Information Management in Rank-Order Tournaments*

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Abstract

We study a rank-order tournament in which employees acquire and use private information for an investment decision. In this environment, competition for promotion can turn employees into “yes men” who make investment decisions that excessively agree with their supervisor’s preconceived notions. Employees become “yes men” when their supervisor’s prior opinion is strong and the parties receive little subsequent information. In response to this inefficiency, the firm may intensify the tournament’s incentives (e.g., increase the wage raise from promotion), increase the correlation of employees’ information (e.g., use tournaments for employees handling similar tasks), or reduce the importance of any individual supervisor’s prior opinion (e.g., evaluate employees using a committee).

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1 Introduction

Firms often use a rank-order promotion tournament to motivate their employees. In this paper, we demonstrate that such an arrangement may cause employees to mismanage information on behalf of their employer. In particular, we show that the competitive pressure from her peers may prevent an employee from using valuable private information in full. In short, a rank-order tournament creates “yes men,” employees who make decisions that agree with the preconceived notions of their superiors. Employees become “yes men” when their superior’s prior opinion is strong and the parties receive little additional information.

A vast literature on rank-order tournaments already exists (surveys include Gibbons 1997, Gibbons and Waldman 1999, McLaughlin 1988, and Prendergast 1999). Early contributions by Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) demonstrated that a tournament can implement the efficient outcome under risk-neutrality and in the absence of wealth constraints. When employees are risk averse, tournaments — like explicit compensation contracts — create a tension between incentive provision and risk-sharing that prevents efficiency.

A question that has drawn a lot of attention is why tournaments are used at all. The most striking feature of a tournament is that it uses only relative performance as the basis for the distribution of rewards. This does indeed have certain advantages: It simplifies performance evaluation and reduces risk-sharing costs by filtering out common risk, a fact that makes competition among employees with highly correlated uncertainty beneficial (Holmström 1979). However, since nothing in principle prevents an explicit contract from replicating them, these advantages cannot by themselves account for the use of tournaments. But there are other aspects of tournaments that may. First, a tournament can kill two birds with one stone in that it not only provides incentives, but also allocates workers to jobs (Baker, Jensen, and

1 But relative performance evaluation also has the serious disadvantage of discouraging cooperation among employees: winning can be accomplished not only by improving one’s own performance, but by sabotaging that of one’s competitors as well (Dye 1984, Lazear 1989, and Drago and Garvey 1998).
Murphy 1988, Fairburn and Malcomson 2001, and Prendergast 1993a). Second, because it circumscribes supervisor discretion, a tournament may discourage inefficient influence activities (Fairburn and Malcomson 2001). Finally, a tournament may be a more effective commitment device for the firm than an explicit contract. The reason is that a tournament determines only the distribution of a set reward across different employees, thus removing the benefit to the firm of denying any one employee the reward she has been promised (Bhattacharya 1983, Carmichael 1983, and Malcomson 1984).

We make three contributions to the tournament literature. First, we identify a previously unrecognized short-coming of rank-order tournaments — they may create “yes men” — and characterize the conditions under which such distortions appear.

Second, we analyze how the “yes man” problem affects the design of the tournament. We find that firms may respond by ratcheting up the incentive intensity in the tournament. The reason is that when employees collect more information they are more likely to use it efficiently. We also demonstrate that firms can alleviate the “yes man” problem by comparing employees who perform tasks with a lot of common uncertainty. Designing tournaments in this way improves employees’ management of information because the correlated information effectively makes their superiors better informed. Hence, this provides a new motivation for why competition among employees with common uncertainty is desirable.

Finally, the importance of the timing of the employer’s information that we highlight suggests other ways to mitigate the inefficiency as well. Because information that firms get after the employees make their decision allows for a more accurate ultimate evaluation, it weakens the “yes man” incentive. By contrast, information that arrives before those decisions are made may actually aggravate the problem because it gives employees a stronger prior opinion to cater to. This points to the possibility that the firm may choose to let a committee or an outsider with unknown

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2This may be a mixed blessing because these two objectives can sometimes come into conflict with one another.
biases make promotion decisions. By diminishing the extent to which the known preconceived opinion of any one individual comes to bear on such decisions, such an organizational arrangement could discourage “yes man” behavior.

Our work also speaks to the relationship between information management and incentive provision more generally. Our analysis naturally complements Prendergast (1993b) and Gentzkow and Shapiro (2005), who show, respectively, that explicit compensation contracts and reputation concerns can create “yes men.” In addition, we build on our previous research that demonstrates that competitive pressure can make producers pay too much attention to the opinions of their consumers and not enough attention to their own information (Cummins and Nyman 2005). In this paper, we extend our previous analysis in several important directions and apply our insights to the tournament setting.

The rest of the paper is organized as follows. In the next section, we introduce the model, which is a variation on the classic setup in Lazear and Rosen (1981). We enrich the basic model by adding a more intricate production decision — based on Brandenburger and Polak (1996) and Cummins and Nyman (2005) — in the form of an investment problem that employees can use private information to solve. After setting up the model, we derive the standard benchmark by assuming complete information. We then turn to the main analysis in which we assume that employees have private information about the investment opportunities. Finally, we investigate how the correlation of employees’ information affects their incentive to use it efficiently. A short discussion of the analysis concludes.

2 Model Setup

There are two ex ante identical employees, indexed by $i$ and $j$, and one supervisor. Each employee is charged with the task of picking one of two mutually exclusive uncertain alternatives which we will think of as investment projects. To guide this choice, the employees exert effort to collect information. Their incentive to do so is provided by a rank-order tournament. There are two time periods, with the
employees acting in the first period and the supervisor acting in the second period. In the first period, the employees first decide on how much costly effort to put into learning about the profitability of the two projects. The effort then generates information in the form of a signal, and based on this signal each employee chooses one of the projects. The employees make each decision simultaneously. In the second period, the supervisor receives signals of the profitability of the choices that the employees have made, and subsequently allocates two promotion tournament prizes to the employees. The performance evaluation measure that dictates this promotion decision contains the supervisor’s assessment of the expected profit of the two projects, given her own signals and whatever else she knows about the employees’ information.

2.1 The Investment Decision

The employees face the identical problem in terms of project choice. The two projects are referred to as the “green” and the “red.” Employee i’s project choice is denoted by $z_i \in \{g, r\}$. There are two possible states of the world, the “Green” and the “Red,” that determine the profit from the projects. A project generates a higher profit (net of investment) if it matches state. For simplicity, both projects have the same high and low payoff realizations, normalized to unity and zero, respectively. The profit of employee i’s project choice, $\pi_i$, is thus equal to

$$
\pi_i (z_i, Z) = \begin{cases} 
1 & \text{if } (g, G) \text{ or } (r, R) \\
0 & \text{if } (g, R) \text{ or } (r, G). 
\end{cases}
$$

The prior belief about the uncertainty, which is shared by everyone and is common knowledge, weakly favors the Green state: $\Pr (Z = G) = \mu \geq \frac{1}{2}$. When making their project choice, the employees can use mixed strategies, and we denote the probability that employee i chooses the green project by $\zeta_i$.

2.2 The Effort Decision

The employees can acquire additional information to guide their choice of project. This information comes in the form of a signal that gives an indication about the
true state of the world. Employee $i$’s signal is denoted by $e_i \in \{\gamma, \rho\}$, with $\gamma$ and $\rho$ indicating, with equal accuracy, that the state is Green and Red, respectively. The signals are conditionally independent across employees.

The employees’ information is not free, but requires effort. For simplicity, employee $i$’s effort is equal to the accuracy of her signal and is denoted by $\varepsilon_i = \Pr (e_i = \gamma | G) = \Pr (e_i = \rho | R)$. Since a completely uninformative signal is one for which $\varepsilon_i = \frac{1}{2}$, the set of possible effort levels is $\varepsilon_i \in \left[\frac{1}{2}, 1\right]$. To create a conflict of interest between employee and employer, effort is costly. Both employees have the same cost of effort, denoted by $C(\varepsilon_i)$, which satisfies the Inada conditions. To create an agency problem, the employees’ effort choice is private information.

2.3 The Promotion Decision

The employees’ incentive to invest in effort is provided by a rank-order tournament for promotion. In the second period, after having observed their choice of project, the supervisor, who is risk-neutral and acts in the interest of the firm’s shareholders, promotes one of the employees. Her strategy is denoted by $\beta_i (\cdot) = \Pr (\text{Employee } i \text{ gets promoted} | \cdot)$. The supervisor evaluates the employees’ project choices by her expectation of their profitability, which represents a summary measure of how the employees have managed this task on the shareholders’ behalf. These assessments, which are denoted by $\hat{\pi}_i$ and $\hat{\pi}_j$, respectively, are aided by two conditionally independent signals that indicate the profitability of each of the projects chosen. The signal about employee $i$’s project can be either high or low and is denoted by $s_i \in \{h, l\}$. A high (low) signal points to the profit from the project being high (low). Both signal realization are equally informative and their accuracy is parametrized by $\sigma = \Pr (s_i = h | \pi_i = 1) = \Pr (s_i = l | \pi_i = 0) \in \left(\frac{1}{2}, 1\right)$. The supervisor’s signal can be thought of as an early indication of the success of a project, perhaps in the form of an early cash-flow, which it would be natural to assume is positively — but imperfectly — correlated with the total cash flow. The supervisor’s signal makes, in effect, the long-run profitability of their choices matter to
the employees. This allows for a partial alignment of the interests of employer and employee when it comes to project choice.

The evaluation measure upon which the promotion is based is also subject to additional uncertainty unrelated to the investment decision, due, for example, to uncertainty about the supervisor’s state of mind or about the evaluation of other decisions, past or future, for which the employees are held responsible at the time of promotion. Hence, the evaluation measure for employee $i$ is $V_i \equiv \hat{\pi}_i + \eta_i$ where $\eta_i$ is a random variable with an expected value equal to zero. $\eta_i$ and $\eta_j$ are mutually iid as well as independent of the investment uncertainty, $Z$. The promotion decision of the supervisor is to promote employee $i$ (employee $j$) if $V_i > V_j$ ($V_i < V_j$). Apart from the realization of the evaluation measures, the supervisor’s decision is unbiased, so when $V_i = V_j$, the promotion is settled with a coin toss, i.e., $\beta_i = \beta_j = \frac{1}{2}$.\footnote{The one exception to this decision rule of an even split is when the construction of an equilibrium requires a different allocation of the promotion prize.}

We refer to employee $j$’s advantage in terms sheer luck as $\xi \equiv \eta_j - \eta_i$. To keep things simple, we assume that on the interval $[-\frac{1}{2}, \frac{1}{2}]$, $\xi$ is uniformly distributed with $\frac{d\Pr(\xi<x)}{dx} = \phi$. In addition to ensuring that a symmetric Nash equilibrium in effort levels exists, this assumption makes the analysis tractable by allowing the expected value of her own evaluation measure, $V_i$, to be the focus of each employee’s decision-making. We believe, however, that our qualitative conclusions would stand if this assumption were dropped so that the employees instead were forced to make decisions using the first-order stochastic dominance of the distribution of outcomes of $V_i$ as their evaluation criterion.

For both employees, the value of receiving and being denied promotion is equal to $W_1$ and $W_2$, respectively. Promotion is desirable, so $W_1 > W_2$. The outcome of the promotion decision is the only compensation that the employees receive. Furthermore, the employees are also risk-neutral, so their objective is to maximize the expected value of their net utility consisting of the value of the outcome of the promotion decision net of the cost of effort: $E[U_i(\varepsilon_i, \varepsilon_j, z_i, z_j)] =$
\[ W_2 + E[P_i (\hat{\pi}_i (\varepsilon_i, \varepsilon_j, z_i, z_j), \hat{\pi}_j (\varepsilon_i, \varepsilon_j, z_i, z_j))] [W_1 - W_2] - C(\varepsilon_i) \] where \( P_i (\cdot) \) is the probability that employee \( i \) gets promoted.

Before the employment relationship starts, the firm and the employees can credibly commit to an agreement about the rules for the promotion tournament. In this negotiation, the firm has all the bargaining power, except that each employee has access to employment elsewhere that yields a net utility that is normalized to zero. Therefore, before the employees make any decision, the firm can unilaterally choose \( W_1 \) and \( W_2 \) to maximize its expected profit, subject to the participation constraints that it must afford each employee an expected net utility that is non-negative.\(^4\)

### 2.4 Efficiency

The normalization of the profit realizations to zero and one, respectively, makes the expected profit of a project simply equal to the probability that it matches the state. If it is less accurate than the public information, \( \mu \), then employee information is of no value: with \( \varepsilon \leq \mu \), the employee should ignore her signal since expected profit is higher for the green project regardless of what the signal is. But if the signal is more accurate than the public information, then employee information is valuable: with \( \varepsilon > \mu \), the employee should follow her signal since the expected payoff after seeing a \( \rho \)-signal is strictly higher for the red project than for the green one. The ex ante expected profit if the employee follows her signal is equal to \( \varepsilon \). Since the public information is assumed to be available for free, the benefit of efficiently used employee information is equal to \( \varepsilon - \mu \), making its marginal social benefit equal to unity.

A key feature of the model is that the amount of information that the employee has access to is not given. Rather, it is an endogenous choice. Denote the first-best choice of effort by \( \varepsilon^{**} \). To give the employees a role to play, we assume that their efficiently chosen information is superior to the public information, i.e., \( \varepsilon^{**} > \mu \).

\(^4\)To keep the incentive problem simple and allow for a first-best efficient benchmark we assume that the other major obstacle in incentive provision apart from risk-exposure, namely limited-liability or wealth constraints, are absent. Hence, we assume that there is no lower bound on the losing prize, \( W_2 \).
Our assumptions on $C(\varepsilon)$ ensure that there exists an interior solution equating the marginal benefit and marginal cost of effort. Therefore, the condition that $\varepsilon^{**} > \mu$ amounts to an assumption that the total and marginal cost of effort both increase sufficiently slowly.

Finally, the value of the employees’ information also depends on the information the supervisor receives in period two and what she can do with it. If the supervisor’s information were sufficiently accurate, she could overrule the employees’ choices, in which case it would be difficult to motivate why the firms would need the employees in the first place. To preclude this degenerate outcome, we assume that the employees’ project choices are irreversible. Hence, the supervisor can use her information only to evaluate — but not to change — the employees’ decisions. An alternative approach would be to assume that the supervisor’s information is sufficiently inaccurate. However, making investment irreversible has the advantage of imposing fewer restrictions on the information structure, thus allowing for a more thorough analysis.

In summary, an efficient outcome is one in which each employee supplies an effort level of $\varepsilon^{**}$ and uses the information that this creates in full by following the signal received when choosing an investment project.

3 Analysis

In studying the outcome of the model, we will focus on the most efficient sequential equilibrium. We choose this equilibrium-selection rule because more efficient equilibria constitute natural focal points.

3.1 Equilibrium when Signal Realizations are Public Information

We start out by analyzing the model when the employees’ signal realizations are known to the supervisor, i.e., when $\pi_i = \mathbb{E}[\pi_i(\varepsilon_i, z_i) \mid e_i, e_j, s_i, s_j]$. This will serve as a benchmark for the more likely situation in which the supervisor is ignorant about the outcome of the employees’ information collection. It will also confirm that our
model replicates a key result in the literature, namely that the efficient outcome can be implemented when information is symmetric, agents are risk-neutral, and wealth constraints are absent.

Recall that employee $i$ maximizes her expected utility, i.e., $E[U_i(\varepsilon_i, \varepsilon_j, z_i, z_j)] = W_2 + E[P_i(\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j), \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j))][W_1 - W_2] - C(\varepsilon_i)$. The employee with the more favorable evaluation gets promoted, so the probability that this happens to employee $i$ is equal to

$$P_i(\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j), \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j)) = \Pr(V_i(\varepsilon_i, \varepsilon_j, z_i, z_j) > V_j(\varepsilon_i, \varepsilon_j, z_i, z_j)) =$$

$$= \Pr(\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j) - \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j) > \xi|\varepsilon_i, \varepsilon_j, z_i, z_j) =$$

$$= \Pr(\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j) - \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j) > \xi)$$

The last equality holds because the random shocks are independent of the investment uncertainty.

The deviation from the standard tournament framework is that even with perfect knowledge of all the employees' decisions, i.e., $(\varepsilon_i, \varepsilon_j, z_i, z_j)$, as well as of the realization of the promotion-decision uncertainty, $\xi$, the probability of winning the promotion is still subject to uncertainty because it is determined by the realization of the supervisor's signals. However, because of the uniform distribution of $\xi$, $P_i$ is linear in $\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j) - \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j)$ with slope $\phi$. Therefore, from the perspective of the employees, who make their decisions before the supervisor’s signals arrive, the expectation of the probability of winning the tournament for a given set of employee decisions is equal to

$$E[P_i(\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j), \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j))] =$$

$$= E[\Pr(\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j) - \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j) > \xi)] =$$

$$= \Pr(E[\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j) - \hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j)] > \xi) =$$

$$= \Pr(E[\hat{\pi}_i(\varepsilon_i, \varepsilon_j, z_i, z_j)] - E[\hat{\pi}_j(\varepsilon_i, \varepsilon_j, z_i, z_j)] > \xi)$$
Consider first the employees’ last decision of choosing the investment project. Notice that the probability of an employee winning the promotion is increasing in the difference in expected profit from her project and that of her competitor, given the supervisor’s information. The supervisor’s expectations are unbiased and the employee’s choice does not reveal or conceal any of her own information, so the employee’s expectation of the supervisor’s assessment of the expected profit is simply equal to the employee’s own expectation of the profit. Thus, the employee chooses the project that her own signal indicates is the most profitable one. In other words, when the supervisor knows what signal the employees’ have received, the employees fully use their information and choose the efficient project.

Next, consider employee $i$’s effort choice. Her marginal benefit of effort is equal to
\[
\frac{\partial}{\partial \varepsilon_i} \{ \Pr ( E[\pi_i (\varepsilon_i,\varepsilon_j, z_i, z_j)] - E[\pi_j (\varepsilon_i,\varepsilon_j, z_i, z_j)] > \xi) [W_1 - W_2] \} = \\
= \frac{\partial \Pr (\xi < x)}{\partial x} \left\{ \frac{\partial E[\tilde{\pi}_i]}{\partial \varepsilon_i} - \frac{\partial E[\tilde{\pi}_j]}{\partial \varepsilon_i} \right\} [W_1 - W_2] = \\
= \phi \left\{ \frac{\partial E[\tilde{\pi}_i]}{\partial \varepsilon_i} - \frac{\partial E[\tilde{\pi}_j]}{\partial \varepsilon_i} \right\} [W_1 - W_2]
\]

Again, at the time when she chooses effort, employee $i$’s expectation of the supervisor’s expectation of project profits is simply equal to her own expectation. When information is used efficiently, this expectation of the supervisor’s expectations is equal to the accuracy of the employees’ signals, respectively:
\[
E[\tilde{\pi}_i] = E[\pi_i (\varepsilon_i, z_i)] = \varepsilon_i \\
E[\tilde{\pi}_j] = E[\pi_j (\varepsilon_j, z_j)] = \varepsilon_j
\]

This makes employee $i$’s marginal benefit of effort equal to $\phi [W_1 - W_2]$. Her rational choice of effort is the one that equates the marginal benefit and marginal cost of effort: $\phi [W_1 - W_2] = C'(\varepsilon_i^*).$ Notice that employee $i$’s optimal choice of effort is independent of that of employee $j$. This absence of strategic interaction is due to the assumption that the difference in luck is uniformly distributed. Furthermore, since
both employees face the same benefit and cost of effort, the effort stage has a unique Nash equilibrium that is symmetric and given by the above optimality condition. Because the employees end up choosing the same effort level, $\varepsilon_i^* = \varepsilon_j^* = \varepsilon^*$, each of them can expect to win the promotion with probability one half. The equilibrium is illustrated in Figure 1 with $R_i$ and $R_j$ denoting the reaction functions of employee $i$ and $j$, respectively.

Finally, the firm’s profit-maximizing choice of tournament prizes, $W_1^*$ and $W_2^*$, is now immediate. First, the profit-maximizing tournament should implement efficient effort levels, so $W_1^* - W_2^* = \frac{C(\varepsilon^{**})}{\phi} = \frac{1}{\phi}$. Second, profit-maximization implies that the employees’ participation constraints must bind, so $\frac{W_1^* + W_2^*}{2} = C(\varepsilon^{**})$. These standard conditions in the literature on tournaments for efficient effort extraction imply that $W_1^* = C(\varepsilon^{**}) + \left(\frac{1}{2\phi}\right)$ and $W_2^* = C(\varepsilon^{**}) - \left(\frac{1}{2\phi}\right)$. The incentive intensity, $W_1^* - W_2^*$, increases with marginal revenue product of efficient effort, which we
have normalized to unity. Furthermore, it decreases with the extent to which an employee’s marginal effort bolsters her chances of winning the promotion, $\phi$. The expected compensation, $\frac{W_1^* + W_2^*}{2}$, increases with the disutility of effort and the value of the employees’ outside option (which we have normalized to zero).

This analysis of the equilibrium when the supervisor knows what the employees learn is summarized in Proposition 1

Proposition 1: When the supervisor knows the realizations of the employees’ signals, the equilibrium is unique with the employees choosing the same effort level, using their private information fully, and each getting promoted with probability one half. The firm chooses the tournament prizes that induce the employees to choose the efficient level of effort while affording them an expected net utility that is equal to zero, given by the conditions $W_1^* - W_2^* = \frac{1}{\phi}$ and $\frac{W_1^* + W_2^*}{2} = C(\varepsilon^{**})$, respectively.

3.2 Equilibrium when Signal Realizations are Private Information

The more reasonable information structure is that the supervisor does not automatically know what the employees have learned about the profitability of the projects. This means that the supervisor can discern what the employees know only through their choice of project. To analyze this situation, we first look for conditions under which efficient project choice can occur in equilibrium. They turn out to be that the supervisor’s prior opinion is sufficiently weak compared to the added information that the supervisor and the employees receive about the profitability of the projects.

Proposition 2: Efficient project choice can occur in equilibrium only if the supervisor’s and employees’ signals are both accurate enough compared to the supervisor’s prior opinion, as captured by the following conditions:

1. $\left( \frac{\mu}{1 - \mu} \right) < \left( \frac{\sigma}{1 - \sigma} \right)^2 \quad (1)$
2. 
\[ \Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1 - \sigma)^2 > \frac{1}{2} \] (2)

In particular, if the supervisor has no prior opinion, i.e., if \( \mu = \frac{1}{2} \), then efficient project choice is always an equilibrium outcome.

**Proof:** See Appendix.

To convey the intuition of Proposition 2, it is useful to start by considering the case when the supervisor has no a priori preference for one of the projects, i.e., when \( \mu = \frac{1}{2} \). There are three configurations of project choices that may occur: both employees choose red, both employees choose green, and the two choose different projects. Only the last of these outcomes puts the employees in competition with one another since the supervisor’s assessment of their performance must be identical in the first two outcomes. The competitive outcome when different projects are chosen can, in turn, evolve into four distinct events depending on the supervisor’s signal realizations. The first two are that the signals for both projects are the same (i.e., green is high and red is high or green is low and red is low), in which case they indicate different states of the world and therefore cancel. This leaves the supervisor with only her prior belief to rely on. The third possibility is that the green and red projects receive a high and a low signal, respectively, which thus both indicate that the state is Green. The final outcome is the opposite with both signals indicating that the state is Red.

If the prior is balanced, then neither project is per se more competitive than the other when both are chosen.\(^5\) If the supervisor’s signals cancel, then she is indifferent between the two projects, which are therefore treated equally. On the other hand, if the supervisor’s signals are informative, then she can be expected to give the promotion to the project that is favored by the state indicated by the signals. But again, with a balanced prior no project is intrinsically better than the

\(^5\)Keep in mind that when the other employee ends up choosing the same project, both red and green lead to promotion half the time in expectation.
other at winning promotion in this way. Therefore, since employee and supervisor signals are correlated through the state, the employees increase the probability of getting promoted by following their own signal.

But it is quite unlikely that the supervisor’s prior experience leaves her completely indifferent between the two projects. Instead, one would expect that the prior probability indicates that one project is more profitable than the other, i.e., that $\mu > \frac{1}{2}$. This slight and generic perturbation has a dramatic effect on the outcome. The reason is that the supervisor now has a tendency to view the green project as more promising than the red one, and this, in turn, gives the employees an incentive to try to impress the supervisor by pretending that their information agrees. It is thus not surprising that it is when the supervisor’s prior conviction is strong compared to other sources of information that the employees become “yes men”— those who choose the supervisor’s pet project even when their own superior information indicates that they should not.

The economics of the case of an unbalanced prior is illustrated in Figure 2. Out of the four possible outcomes when the employees effectively compete with one another by choosing different projects, three are potentially affected by the supervisor’s biased prior. In the two outcomes in which the supervisor’s signals cancel and therefore are uninformative, the supervisor now favors the green project over the red. In the outcome in which the supervisor’s signals indicate that the state is Red, she still favors the green project if her prior conviction is so strong that even two contradictory signals prove insufficient to overcome it. If this is the case, then the red project always loses against the green. As a result, an employee with a $\rho$-signal would never find it in her best interest to follow her own signal: the red project receives only a tie against the red one, whereas the green one also ties against the green, but wins outright against the red. Therefore, a necessary condition for the efficient outcome to be an equilibrium is that contradictory information can dissuade the supervisor of her initial conviction, captured by condition (1) in Proposition 2.
Figure 2: Efficient and inefficient project choice.
In Figure 2, this is the area of the \((\sigma, \varepsilon)\)-space (for a given \(\mu\)) that lies to the right of the vertical line denoted by \(H_1 = 0\).\(^6\)

However, it is not sufficient that the supervisor receives enough new information to overcome her initial bias towards the green project as captured by condition (1). The probability of the supervisor giving the promotion to the red project because she receives two signals indicating that the state is Red must also be sufficiently high for the \(\rho\)-type employee to refrain from “yes man” behavior. This happens if the correlation between the employees’ and the supervisor’s signals is strong enough, i.e., if either \(\varepsilon\) or \(\sigma\) is large enough. In Figure 2, this is the area above the curve denoted by \(H_2 = 0\).\(^7\)

Hence, in Figure 2, the “yes man” incentive prevents the efficient use of information that contradicts the supervisor’s opinion everywhere except in region I. In regions III and IV, the supervisor’s signal is so inaccurate that not even the most convincing contradictory information can shake her out of her prior conviction that the green project is the best. In region II, the supervisor receives enough information to change her mind, but the low accuracy in the employees’ information makes this too unlikely an event for the \(\rho\)-type employee to follow her own information. As would be expected, an increase in the supervisor’s prior conviction, \(\mu\), shifts \(H_1 = 0\) to the right and \(H_2 = 0\) up, making efficiency more difficult to achieve.

Having determined what it takes for efficient project choice to occur in equilibrium, we now turn to the question of what is an equilibrium when efficiency is not attainable. The equilibrium outside Region I in Figure 2 is an inefficient partially-separating mixed-strategy equilibrium in which the \(\rho\)-type employee invests inefficiently in the green project with some probability. This confuses the supervisor so that she is indifferent between giving the promotion to the green and

\(^6\)\(H_1\) denotes the net preference for the red project in the supervisor’s posterior when both projects are chosen efficiently and both of her own signals indicate that the state is Red.

\(^7\)\(H_2\) denotes the \(\rho\)-type employee’s net benefit of choosing the red project rather than the green one if the other employee invests efficiently and the red project receives the promotion if both the supervisor’s signals indicate that the state is Red.
the red project. She can therefore give the promotion to the red project in an outcome in which it would have been given to the green project in the face of efficient investment. The temptation for the ρ-type employee to deviate to the green project can thus be tempered to the point where she, in turn, is indifferent between the two projects and therefore her mixed strategy is rational.

**Proposition 3:** If efficient use of employee information cannot occur in equilibrium, then the most efficient equilibrium in the subgame starting with the employees’ project choices looks as follows:

1. A γ-type employee always chooses the efficient project: \( \zeta^\gamma = 1 \).

2. In Regions II and III, a ρ-type employee invests inefficiently: \( \zeta^\rho = \frac{\varepsilon(1-\varepsilon)(2\mu-1)}{\varepsilon^2(1-\mu)-(1-\varepsilon)^2\mu} \in (0,1) \). The supervisor splits the promotion between the green and the red project when both are chosen and her own signals cancel: \( \beta^* (z_i = r, z_j = g, s_i = s_j) = \frac{1}{2} \left\{ 1 - \left[ \frac{\sigma^2}{(1-\mu)} \right] \left[ \frac{\varepsilon-\mu}{\varepsilon(1-\mu)-(1-\varepsilon)^2\mu} \right] \right\} \in (0, \frac{1}{2}) \).

3. In Region IV, a ρ-type employee invests inefficiently: \( \zeta^\rho = \frac{\varepsilon(1-\varepsilon)(1-\sigma)^2\mu-\sigma^2(1-\mu)}{\varepsilon^2(1-\mu)(1-\varepsilon)^2\mu(1-\sigma)^2} \in (0,1) \). The supervisor splits the promotion between the green and the red project when both are chosen and her own signals both indicate that the state is Red: \( \beta^* (z_i = r, z_j = g, s_i = s_j) = \frac{1}{2} \left\{ \frac{(1-\mu)\varepsilon+\mu(1-\varepsilon)}{(1-\mu)\varepsilon\sigma^2+\mu(1-\varepsilon)(1-\sigma)^2} \right\} \in \left( \frac{1}{2}, 1 \right) \).

4. The expected payoff from investment is equal to \( E[\pi_i (z_i) | \varepsilon] = \varepsilon - \zeta^\rho (\varepsilon - \mu) \).

**Proof:** See Appendix.

It is the sign of \( H_2 \) that determines what the inefficient equilibrium looks like. If \( H_2 < 0 \), then giving the promotion to the red project when the supervisor’s signals support it is not enough to make the ρ-type employee invest efficiently. Therefore, to make the ρ-type employee indifferent, the promotion must also sometimes be given to the red project when the supervisor’s signals are uninformative because they cancel. Hence, in Regions II and III, it is in this outcome that the supervisor
plays a mixed strategy and, in turn, is made completely confused by the \( \rho \)-type employee’s mixed strategy. In the outcomes where her signals are informative, the supervisor follows them.

By contrast, if \( H_2 > 0 \), then the \( \rho \)-type employee does invest efficiently if she is promoted when the supervisor’s signals support her choice. This means that to make the \( \rho \)-type employee indifferent, the supervisor should sometimes give the promotion to the green project even when her signals indicate that the state is Red. Hence, in Region IV, the supervisor plays a mixed strategy when her signals indicate that the state is Red, while giving the promotion outright to the green project in all other outcomes. This, in turn, is made rational by the \( \rho \)-type employee’s mixed strategy leaving the supervisor with a balanced posterior in the outcome where she mixes.

The extent to which the \( \rho \)-type employee must invest inefficiently, \( \zeta^{\rho} \), is larger in Regions II and III than in Region IV. The reason for this is simply that when \( H_2 < 0 \), the inefficient investment must reveal enough information to overcome the prior, whereas when \( H_2 > 0 \), it must reveal enough information to overcome the prior already tempered by the supervisor’s two signals indicating that the state is Red.

The inefficiency in investment increases with the prior bias towards the green project, \( \mu \). This is because it now takes more contradictory information to shake the supervisor out of her preconceived notions. On the other hand, the inefficiency decreases with the accuracy of the employees’ information, \( \varepsilon \). This is because any given investment strategy now reveals more contradictory information. Finally, the accuracy of the supervisor’s information leaves the extent of inefficiency in investment unaffected in Regions II and III, but decreases it in Region IV. This is because in Region II and III, the inefficient investment is designed to confuse the supervisor when she has no information of her own, whereas in Region IV the supervisor’s own information can substitute for the contradictory information provided through the employees’ inefficient investment strategy.
The expected payoff is strictly increasing in the accuracy of the employees’ information, \( \varepsilon \), because it enhances the potential payoff and decreases the deviation from it. When the accuracy of the supervisor’s information, \( \sigma \), affects behavior in Region IV, the expected payoff increases with \( \sigma \) because it decreases the extent of investment inefficiency. By contrast, the effect of the accuracy of the public information, \( \mu \), is ambiguous. On the one hand, an increase in \( \mu \) allows for better public information to substitute for the private information that the \( \rho \)-type employee wastes in equilibrium. On the other hand, an increase in \( \mu \) also leads to a greater distortion of the \( \rho \)-type employee’s investment choice. Hence, the expected payoff is non-monotonic in \( \mu \), i.e., providing more prior information to the supervisor can actually aggravate the “yes man” problem in the sense of leading to a less efficient outcome.\(^8\) To put it differently, the timing of the supervisor’s information matters: late information always helps, whereas early information may actually hurt.

We now consider the effect of “yes man” behavior on the supply of effort and the firm’s design of the tournament. For any given tournament, a lower value of information decreases the employees’ incentive to supply effort. In fact, the employee’s private benefit from effort reflects the entire decrease in the value of the information that they collect. Therefore, there is no need for the firm to change the profit-maximizing incentive intensity, which remains at \( W_1^* - W_2^* = \frac{1}{\phi} \). However, even though any given level of effort has a lower value, at the margin effort can now have a higher value because it can also make the use of employee information more efficient. When this happens, the profit-maximizing incentive intensity actually exceeds the efficient one, \( W_1^* - W_2^* = \frac{1}{\phi} \).

**Proposition 4:** Inefficient use of information decreases the equilibrium level of employee effort. However, it does not affect the profit-maximizing incentive intensity, which remains at \( W_1^* - W_2^* = \frac{1}{\hat{\sigma}} \), except for high enough levels of \( \sigma \) in Region II. Above a threshold level of accuracy in the supervisor’s information, \( \hat{\sigma} \), the inefficient use of employee information leads to a higher incentive intensity, \( W_1^* - W_2^* > \frac{1}{\phi} \). If

\(^8\)Cummins and Nyman (2005) and Heidhues and Lagerlöf (2003) reach the same conclusions.
it occurs, the extent of this over-extraction of effort decreases with the accuracy of the supervisor’s information.

Proof: See Appendix.

When employees have a tendency to become “yes men,” the expected value of their information decreases. If this is taken as a *fait accompli*, then the rational response from the firm is to extract a smaller amount of effort to collect information from the employee. However, in Region II, a high enough level of employee effort takes us from Region II to Region I, thus inducing the employees to use their information efficiently. Hence, for high levels of $\sigma$ (taking inefficient use of information as given), the amount of excess information needed to eradicate the “yes man” behavior, and therefore also the cost of getting it, is small. But the amount of effort required to take us to Region I increases as $\sigma$ falls, and eventually outweighs the added benefit of inducing the employees to utilize all their information. The mirror-image of this in terms of incentive intensity is illustrated in Figure 3. At the threshold, $\hat{\sigma}$, inducing efficient investment becomes feasible, so the incentive intensity jumps up discontinuously above the efficient level. From there, it decreases with $\sigma$ as more supervisor information substitutes for employee information until it reaches the efficient level at the boundary between Region II and Region I, $\bar{\sigma}$.

3.3 Learning About Different Sources of Uncertainty

In the baseline model, the two employees collect information about the same uncertainty. One way to interpret this is that the employees’ tasks are similar. It is natural to ask whether “yes man” incentives change if an employee competes for promotion against someone who is charged with a very different task. Economic intuition suggests that such a modification might increase the temptation to indulge in “yes man” behavior. When another employee has unrelated information, the supervisor can infer less about the profitability of each employee’s project choice.
Incentive intensity when signals are private information.

Since a well-informed supervisor is an effective way to discourage “yes man” behavior, one might conjecture that it is more difficult to induce the employees to use their private information efficiently when they have different tasks.

To address this issue, in this section we generalize the uncertainties that the employees learn about to allow for any arbitrary level of correlation between them.\textsuperscript{9} All signal processes remain independent of one another, conditional on the state of the world. We now denote the state of the world governing the profitability of the employee’s investment projects by $Z_i \in \{G, R\}$ and $Z_j \in \{G, R\}$, respectively. We parametrize the correlation between the payoffs from the employees’ investment projects by $\kappa \in [0, 1]$.\textsuperscript{10} The following table illustrates the joint probability distribution of $Z_i$ and $Z_j$.

\textsuperscript{9}This generalization is taken from Heidhues and Lagerl"of (2003).
\textsuperscript{10}Notice, however, that $\kappa$ is not the correlation coefficient.
A weaker correlation between the employees’ sources of uncertainty aggravates the “yes man” problem because it effectively dilutes the information content in the signals of the employees and the supervisor alike. This is illustrated in Figure 4, which is the analogue of Figure 2 for the case of less-than-perfectly uncorrelated investment uncertainties. Uncoupling the uncertainties erodes the efficiency of the tournament outcome in two ways. The first is to make it more difficult to sustain efficient investment choice as an equilibrium. A weaker correlation weakens the supervisor’s signal relative to her prior. This makes it more difficult to shake her out of her initial conviction that green projects are better and therefore shifts the boundary \( H_1 = 0 \) to the right. Moreover, because it effectively decreases the accuracy of everyone’s signals relative to the prior beliefs, a weaker correlation between the investment uncertainties also erodes the expected net benefit to the \( \rho \)-type employee of choosing the efficient red project, i.e., shifts the boundary \( H_2 = 0 \) upward. Hence, the subset of the \((\varepsilon,\sigma)\)-space where efficient choice of investment project is an equilibrium outcome, denoted Region I, becomes strictly smaller as the correlation between the investment uncertainties weakens.

The second way in which having the employees deal with different tasks rather than the same one makes “yes man” behavior a more serious problem is by making the equilibrium in Regions II, III, and IV, more inefficient. The reason for this is once again that a lower correlation between the employees’ investment uncertainties leaves the supervisor with less information about them. As a consequence, the supervisor’s initial bias towards green projects plays a more prominent role in the formation of her posterior beliefs. But this means that it takes more contamination in the form of inefficient investment by the \( \rho \)-type employee to dissuade the supervisor of her initial conviction that green is the better project. Moreover, when the accuracy of the employees’ signals and the correlation between their uncertainties are both
Figure 4: Efficient and inefficient project choice with less-than-perfectly correlated investment uncertainties.
sufficiently low, this greater extent of inefficient investment in equilibrium translates into a perfectly pooling equilibrium in which the \( \rho \)-type employee wastes all her private information by investing in the green project with certainty. In Figure 4, this happens in the new Region V that opens up and expands as the correlation between uncertainties becomes weaker.

The preceding discussion is formalized in the following proposition:

*Proposition 5*: As the correlation between the sources of uncertainty that the employees learn about grows weaker, “yes man” behavior becomes a more serious problem: the region of the parameter space where efficient investment is an equilibrium outcome shrinks and outside of it the \( \rho \)-type employee wastes more of her valuable information. In fact, all information is wasted in a perfectly pooling equilibrium if \( \kappa \) and \( \varepsilon \) are both sufficiently small.

*Proof*: See Appendix.

One important conclusion in the theory of incentives is that comparing the performance of employees with similar tasks is beneficial because it shields them from common risk. We propose a second benefit from comparing employees with similar tasks when they manage private information on behalf of their employer, namely that it gives the employer a more accurate estimate of what that information might be. This second benefit from incorporating information from employees with similar tasks therefore has nothing to do with shielding the employees from risk. Rather, it allows the supervisor to substitute preconceived notions with a better informed evaluation, something that in turn lets the employees’ own knowledge inform their choice of investment project.

4 Discussion

Our analysis illustrates how a rank-order tournament compromises the ability of employees to act with integrity and, in fact, can turn them into “yes men” who
make decisions that conform with their superior’s preconceived — but relatively
ill-informed — notions. Hence, the competitive pressure that provides employees
with the incentive to collect valuable information also discourages them from using
it efficiently.

The same tension between incentive provision and information management that
we study is also the focus of Prendergast (1993b), who demonstrates that the ten-
dency to conform to the opinions of those who administer rewards is nothing unique
to a tournament, but appears when incentives are provided with an explicit contract
as well. Apart from the incentive mechanisms studied, the main difference between
Prendergast’s analysis and ours lies in the perception of the information that the
supervisor has access to.\footnote{Prendergast’s model also lacks any countervailing
efficiency cost to “yes man” behavior of the type that the supervisor’s signal provides in our model.} In Prendergast’s model, the supervisor collects informa-
tion at only one point in time. By contrast, our model makes a sharp distinction
between what the employees know and what they do not know about the super-
visor’s information because of when it arrives. This highlights the importance of
the timing of the supervisor’s information. Information that the supervisor receives
before the employees’ decisions makes the tournament unfair in that it favors one
project over the other. Unfortunately, the employees can react to such early infor-
mation by choosing away from alternatives with the deck stacked against them and
instead choose the supervisor’s favorite alternative. Information that the supervisor
receives after the employees have committed to an alternative, on the other hand,
discourages “yes man” behavior because the employees can expect it to re-enforce
the tendency for the supervisor to ultimately see the world the way they do.

The source of the “yes man” incentive in tournaments is the supervisor’s prior
opinion. Therefore, to the extent that her objective is to make employees behave in
a way that maximizes the firm’s profit, the supervisor has an incentive to ignore her
prior information: with a balanced prior, the employees have no reason to privilege
one project over the other unless their own information about profitability tells them
that they should. However, we think of the supervisor’s prior beliefs and opinions as capturing not a fleeting impression that can be erased at will, but rather past information that has shaped the very core of the supervisor’s view of the world in a way that can be reversed only by new information.

But even if it may be difficult for the individual supervisor to prevent her preconceived notions from influencing promotion decisions, the firm as an organization may be able to take measures to do so. Notice that it is the employees perception of the supervisor’s initial bias that matters. Therefore, institutional arrangements that make it more difficult for employees to discern which prior they should cater to would benefit the management of private information inside the firm. One way to accomplish this might be to let a group make the promotion decision: compared to gauging the biases of a single person, it is likely to be more difficult for employees to predict which prior opinions will carry the day in a group decision. Similarly, engaging an outside evaluator may serve the same purpose of confusing the employees as to which direction they should turn their coats. This provides a new rationale — in addition to discouraging various inefficient influence activities — for taking the distribution of rewards and punishment to employees out of the hands of their immediate supervisors.

The model we develop is extremely simple, but many of the assumptions that we make are simplifications that do not alter the qualitative economic results. For example, this is the case with the restriction to only two investment projects and the normalization of their payoffs as well as the conditional independence of the signals, the rudimentary strategic interaction in the effort-stage of the game, and the fact that the employees use their information in a real decision rather than simply report it to their supervisor. Furthermore, it is our conjecture that the restriction on the extraneous uncertainty to a uniform distribution is innocuous as well. While it is standard practice in the tournament literature to make distributional assumptions to ensure that a symmetric Nash equilibrium in effort levels exists, we are more restrictive and assume a uniform distribution. The role of this assumption is to allow the
employees to maximize the expected value of the supervisor’s profit estimate when choosing their investment project. However, we believe that it could be dispensed with because the incentive to neglect information that contradicts the supervisor’s prior bias seems likely to remain even if employees instead were to choose projects using the general criterion of first-order stochastic dominance of the distribution of supervisor profit estimates.

By contrast, other assumptions are necessary for our results. One of those is that we have let the employees contribute to the firm’s investment decision by putting effort into collecting information about it. Alternatively, the employees could put effort into generating investment payoff. We believe that most of our conclusions would stand in the face of such a modification of the model. However, one interesting result that would not is that inefficient use of information ratchets up the incentive intensity above the efficient level. The reason is that such over-extraction of effort is an attempt to achieve a higher correlation between employee and supervisor signals by making the former more accurate. If effort were to produce payoffs rather than information, then this benefit would obviously disappear.

Because of its empirical and theoretical importance, we take the tournament as given and study how information is managed in the face of that particular incentive mechanism. Alternative mechanisms that could mitigate the “yes man” tendencies might of course exist. In light of Prendergast’s work, however, an individual bonus contract certainly seems to be prone to produce its own “yes man” problems. Furthermore, in the context of voluntary employment that we are considering, it seems likely that participation and limited liability constraints would hamper the use of harsh punishments to exploit the correlation of the employees’ types to implement efficient investment. But the important and interesting topic of whether the “yes man” problem can be eliminated through the design of alternative mechanisms more generally must be left for future research.

An equally crucial assumption that we make about the incentive mechanisms is that the expected profit is used to evaluate the employees’ investment decisions.
This seems reasonable if the task that the employees handle is complex because then expected profit serves as an overall measure of the effects of what the employees do on the welfare of the firm’s shareholders. But the “yes man” incentive highlighted in this paper certainly constitutes a drawback to rewarding the employee according to the shareholders’ less-than-perfectly-informed preferences over her choices.

In our model, we do not allow for the possibility that employees are heterogeneous in their innate ability to collect information. If this were the case, then the firm may want to promote more able employees. To the extent that the employees know their own ability, this would give them an incentive to try to signal their ability to collect information. One way this could be done is by expressing an opinion that in fact contradicts the conventional wisdom in general, and the preconceived notions of the supervisor in particular. Hence, the desire to signal innate learning ability may introduce a countervailing incentive for employees to become not “yes men,” but rather “no men” (Kim and Ruy 2003 and Prendergast and Stole 1996).

Finally, we restrict what the employees can do in a crucial way in that they cannot choose how to allocate their effort across the different projects: an increase in effort increases the accuracy of the signals about both projects by the same amount. An interesting question is whether the competitive pressure of a promotion tournament not only makes employees neglect information that they have, but also distorts their allocation of information-collection effort across different projects. We hope to address this issue in future research.
Appendix

Proof of Proposition 2: In terms of the profitability of the chosen projects, the supervisor has a non-trivial decision to make only when the two employees choose different projects. In this event, there are four different realizations of the supervisor’s signals to consider. With a balanced prior, the promotion is split evenly between the employees if the supervisor’s signals cancel and otherwise given to the project that the signals indicate is the more profitable one. Since the signal of each employee is correlated with the signals of the supervisor, the expected probability of getting the promotion is maximized by each employee following her own signal, making efficient project choice an equilibrium outcome.

With an unbalanced prior, the promotion is given to the green project if the supervisor’s signals cancel and therefore, a fortiori, also if the signals indicate that the state is Green. If the signals indicate that the state is Red, the promotion is given to the red project if the net preference for the red project in the supervisor's posterior, denoted by $H_1$, is strictly positive:

$$H_1 \equiv \frac{1}{2} - \Pr (G|e_i = \rho, e_j = \gamma, s_i = h, s_j = l) > 0$$

$$\frac{\mu \varepsilon (1 - \varepsilon) (1 - \sigma)^2}{\mu \varepsilon (1 - \varepsilon) (1 - \sigma)^2 + (1 - \mu) (1 - \varepsilon) \varepsilon \sigma^2} < \frac{1}{2}$$

$$\frac{\mu}{1 - \mu} < \left( \frac{\sigma}{1 - \sigma} \right)^2$$

$H_1$, is obviously strictly decreasing in $\mu$. Efficient project choice is now an equilibrium outcome if this allocation of promotion gives an employee with a $\rho$-signal a positive net benefit from choosing the red project rather than the green one, denoted by $H_2$:

$$H_2 \equiv \E \{ \hat{\pi}_i (r) | \rho \} - \E \{ \hat{\pi}_i (g) | \rho \} \geq 0$$

$$\Pr (R|\rho) \left\{ \frac{1}{2} \Pr (z_j = r|R) + \Pr (z_j = g, s_i = h, s_j = l|R) \right\} +$$
\[
+ \Pr(G|\rho) \left\{ \frac{1}{2} \Pr(z_j = r|G) + \Pr(z_j = g, s_i = h, s_j = l|G) \right\} \geq \\
\geq \Pr(R|\rho) \left\{ \frac{1}{2} \Pr(z_j = g|R) + [\Pr(z_j = r|R) - \Pr(z_j = r, s_i = l, s_j = h|R)] \right\} + \\
+ \Pr(G|\rho) \left\{ \frac{1}{2} \Pr(z_j = g|G) + [\Pr(z_j = r|G) - \Pr(z_j = r, s_i = l, s_j = h|G)] \right\} \\
\frac{1}{2} (1 - \varepsilon) + \Pr(R|\rho) \frac{1}{2} (2\varepsilon - 1) + \left\{ \Pr(R|\rho) (1 - \varepsilon) \sigma^2 + \Pr(G|\rho) \varepsilon (1 - \sigma)^2 \right\} \geq \\
\geq \frac{1}{2} (1 - \varepsilon) - \Pr(G|\rho) \frac{1}{2} (2\varepsilon - 1) + \varepsilon - \left\{ \Pr(R|\rho) \varepsilon \sigma^2 + \Pr(G|\rho) (1 - \varepsilon) (1 - \sigma)^2 \right\} \\
\Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1 - \sigma)^2 \geq \frac{1}{2}
\]

The left-hand side is strictly increasing in both \( \sigma \) and \( \varepsilon \), so it follows from the Implicit Function Theorem that the boundary in \( \sigma - \varepsilon \) space implicit defined by \( H_2 = 0 \) has a negative slope. Moreover, condition (2) holds for any value of \( \sigma \) if \( \varepsilon \) is large enough, as well as for any value of \( \varepsilon \) if \( \sigma \) is large enough. And as \( \sigma \) (\( \varepsilon \)) approaches \( \frac{1}{2} \), the \( \varepsilon \) (\( \sigma \)) required to make condition (2) approaches unity. Finally, it is easy to confirm that \( H_2 \) is strictly decreasing in \( \mu \).

If condition (1) is violated so that \( H_1 = 0 \), then the supervisor splits the promotion evenly when her signals indicate that the state is Red, thus taking away half the net benefit that the red project gets in this outcome. This lost net benefit consists of the expected benefit taken away from the red project and the expected benefit given to the green project:

\[
\frac{1}{2} \left\{ [\Pr(z_j = g, s_i = h, s_j = l|R) \Pr(R|\rho) + \Pr(z_j = g, s_i = h, s_j = l|G) \Pr(G|\rho)] + \\
+ [\Pr(z_j = r, s_i = l, s_j = h|R) \Pr(R|\rho) + \Pr(z_j = r, s_i = l, s_j = h|G) \Pr(G|\rho)] \right\} = \\
= \frac{1}{2} \left\{ \left[ \Pr(R|\rho) (1 - \varepsilon) \sigma^2 + \Pr(G|\rho) \varepsilon (1 - \sigma)^2 \right] + \left[ \Pr(R|\rho) \varepsilon \sigma^2 + \Pr(G|\rho) (1 - \varepsilon) (1 - \sigma)^2 \right] \right\} = \\
= \frac{1}{2} \left\{ \Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1 - \sigma)^2 \right\}
\]
The net benefit of choosing the red project to an employee with a $\rho$-signal therefore looks as follows:

$$H_2 - \frac{1}{2} \left\{ \Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1-\sigma)^2 \right\} =
\Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1-\sigma)^2 - \frac{1}{2} \left\{ \Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1-\sigma)^2 \right\} =
= \frac{1}{2} \left\{ \left[ \Pr(R|\rho) \sigma^2 + \Pr(G|\rho) (1-\sigma)^2 \right] - 1 \right\} < 0$$

It follows that efficient project is not an equilibrium outcome when $H_1 = 0$, and therefore, a fortiori, not when $H_1 < 0$ either.

Proof of Proposition 3: Consider first Regions II and III. Since $H_2 < 0$, giving the employee the promotion when the supervisor’s signals indicate that the state is Red is insufficient to make the $\rho$-type employee invest efficiently. Therefore, in addition to getting the promotion when the supervisor’s signals support her choice, the employee with the red project must also get promoted with strictly positive probability when the supervisor’s signals cancel. When the supervisor’s signals cancel, her posterior looks as follows:

$$\Pr(G|z_i = r, z_j = g, s_i = s_j) = \frac{\mu (1-\varepsilon) (1-\zeta^\rho)}{\mu (1-\varepsilon) (1-\zeta^\rho) [\varepsilon + (1-\varepsilon) \zeta^\rho] + (1-\mu) \varepsilon (1-\zeta^\rho) [(1-\varepsilon) + \varepsilon \zeta^\rho]}$$

This expression is strictly decreasing in $\zeta^\rho$, strictly greater than $\frac{1}{2}$ when $\zeta^\rho = 0$, and strictly smaller than $\frac{1}{2}$ when $\zeta^\rho$ approaches unity. Hence, there exists a unique $\zeta^\rho^*$ that balances the supervisor’s posterior:

$$\frac{\mu (1-\varepsilon) (1-\zeta^\rho^*) [\varepsilon + (1-\varepsilon) \zeta^\rho^*] + (1-\mu) \varepsilon (1-\zeta^\rho^*) [(1-\varepsilon) + \varepsilon \zeta^\rho^*]}{\mu (1-\varepsilon) (1-\zeta^\rho^*) [\varepsilon + (1-\varepsilon) \zeta^\rho^*] + (1-\mu) \varepsilon (1-\zeta^\rho^*) [(1-\varepsilon) + \varepsilon \zeta^\rho^*]} = \frac{1}{2}$$

$$\zeta^\rho^* = \frac{\varepsilon (1-\varepsilon) (2\mu - 1)}{\varepsilon^2 (1-\mu) - (1-\varepsilon)^2 \mu}$$

Furthermore, if the posterior is balanced when the supervisor’s signals cancel, it must strictly favor the Red (Green) state if the supervisor’s signals indicate that the state is Red (Green).
Next, turn to the supervisor’s equilibrium strategy. The \( \rho \)-type employee’s net benefit from choosing the red project rather than the green as a function of the supervisor’s mixed strategy, \( \beta_i (z_i = r, z_j = g, s_i = s_j) \) looks as follows:

\[
E \{ \hat{\pi}_i (r) \mid \rho \} - E \{ \hat{\pi}_i (g) \mid \rho \} = \\
= \Pr (R \mid \rho) \sigma^2 + \Pr (G \mid \rho) (1 - \sigma)^2 - \frac{1}{2} + 2\sigma (1 - \sigma) \beta_i (z_i = r, z_j = g, s_i = s_j)
\]

This net benefit is strictly increasing in \( \beta \). Since \( H_2 < 0 \), it is strictly negative for \( \beta = 0 \). Furthermore, it is strictly positive for \( \beta = 1 \):

\[
\Pr (R \mid \rho) \sigma^2 + \Pr (G \mid \rho) (1 - \sigma)^2 - \frac{1}{2} + 2\sigma (1 - \sigma) = \\
= \frac{1}{2} - \Pr (R \mid \rho) \sigma^2 + \Pr (G \mid \rho) (1 - \sigma)^2 = -H_2 > 0
\]

Therefore, a unique \( \beta^* \) that makes the \( \rho \)-type employee indifferent exists and is given by:

\[
\Pr (R \mid \rho) \sigma^2 + \Pr (G \mid \rho) (1 - \sigma)^2 - \frac{1}{2} + 2\sigma (1 - \sigma) \beta_i^* (z_i = r, z_j = g, s_i = s_j) = 0
\]

\[
\beta_i^* (z_i = r, z_j = g, s_i = s_j) = \frac{1}{2} \left\{ 1 - \left[ \frac{\sigma - \frac{1}{2}}{\sigma (1 - \sigma)} \right] \left[ \frac{\varepsilon - \mu}{\varepsilon (1 - \mu) - (1 - \varepsilon) \mu} \right] \right\} \in \left( 0, \frac{1}{2} \right)
\]

These strategies for the \( \rho \)-type employee and the supervisor makes it rational for the \( \gamma \)-type employee to invest efficiently since her net benefit from investing in the green project exceeds that of the \( \rho \)-type employee, which is by construction equal to zero.

A partially separating equilibrium Pareto-dominates a perfectly pooling one, and the above equilibrium is the only mixed-strategy equilibrium that exists. To see that this is true, notice that \( \zeta^\rho \) and \( \beta^* \) are obviously unique if \( \zeta^\gamma = 1 \). Furthermore, if both projects are chosen with positive probability, then net benefit of choosing green over red is strictly higher for the \( \gamma \)-employee than for the \( \rho \)-employee. Therefore, if \( \zeta^\gamma \in (0, 1) \), then \( \zeta^\rho = 0 \). But that means that green reveals the \( \gamma \)-signal perfectly, so the \( \gamma \)-employee strictly prefers green to red, making \( \zeta^\gamma \in (0, 1) \) not sequentially rational.
The argument establishing the equilibrium in Region IV is analogous. Since it is now the case that \( H_2 > 0 \), giving the employee the promotion when the supervisor’s signals indicate that the state is Red suffices to make the \( \rho \)-type employee invest efficiently. Therefore, to make her indifferent between the two projects, in this outcome the \( \rho \)-type employee should be denied the promotion with some probability. With signals supporting the state being Red, the supervisor’s posterior looks as follows:

\[
\Pr (G | z_i = r, z_j = g, s_i = h, s_j = l) = \frac{\mu (1 - \varepsilon) (1 - \xi^*) \varepsilon [1 + (1 - \varepsilon) \xi^*] (1 - \sigma)^2}{\mu (1 - \varepsilon) (1 - \xi^*) \varepsilon [1 + (1 - \varepsilon) \xi^*] (1 - \sigma)^2 + (1 - \mu) \varepsilon (1 - \xi^*) \varepsilon [1 + (1 - \varepsilon) \xi^*] (1 - \sigma)^2 + (1 - \mu) \varepsilon (1 - \xi^*) \varepsilon [1 + (1 - \varepsilon) \xi^*] (1 - \sigma)^2}
\]

Since \( H_1 < 0 \), this posterior favors Green with perfect separation, which implies that a unique level of inefficient investment by the \( \rho \)-type employee balances the supervisor’s posterior:

\[
\xi^* = \frac{\varepsilon (1 - \varepsilon) [1 - \sigma^2] \mu - \sigma^2 (1 - \mu)}{\varepsilon^2 (1 - \mu) \sigma^2 - (1 - \varepsilon)^2 \mu (1 - \sigma)^2}
\]

Next, turn to the supervisor’s equilibrium strategy. The \( \rho \)-type employee’s net benefit from choosing the red project rather than the green as a function of the supervisor’s mixed strategy, \( \beta (z_i = r, z_j = g, s_i = h, s_j = l) \) looks as follows:

\[
E \{ \hat{\pi}_i (r) | \rho \} - E \{ \hat{\pi}_i (g) | \rho \} = \beta (z_i = r, z_j = g, s_i = h, s_j = l) \left[ \Pr (R | \rho) \sigma^2 + \Pr (G | \rho) (1 - \sigma)^2 \right] - \frac{1}{2}
\]

This net benefit is strictly increasing in \( \beta (z_i = r, z_j = g, s_i = h, s_j = l) \). It is strictly negative for \( \beta = 0 \) and, since \( H_2 > 0 \), strictly positive for \( \beta = 1 \). Therefore, a unique \( \beta^* \) that makes the \( \rho \)-type employee indifferent exists and is given by:

\[
\beta^* (z_i = r, z_j = g, s_i = h, s_j = l) = \frac{1}{2} \left\{ \frac{1}{\Pr (R | \rho) \sigma^2 + \Pr (G | \rho) (1 - \sigma)^2} \right\} \in \left( \frac{1}{2}, 1 \right)
\]
Again, since the ρ-type employee is indifferent between the two projects, the γ-type employee must strictly prefer the green project to the red one. Furthermore, by the same argument as above, no other mixed-strategy equilibrium exists.

Finally, for a given symmetric effort level, the expected payoff from investment is equal to:

\[ E\{\pi_i\} = \mu[\varepsilon + (1 - \varepsilon) \zeta^\rho] + (1 - \mu) \varepsilon \left(1 - \zeta^\rho\right) = \varepsilon - \zeta^\rho (\varepsilon - \mu) \]

\[ \equiv \]

**Proof of Proposition 4:** Taking inefficient use of the employee’s information as given, the expected social marginal benefit of effort is equal to the expected value of information, i.e., \(1 - \zeta^\rho - (\varepsilon - \mu) \frac{\partial \zeta^\rho}{\partial \varepsilon}\). The symmetric equilibrium level of effort, \(\varepsilon^*\), equates the employees’ expected private marginal benefit and cost of effort:

\[
[W_1 - W_2] \phi \left\{1 - \zeta^\rho - (\varepsilon^* - \mu) \frac{\partial \zeta^\rho}{\partial \varepsilon}\right\} = C'(\varepsilon^*).
\]

This equilibrium effort level is smaller than when information is used efficiently, given by \([W_1 - W_2] \phi = C'(\varepsilon^*)\). The profit-maximizing choice of incentive intensity, \(W_1^* - W_2^*\), makes the employees’ expected private marginal benefit equal to the expected social marginal benefit, i.e., \(W_1^* - W_2^* = \frac{1}{\delta}\).

When \(H_1 < 0\), efficient project choice cannot be an equilibrium outcome no matter how accurate is the employees’ information. However, when \(H_1 > 0\), then it is always the case that sufficient employee effort induces efficient use of employee information. Hence, in Region II it may be worthwhile for the firm to extract excess effort above and beyond the equilibrium level derived above to move into Region I, where employee information is used in full. For any given level of \(\sigma\), denote the level of effort required to reach the boundary \(\hat{\varepsilon}\). Since effort is costly, implementing an \(\varepsilon > \hat{\varepsilon}\) cannot be optimal. The expected net benefit from staying in Region II and from instead moving into Region I is equal to \(\varepsilon^* - \zeta^\rho (\varepsilon^* - \mu) - C(\varepsilon^*)\) and \(\hat{\varepsilon} - C(\hat{\varepsilon})\), respectively. Hence, implementing \(\hat{\varepsilon}\) is optimal whenever \(\zeta^\rho (\varepsilon^* - \mu) > [C(\hat{\varepsilon}) - C(\varepsilon^*)] - [\hat{\varepsilon} - \varepsilon^*]\). Notice that in Region II, the extent of
inefficient investment, \( \zeta^\sigma \), is independent of \( \sigma \), which implies that the same is true of \( \varepsilon^* \). Hence, the left-hand side of the above expression is constant across Region II. By contrast, the right-hand side is strictly decreasing in \( \sigma \), and falls to zero at the boundary to Region I, given \( \varepsilon^* \). Denote this boundary-level \( \bar{\sigma} \). It now follows that there exists a unique threshold level of \( \sigma \), call it \( \hat{\sigma} \), such that for all \( \sigma \in (\hat{\sigma}, \bar{\sigma}) \) it is optimal to implement over-investment in effort by choosing a higher incentive intensity, i.e., \( W_1^* - W_2^* > \frac{1}{\phi} \). Moreover, since \( \hat{\varepsilon} \) decreases with \( \sigma \), so does the optimal incentive intensity for \( \sigma \in (\hat{\sigma}, \bar{\sigma}) \), reaching \( W_1^* - W_2^* = \frac{1}{\phi} \) at \( \sigma = \bar{\sigma} \).

Proof of Proposition 5: It is still the case that in equilibrium the employees choose the same effort level, denoted by \( \varepsilon \). Consider employee \( i \)'s incentive to choose the red project if her own signal indicates that it is profitable. Just as with perfectly correlated uncertainty, a necessary condition for such a \( \rho \)-type employee to choose the red project is that it earns her the promotion if both the supervisor’s signals indicate that red is a good choice and green is a bad choice. Again, denote the supervisor’s net preference for the red project in this outcome by \( H_1 \).

\[
H_1 \equiv \Pr (Z_i = R | e_i = \rho, e_j = \gamma, s_i = h, s_j = l) - \\
- \Pr (Z_j = G | e_i = \rho, e_j = \gamma, s_i = h, s_j = l) = \\
= \Pr (Z_i = R, Z_j = R | e_i = \rho, e_j = \gamma, s_i = h, s_j = l) - \\
- \Pr (Z_i = G, Z_j = G | e_i = \rho, e_j = \gamma, s_i = h, s_j = l) = \\
= \frac{\left[ \varepsilon^2 - \left( \mu + \kappa \mu (1 - \mu) \right) \sigma^2 \right] \left( 1 - \varepsilon \right) \sigma \left( 1 - \varepsilon \right) \sigma^2}{ \left( 1 - \mu \right) \varepsilon + \mu \left( 1 - \varepsilon \right)} = \\
= \frac{\varepsilon (1 - \varepsilon) \left\{ [(1 - \mu) + \kappa \mu] (1 - \mu) \sigma^2 - [\mu + \kappa (1 - \mu)] \mu (1 - \sigma)^2 \right\}}{\left( 1 - \mu \right) \varepsilon + \mu \left( 1 - \varepsilon \right)}
\]

This expression converges to the expression for derived in Proposition 1 when \( \kappa \) approaches unity. Furthermore, it is independent of \( \varepsilon \), strictly increasing in \( \sigma \) and \( \kappa \), and strictly decreasing in \( \mu \). It therefore follows from the Implicit Function

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Theorem that an increase in $\kappa$ shifts the boundary in the $(\sigma, \varepsilon)$ parameter space where $H_1 = 0$ to the left, expanding Region I where efficient investment can occur.

Next, consider the $\rho$-employee’s expected net benefit from choosing the red project rather than the green one given that $H_1 > 0$, again denoted by $H_2$. With less than perfect correlation, if the employees choose the same project, then the promotion is split evenly if they receive the same signal and otherwise is given outright to the one with the high signal, whereas if they choose different project, then the promotion is given to the employee with the green project unless she herself receives a low signal and her competitor receives a high signal. Furthermore, notice that $\Pr(Z|e_i = \rho) = \frac{\Pr(e_i = \rho|Z) \Pr(Z)}{\Pr(e_i = \rho)}$ and that the symmetry of the employees’ signal process implies the following four equalities:

\[
\begin{align*}
\Pr(s_i = l, s_j = h|z_i = g, z_j = r, Z) &= \Pr(s_i = h, s_j = l|z_i = r, z_j = g, Z) \\
\Pr(s_i = l, s_j = h|z_i = r, z_j = r, Z) &= \Pr(s_i = h, s_j = l|z_i = g, z_j = g, Z) \\
\Pr(s_i = h, s_j = h|z_i = r, z_j = r, Z) &= \Pr(s_i = l, s_j = l|z_i = g, z_j = g, Z) \\
\Pr(s_i = l, s_j = l|z_i = r, z_j = r, Z) &= \Pr(s_i = h, s_j = h|z_i = g, z_j = g, Z)
\end{align*}
\]

This implies that, after some algebraic manipulation, the expression for $H_2$ simplifies to

\[
H_2 \equiv E \{ \pi_i (r) | \rho \} - E \{ \pi_i (g) | \rho \} = \\
= \frac{1}{\Pr(e_i = \rho)} \sum_Z \Pr(e_i = \rho|Z) \Pr(Z) \{ \Pr(s_i = l, s_j = h|z_i = g, z_j = r, Z) - \\
- \Pr(s_i = l, s_j = h|z_i = r, z_j = r, Z) - \frac{1}{2} \Pr(s_i = h, s_j = h|z_i = r, z_j = r, Z) - \\
- \frac{1}{2} \Pr(s_i = l, s_j = l|z_i = r, z_j = r, Z) \} = \\
= \left[ \frac{1}{\varepsilon (1 - \mu) + (1 - \varepsilon) \mu} \right] \{ \varepsilon (1 - \mu) [(1 - \mu) + \kappa \mu] \sigma^2 + \\
(1 - \varepsilon) \mu [\mu + \kappa (1 - \mu)] (1 - \sigma)^2 \} - \frac{1}{2} \varepsilon (1 - \mu) [(1 - \mu) + \kappa \mu] + \\
+ (1 - \varepsilon) \mu [\mu + \kappa (1 - \mu)] + 2 (1 - \kappa) \mu (1 - \mu) \{ \varepsilon (1 - \sigma)^2 + (1 - \varepsilon) \sigma^2 \} \}
\]
$H_2$ is continuous in $\kappa$ because the limit of this expression as $\kappa$ approaches unity is the expression for $H_2$ derived in Proposition 2. Moreover, it is straightforward to show that at the boundary where its value is equal to zero, $H_2$ is strictly increasing in $\varepsilon$, $\sigma$, and $\kappa$, and strictly decreasing in $\mu$. It therefore follows from the Implicit Function Theorem that an increase in the correlation between the employees’ sources of uncertainty shifts the boundary where $H_2 = 0$ down/left, expanding Region I where efficient investment can occur.

Finally, it remains to be shown that when efficient investment is not possible, correlation between the employees’ sources of uncertainty decreases the extent of inefficient investment in equilibrium. Consider first Regions II and III. In these regions, a mixed-strategy equilibrium involves mixing when the employees choose different projects and the supervisor receives the same signal for both of them. A (non-degenerate) equilibrium mixed investment strategy for an employee with $\rho$ signal, $\zeta^{\rho'}$, is given by the indifference condition that the supervisor considers the two different projects equally profitable:

$$
\Pr(Z_i = R|e_i = \rho, e_j = \gamma, s_i = h, s_j = h) - \Pr(Z_j = G|e_i = \rho, e_j = \gamma, s_i = h, s_j = h) = 0 \iff \frac{[(1 - \mu)^2 + \kappa \mu (1 - \mu)] \varepsilon (1 - \zeta^{\rho'}) (1 - \varepsilon + \varepsilon \zeta^{\rho'}) \sigma (1 - \sigma)}{\Pr(e_i = \rho, e_j = \gamma, s_i = h, s_j = h)} - \frac{[\mu^2 + \kappa \mu (1 - \mu)] (1 - \varepsilon) (1 - \zeta^{\rho'}) (\varepsilon + (1 - \varepsilon) \zeta^{\rho'}) (1 - \sigma) \sigma}{\Pr(e_i = \rho, e_j = \gamma, s_i = h, s_j = h)} = 0 \iff 
$$

$$
\zeta^{\rho'} = \frac{\varepsilon (1 - \varepsilon) \{[\mu + \kappa (1 - \mu)] \mu - [(1 - \mu) + \kappa \mu] (1 - \mu)\}}{[(1 - \mu) + \kappa \mu] (1 - \mu) \varepsilon^2 - [\mu + \kappa (1 - \mu)] \mu (1 - \varepsilon)^2}
$$

$\zeta^{\rho'}$ is strictly decreasing in $\kappa$ and converges to the corresponding equilibrium mixed strategy in Proposition 3 when $\kappa$ approaches unity. $\zeta^{\rho'}$ is strictly greater than zero: the numerator is strictly positive because $\mu > \frac{1}{2}$, while the denominator is strictly positive because $\varepsilon > \mu$. Furthermore, it can be shown that $\zeta^{\rho'}$ is strictly below unity so that a mixed-strategy equilibrium exists if the following condition is
satisfied:

\[
\frac{\mu + \kappa (1 - \mu)}{(1 - \mu) + \kappa \mu} \left( \frac{\mu}{1 - \mu} \right) < \left( \frac{\varepsilon}{1 - \varepsilon} \right)
\]

If this condition is violated, which happens if the employees’ signals are sufficiently inaccurate and the correlation between their sources of uncertainty is sufficiently low, then the only equilibrium is a perfectly pooling pure-strategy equilibrium in which the employees invest with certainty no matter what signal they get. A universally-divine out-of-equilibrium belief that holds together this equilibrium is that the supervisor believes with certainty that deviation to the red project is done by an employee with a \( \rho \) signal.

In Region IV, a mixed strategy equilibrium involves mixing by the supervisor and an employee with a \( \rho \) signal when the employees choose different projects and the supervisor’s signals both indicate that the red project is the better choice. The indifference condition that defines the \( \rho \)-type employee’s (non-degenerate) mixed equilibrium investment strategy looks as follows:

\[
\Pr (Z_i = R | e_i = \rho, e_j = \gamma, s_i = h, s_j = l) - \Pr (Z_j = G | e_i = \rho, e_j = \gamma, s_i = h, s_j = l) = 0 \iff \frac{\varepsilon (1 - \varepsilon) \left( \frac{\mu + \kappa (1 - \mu)}{(1 - \mu) + \kappa \mu} \right) \left( 1 - \zeta^* \right) \left( 1 - \varepsilon \right)}{\Pr (e_i = \rho, e_j = \gamma, s_i = h, s_j = l)} - \frac{\varepsilon (1 - \varepsilon) \left( \frac{\mu + \kappa (1 - \mu)}{(1 - \mu) + \kappa \mu} \right) \left( 1 - \zeta^* \right) \left( 1 - \varepsilon \right)}{\Pr (e_i = \rho, e_j = \gamma, s_i = h, s_j = l)} = 0 \iff \zeta^* = \frac{\varepsilon (1 - \varepsilon) \left( \frac{\mu + \kappa (1 - \mu)}{(1 - \mu) + \kappa \mu} \right) \left( 1 - \varepsilon \right)}{\left( \frac{\mu + \kappa (1 - \mu)}{(1 - \mu) + \kappa \mu} \right) \varepsilon^2 \sigma^2 - \left( \frac{\mu + \kappa (1 - \mu)}{(1 - \mu) + \kappa \mu} \right) \mu (1 - \varepsilon)^2 (1 - \sigma)^2}
\]

\( \zeta^* \) is strictly decreasing in \( \kappa \) and converges to the corresponding equilibrium mixed strategy in Proposition 3 when \( \kappa \) approaches unity. \( \zeta^* \) is strictly greater than zero: the numerator is strictly positive because \( H_1 < 0 \) in Region IV, while the denominator is strictly positive because \( \varepsilon \) and \( \sigma \) both exceed \( \mu \). Finally, it can be shown that the fact that \( H_2 > 0 \) in Region IV implies that \( \zeta^* \) is strictly below unity so that a mixed-strategy equilibrium exists.
References


