rate | Real exchange rate | Nominal exchange rate |
| 0.9 | 3 | 7 | 0.498 | 1.029 |
| 0.8 | 2 | 5 | 0.540 | 1.120 |
| 0.65 | 2 | 3 | 0.571 | 1.319 |
| 0.5 | 2 | 2 | 0.574 | 1.552 |
| 0.2 | initial period | initial period | 0.609 | 2.116 |

Notes: Different weights of the interest rate smoothing parameter $\rho_i$ in the monetary policy rule. For the nominal exchange rate the magnitude of overshooting is expressed by the peak relative to its long run level while for the real exchange rate it refers to the absolute peak.

| Table 5 | | | | |
|---|---|---|---|---|
| | Timing (peak) of Overshooting (in Period) | | Magnitude of Overshooting | |
| $\rho_y/\rho_\pi$ | Real exchange rate | Nominal exchange rate | Real exchange rate | Nominal exchange rate |
| 0.05/1.95 | 2 | 3 | 0.684 | 1.308 |
| 0.15/1.85 | 2 | 3 | 0.571 | 1.319 |
| 0.90/1.10 | 2 | 3 | 0.045 | 1.370 |
| 0.95/1.05 | 2 | 3 | 0.022 | 1.373 |

Notes: Different weights on the output gap and inflation in the monetary policy rule ($\rho_y/\rho_\pi$). For the nominal exchange rate the magnitude of overshooting is expressed by the peak relative to its long run level while for the real exchange rate it refers to the absolute peak.

| Table 6 | | | | |
|---|---|---|---|---|
| | Timing (peak) of Overshooting (in Period) | | Magnitude of Overshooting | |
| $\alpha$ | Real exchange rate | Nominal exchange rate | Real exchange rate | Nominal exchange rate |
| 0.9 | 3 | 17 | 0.039 | 1.000 |
| 0.6 | 2 | 8 | 0.174 | 1.013 |
| 0.4 | 2 | 5 | 0.287 | 1.052 |
| 0.2 | 2 | 4 | 0.426 | 1.141 |
| 0.1 | 2 | 3 | 0.509 | 1.207 |

Notes: Different degrees of trade openness (the share of foreign goods in the domestic consumption basket), $\alpha$. For the nominal exchange rate the magnitude of overshooting is expressed by the peak relative to its long run level while for the real exchange rate it refers to the absolute peak.

| Table 7 | | | | |
|---|---|---|---|---|
| | Timing (peak) of Overshooting (in Period) | | Magnitude of Overshooting | |
| $\theta_F$ | Real exchange rate | Nominal exchange rate | Real exchange rate | Nominal exchange rate |
| 0.00 | 2 | 5 | 0.287 | 1.052 |
| 0.20 | 2 | 5 | 0.325 | 1.055 |
| 0.50 | 2 | 4 | 0.430 | 1.086 |
| 0.75 | 2 | 3 | 0.571 | 1.319 |
| 0.95 | 2 | 3 | 0.753 | 1.905 |

Notes: Different degrees of exchange rate pass through. Full pass through refers to the case when $\theta_F \rightarrow 0$, intermediate pass through corresponds to $\theta_F = 0.75$ while for no pass through $\theta_F = 0.95$. For the nominal exchange rate the magnitude of overshooting is expressed by the peak relative to its long run level while for the real exchange rate it refers to the absolute peak.