Labor Participation, Human Capital Accumulation, and the Business Cycle

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Abstract

We explore labor force participation in a search model where overlapping generations make endogenous education decisions and where their skills can grow while working. Our model closely matches U.S. labor force participation and its response to GDP growth across different age groups and education levels, and it sheds light on the persistent decrease in participation following the pandemic recession. We find that subsidizing job search may lower welfare, as it discourages education and negatively impacts skill development in the long run. However, an age-based subsidy can raise overall welfare.

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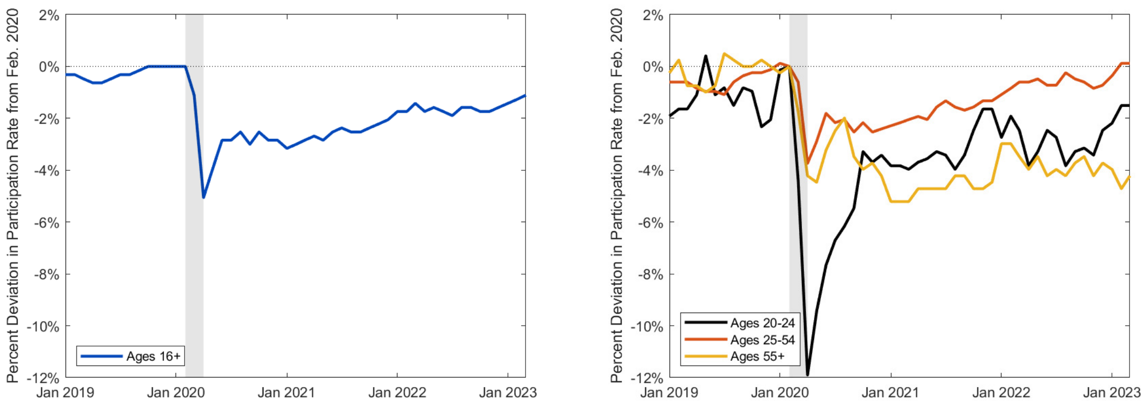
1 Introduction

Conventional labor market models primarily emphasize the dynamics between employment and unemployment, often disregarding the labor force participation margin. This emphasis is due to the perception that the labor force participation rate is relatively stable over the business cycle, even though some secular trends may be observed. However, this was not the case, at least during the recent pandemic-induced recession, where the labor force participation rate experienced a significant decline with a high level of persistence (*Figure 1*).[1]

Figure 1: Overall and Age-specific Labor Participation Rates



This striking pattern leads us to question the conventional wisdom that labor participation is irrelevant to business cycle dynamics. In line with recent research (Krusell et al., 2011, 2017, 2020), we explore endogenous labor force participation in a heterogeneous-agent model incorporating search and matching frictions, with a focus on the previously under-explored aspect of participation over the life cycle. *Figure 1* reveals that labor participation of different age groups responded differently to the pandemic recession. Both the young (20-24) and old (55+) populations exhibited a more significant decline in labor participation compared to prime-aged (25-54) individuals. What factors contribute to the age-related dynamics of labor participation? Why do certain age groups appear more susceptible to business cycle shocks? Moreover, can government intervention, such as workforce subsidies, improve overall welfare?

Our paper addresses these questions from both empirical and quantitative perspectives. We begin by documenting micro-level evidence on age-based labor force participation patterns using data from the Current Population Survey (CPS). Our findings reveal an inverted U-shape in the participation rates across various age groups, with the youngest and oldest populations exhibiting the lowest labor participation rates relative

[1] Persistence in the decline of labor force participation is also observed in other recessions such as the 2007-09 Great Recession.

to prime-aged individuals. This pattern is consistent across different education levels (college vs. non-college), although college graduates generally display a higher propensity to participate in the labor force throughout the entire life cycle. We then investigate how labor force participation responds to shocks to aggregate GDP growth. Utilizing vector autoregression (VAR) methods, we find that the responses exhibit a U-shape, with the youngest and oldest populations being the most responsive to changes in GDP growth rates. This pattern persists across different education levels, although individuals with college degrees generally demonstrate lower elasticity with respect to GDP growth changes.

To explain these empirical findings, we develop an overlapping generations model with endogenous college choices and on-the-job human capital accumulation. In the model, attending college is costly but boosts expected lifetime income. Endogenizing the college enrollment decision allows us to match the relatively low labor participation among younger populations (18-24). At the same time, on-the-job learning helps explain the persistently high labor participation among prime-aged populations. The model predicts that older populations exhibit lower participation due to a horizon effect - they are closer to retirement. Hence, the expected return from search is lower, making them less likely to participate in the labor force.

Regarding the participation margin, we assume that each non-employed individual draws a stochastic cost of participation in each period and decides whether to participate in labor market search. The cost distribution is the key object to be disciplined in our quantitative exercise, and we calibrate it to match the overall participation-age profile documented in the empirical section. Given that our model features an endogenous college sector, we match statistics on college enrollment, graduation, and the college wage premium. Finally, we replicate key labor market statistics, such as unemployment and vacancy posting rates. The calibration produces a relatively compressed cost distribution, indicating that older populations are almost indifferent regarding participation in the labor market. The identification comes from the sharp drop in the labor force participation rate among older populations.

We validate the quantitative model by confronting its empirical predictions with a set of untargeted, conditional moments. First, our model replicates the hump-shaped participation-age profile, *conditional on education*.[2] It predicts that, among older individuals, the labor participation rate decreases more rapidly for non-college workers compared to college graduates, which aligns well with the data. This sharper decline in participation as individuals approach retirement age among those without a college degree is due to different human capital dynamics among the two groups. Non-college workers typically experience higher unemployment rates, and due to fewer employment

[2] In the calibration, we match the overall participation-age profile and college wage premium without directly targeting any moments from the conditional participation-age profile.

opportunities, their human capital grows at a slower rate. Hence their participation rate declines at a faster rate as they age because they do not benefit as much from employment.

Second, the model accounts well for the response of labor participation to GDP growth shocks, conditional on both age and education. The stochastic participation cost plays a key role here. Highly educated and/or prime-aged individuals have the highest expected values of work, leading to high participation thresholds. Hence their labor participation choices exhibit low sensitivity to alterations in economic fundamentals. In contrast, non-college graduates, younger, and older populations have low relative values of work, which can be attributed to either less skill, the need to attend college, or being close to retirement. Consequently, their participation threshold is relatively low, indicating that their participation decisions display greater elasticity in response to changes in economic fundamentals. Overall, our micro-founded quantitative model successfully replicates various aspects of the participation profile, and its responses to business cycle shocks. This lends us confidence in its validity in serving as a lab to evaluate various counterfactuals and policy analyses related to labor force participation.

We examine the long-term welfare implications of introducing subsidies to incentivize job-seeking. These subsidies can be thought of as capturing unemployment insurance, job search assistance, or general workfare programs aimed at enhancing the job prospects of recipients (Pavoni and Violante, 2007). In the model, these subsidies have the potential to improve overall welfare by closing the consumption gap between the employed and non-employed while simultaneously promoting job search and human capital development.

Our first experiment studies a uniform subsidy given to all job-seekers regardless of age. Surprisingly, we find that such a policy reduces steady-state welfare, even if it narrows consumption gaps and enhances aggregate employment. This result is driven by the college attendance margin. In the model, young individuals must choose between seeking employment or attending college. Job search subsidies inadvertently discourage college attendance among young people, reducing aggregate human capital, consumption, and overall welfare.

In a setup where only the directed search decisions of agents respond to the search subsidy, the model predicts that subsidizing the search of all ages would increase welfare. We demonstrate that by excluding the participation and college attendance channels from our model, predictions regarding the effects of providing this search subsidy are substantially altered. This decomposition analysis aims to highlight the disparities between our model's outcomes and those of a conventional framework. The results of this decomposition yield significant quantitative variations in the model's predictions and unveil qualitatively distinct outcomes and policy implications, underscoring the critical influence of considering the effect of search subsidies on participation and college attendance.

Considering that college enrollment primarily impacts younger cohorts, we also explore age-based subsidies that target job-seekers aged 25-54 and 55+. Both policies result in higher welfare than the baseline, with the 55+ policy delivering the highest welfare. Subsidizing the old is desirable because it is cost-effective. Recall that our inferred search cost distribution is compressed, and older workers are often on the verge of participating or not participating in the labor market. As a result, their labor force participation decisions are sensitive to even small increases in job search subsidies. In contrast, prime-aged workers (25-54) are mostly already participating in the labor market, and their participation decisions are less responsive to subsidies. Therefore, subsidizing only the older cohort achieves the most significant increase in labor participation with the least tax burden on the working population.

Our welfare findings offer new insights into the debate surrounding the desirability of workfare programs, such as those implemented in the US, UK, Canada, and other countries. In general, these programs either overlook the age aspect or target relatively young populations. For instance, the UK's Welfare-to-Work program allocates most of its funding to the New Deal for Young People (NDYP), a program targeting individuals aged 19 to 24 whose unemployment rate was high in the 1990s. Our findings suggest that such a policy could inadvertently discourage college attendance, negatively affecting the economy in the long run. In contrast, subsidizing older populations may be a cost-effective strategy for increasing long-term output and welfare.

Our final experiments focus on how the financial assistance to job seekers, or search subsidy, should vary over the business cycle. In particular, would providing a more generous search subsidy in recessions improve welfare? Our findings demonstrate that such a policy contributes to a negligible decline in welfare relative to the case where the optimal fixed subsidy is provided to agents 55 and older. The main rationale for increasing the subsidy amount in recessions might be to smooth consumption over the business cycle. However, this benefit is relatively small in our model given the modest volatility of aggregate productivity, especially compared to the idiosyncratic job separation risk. Furthermore, the decision to bolster search subsidies during recessions introduces an inefficiency by inadvertently urging workers to intensify their job search efforts during periods when vacancy posting is lower. Our findings suggest that while such a policy may temper fluctuations in output, it also engenders a decline in average aggregate consumption due to the efficiency loss. This, in turn, precipitates negligible, or potentially even negative, shifts in welfare.

Literature Review The paper contributes to the literature on labor market participation over the business cycle. Early works, such as Tripier (2003) and Veracierto (2008), predict that standard RBC models extended with a frictional labor market and a participation margin generate counterfactual labor market outcomes. In contrast, the works by Krusell et al. (2011), Elsby et al. (2015), Krusell et al. (2017, 2020) focus on gross flows in the labor market and find that a model with wealth heterogeneity, along the line of Chang and Kim (2006), can match those flows well. Christiano, Trabandt, and Walentin (2021) find that introducing a labor participation margin into a standard monetary model enhances its performance against business cycle shocks. Unlike these previous studies, this paper emphasizes the role of age and human capital accumulation in driving labor participation decisions, providing a novel perspective in the field. Furthermore, our work generates new policy implications, while previous works primarily focus on positive analysis.

This paper also connects to the literature on college enrollment and labor markets. By examining the sensitivity of college enrollments to business cycles and labor market policy changes, it aligns with empirical works by Dellas and Sakellaris (2003), Barrow and Davis (2012), Barr and Turner (2015), and Long (2015). The structural approach used in the paper allows us to explore how policy variations in the labor market could impact welfare, taking into account their impact on college enrollment decisions.

Finally, the paper's policy analysis is related to research on Welfare-to-Work programs, such as the works of Pavoni and Violante (2007) and Pavoni et al. (2016). Similar to Pavoni and Violante (2007), this paper investigates the influence of human capital on optimal labor-market policy design. However, the paper simplifies the analysis by considering a straightforward subsidy to join the labor force, allowing for a focused discussion on the novel aspects of the model, including life cycles, human capital growth, and endogenous college choices.

2 Empirical Motivation

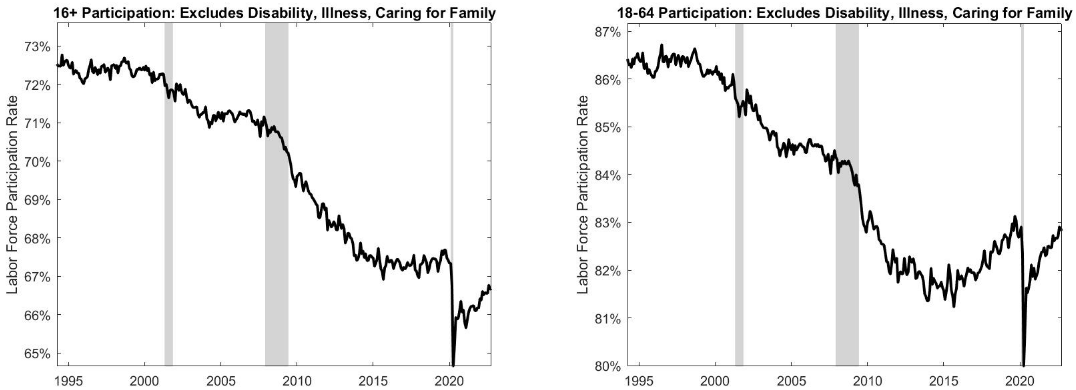
2.1 Labor Participation Across Age and Education

Data on labor force participation in the U.S. is available at a monthly frequency from the Current Population Survey (CPS) starting in January 1976. However, information on the reason why an individual reports being out of the labor force, such as illness or disability, is available only from January 1994 onward. *Figure 2* shows labor force participation after excluding those who choose not to participate due to disability, illness, or because they report caring for their house or family. Even after excluding those who choose not to participate due to family or health concerns, labor force participation dropped notably during the 2020 pandemic recession and exhibits a possibly procyclical behavior over other time-frames as well.

We now turn to study how labor participation varies over the life cycle. *Figure 3* plots the participation rate across different ages and educational attainment groups using CPS data, where we continue to exclude those not participating due to disability, illness, or because they were caring for their house or family.[3] Throughout their entire life cycle, individuals with higher educational attainment demonstrate a higher likelihood of partic-

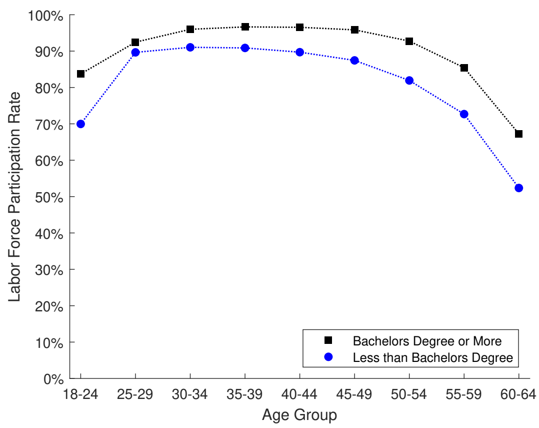
[3]Note that we exclude those who are 65 and beyond, as retirement is not the focus of this paper.

Figure 2: Labor Force Participation Rate



ipating in the labor force. *Figure 3* illustrates that individuals holding a Bachelor's degree or higher are approximately 10 percent more likely to be in the labor force across various age groups. Labor participation begins to decline around the age of 40 for both education groups; however, the rate of decline is more pronounced for non-college graduates than college graduates. At ages 40-44, the difference in the participation rate between the two groups is under 10 percent, but this gap expands to nearly 20 percent by ages 60-64.

Figure 3: Participation by Educational Attainment over the Life-Cycle



2.2 Responsiveness of Labor Force Participation to Aggregate Shocks

This subsection investigates the relationship between participation and aggregate fluctuations. First, we collect data on state-level GDP available from the Bureau of Economic Analysis (BEA), state-level labor force participation, employment, and unemployment data from the Local Area Unemployment Statistics (LAUS) program, and finally, state-level data on job openings, total separations, layoffs, quits, and hires available from the Job Openings and Labor Turnover Survey (JOLTS). All data except state-level GDP

Table 1: Panel Regression with State Fixed Effects

Percentage Change in Labor Force Participation Rate	Coefficient	Robust Standard Error
GDP Growth	**0.0743*****	0.0195
GDP Growth (t-1)	**-0.0021**	0.0117
GDP Growth (t-2)	**0.0056**	0.0072
GDP Growth (t-3)	**0.0198*****	0.0074
GDP Growth (t-4)	**0.0082**	0.0061
Unemployment Rate % Change	**-0.0059*****	0.0010
Unemployment Rate % Change (t-1)	**0.0014**	0.0010
Unemployment Rate % Change (t-2)	**-0.0019*****	0.0006
Unemployment Rate % Change (t-3)	**0.0009**	0.0006
Unemployment Rate % Change (t-4)	**0.0006**	0.0004
Participation Rate % Change (t-1)	**0.127*****	0.0459
Participation Rate % Change (t-2)	**-0.0464**	0.0248
Participation Rate % Change (t-3)	**-0.0054**	0.0181
Participation Rate % Change (t-4)	**-0.0249**	0.0149
Observations	3,150	
R^2	0.2826	

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

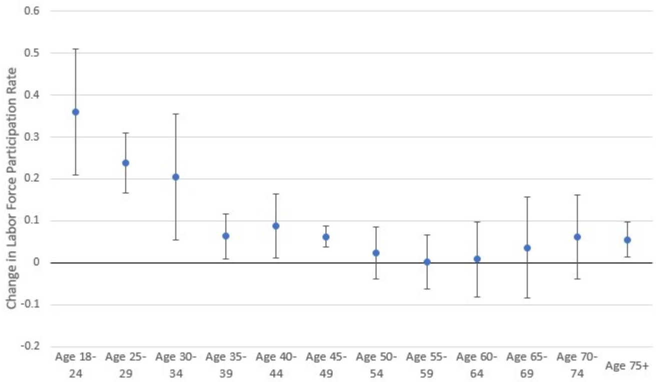
growth is available at a monthly frequency. However, because GDP growth is quite significant as an explanatory variable, we use quarterly averages of these variables to link with GDP growth data, which is available only at a quarterly frequency. *Table 1* shows that positive GDP growth is significantly and positively associated with growth in labor force participation. Positive changes in unemployment are negatively associated with growth in labor force participation. Finally, growth in labor force participation is also significantly and positively associated with its own lagged growth from one quarter in the past.

Labor force participation is responsive to aggregate fluctuations. Additionally, different groups of individuals have distinct levels of opportunity cost associated with labor force participation, which are likely differently affected by aggregate shocks. We investigate how business cycle fluctuations affect different age groups' participation rates. We compute participation rates for twelve age groups using data from the CPS from 2000 through the third quarter of 2022.[4] After estimating a VAR of the participation rate of each group on GDP growth and the past four lags of both GDP growth and participation, it is apparent that the participation of younger and older workers is more responsive to changes in GDP growth than that of middle-aged workers. *Figure 4* displays the change

[4]In this data we exclude non-participants who report not participating due to illness, disability, or because they were taking care of their home or family.

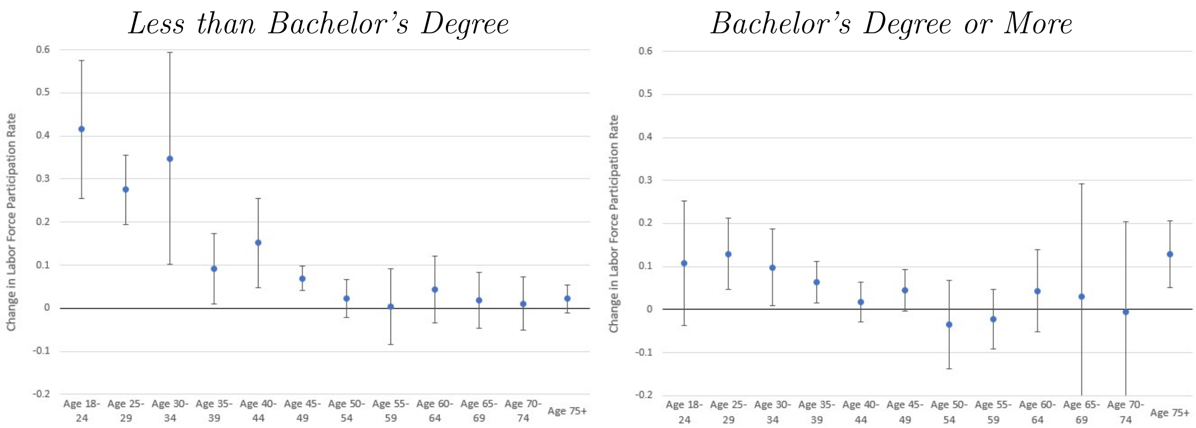
in the participation rate of each group associated with a 1% increase in GDP growth in the same quarter. In other words, the figure displays the immediate (same-quarter) change in participation associated with 1% higher GDP growth, while there are lagged responses to the change not displayed.

Figure 4: Same-Quarter Change in Participation Rate Associated with 1% Higher GDP Growth



Given that the value of participation varies significantly for individuals with different education levels, it is unsurprising that the participation of those without a Bachelor's degree is much more responsive to aggregate fluctuations than those with a Bachelor's or more. *Figure 5* shows the estimated same-quarter change in the participation rate of individuals segmented both by age and education associated with a 1% increase in GDP growth. After controlling for education, we see that it is primarily younger individuals without a Bachelor's degree that account for much of the change in participation following aggregate fluctuations.

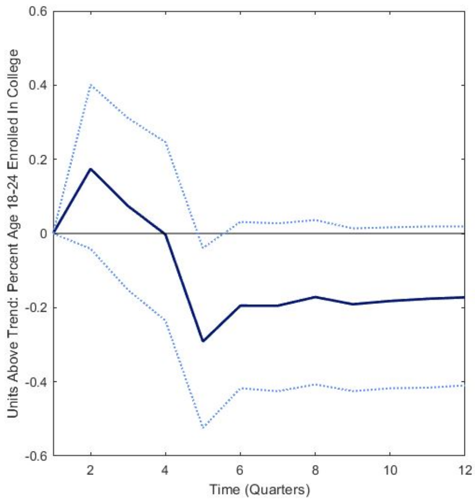
Figure 5: Same-Quarter Change in Participation Rate Associated with 1% Higher GDP Growth by Education Group



2.3 College Enrollment Over the Business Cycle

A modest literature exists estimating the change in college enrollment over the business cycle. The literature generally agrees that enrollment typically increases during recessions. To investigate the responsiveness of college enrollment to business cycle fluctuations, we take the quarterly average of the monthly enrollment rate and combine it with quarterly GDP data.[5] *Figure 6* shows an impulse response function (IRF) estimating the response of the college enrollment rate for individuals aged 18-24 following a 1% *increase* in GDP growth. To compute this impulse response function, we first estimate a bivariate VAR in GDP growth and the 18-24-year-old enrollment rate using data from 1990 through 2019.[6] In estimating this VAR, a lag length of four was selected for the quarterly data. We then use the Cholesky decomposition to separate the variance of the enrollment rate due to changes in GDP growth.[7]

Figure 6: College Enrollment Response to a 1% GDP Growth Increase



[5]Data on college enrollment (including both full- and part-time enrollment) is taken from CPS and spans from January 1990 through December 2022, and this data was seasonally adjusted using the X-13ARIMA-SEATS seasonal adjustment software produced by the U.S. Census Bureau. See census.gov/srd/www/x13as/ for details. Data on U.S. real GDP is from the U.S. Bureau of Economic Analysis GDPC1 series and was obtained from the Federal Reserve Bank of St. Louis FRED site.

[6]The response of college enrollment appeared quite different during the 2020 recession compared to the rest of the sample, possibly due to extraneous factors, including many classes having been moved online. We estimated the same IRF using data through the end of 2022 and found the same general pattern displayed in *Figure 6*, but with significantly wider confidence bands. Because of the extraneous factors that may have altered the enrollment response during the 2020 recession, we limit our data set to pre-pandemic observations when estimating the size of this response.

[7]In *Figure 6*, the dotted lines indicate the 95% confidence bands. These bands were computed using the VAR toolbox created by Ambrogio Cesa-Bianchi in Matlab. The code in the VAR toolbox computes the confidence bands by first using the bootstrap method to generate artificial data starting with observation 5 to the last observation, the first 4 observations (this is the number of lags used) remain as they are in the original data. A VAR is then estimated on this artificial data, and an impulse response is computed. This is done 10,000 times, and the confidence bands are then set to indicate the area that 95% of these 10,000 impulse responses fall within.

The IRF shows that a 1% increase in GDP growth is expected to result in a decrease in the enrollment rate by about 0.3 percentage points after five quarters. There is a lag between the change in GDP growth and the enrollment change, likely due to the time needed to apply before enrolling in college classes. Once it occurs, this estimated change in enrollment is persistent and remains around the same level even 12 quarters following the shock. These results suggest that there is usually a college enrollment channel present over the business cycle, whereby young individuals are more likely to enroll during recessions when the opportunity cost of enrollment is low. They are less likely to enroll during expansions when the opportunity cost is high.

3 Model

3.1 Model Environment

To investigate the labor force participation decision and college enrollment choice over the business cycle for individuals of different ages, we develop an overlapping generations model where agents can be in one of four states. In any period, an agent could be employed, unemployed, out of the labor force attending college, or out of the labor force engaging in no activity. Agents in the model acquire productive skills both through college attendance and through learning on-the-job, where the rate of learning on-the-job is allowed to vary by education group to match differences in wage growth observed in the data. Aggregate shocks in the economy influence individuals' participation and college enrollment decisions, and therefore can have persistent effects on worker productivity.

3.1.1 Setting

Agents with the option of labor force participation for 188 periods, where each period represents one quarter, populate the model. This setup portrays an environment where individuals can participate in the labor force from age 18 until age 65. Every period, a unit mass of agents enter the model one quarter before they turn 18 and decide if they want to search for a job, attend college, or do nothing in the next period when they are 18. Additionally, a unit mass of age 65 agents exit the model each period. In any period, an agent can be in one of four positions; employed, unemployed and searching for a job, attending college, or out of the labor force and not attending college. The model also contains an endogenous mass of firms determined via free entry.

3.1.2 Productivity and Skill Accumulation

Agents in the model are heterogeneous in terms of their age, skill level z, and education level. Each agent's education level affects their skill accumulation on the job. When employed, the output of a worker with skill z_t at time t is $f(Z_t, z_t)$, where Z_t is the level of aggregate productivity. Aggregate productivity follows an AR(1) process where $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. A worker who is not employed enjoys leisure benefit $b(z_t)$.

Let τ denote the education level of each agent, where $\tau = 1$ represents an agent without a college degree and $\tau = 2$ signifies an agent who has graduated from college. All agents enter the model without a college degree ($\tau = 1$) and with $z = 0$. Workers can acquire skill on-the-job, and the rate of skill acquisition depends on their education level. Specifically, if a worker with education τ is employed, their skill level evolves according to the following process.[8]

$$z_{t+1} = \begin{cases} z_t & \text{with probability } 1 - \pi_\tau \\ z_t + \Delta_z & \text{with probability } \pi_\tau. \end{cases} \tag{1}$$

3.1.3 Endogenous College Attendance and Labor Force Participation Decisions

The decision to attend college is endogenous, and agents can attend college at any age. A non-employed worker may pay a fixed cost κ to attend college in the next period. With probability g per period, an agent attending college graduates. Upon graduation, an agent's education level becomes $\tau = 2$, and they receive a one-time skill increase of Δ_g. At the same time, workers also make an endogenous labor force participation decision. A non-employed worker can pay c_w to search in the next period, where c_w is drawn independently log-normal$(\mu_{cw}, \sigma_{cw}^2)$. An agent may not simultaneously attend college and search for a job.

3.1.4 Search

Non-employed agents who paid cost c_w in the preceding period direct their search to a job offering wage w. Firms may post a vacancy at cost c_f. In this environment, a sub-market is a collection of vacancies offering the same wage to workers with the same characteristics. Define sub-market tightness as $\theta \equiv \frac{v}{u}$, where v is the mass of vacancies in a sub-market and u is the mass of agents searching in that sub-market. Free entry of firms decides the tightness of every sub-market in the model. Matches occur according to a constant returns to scale matching function $M(u, v) = \frac{uv}{\left(u^\ell + v^\ell\right)^{1/\ell}}$ as in den Haan, Ramey and Watson (2000). Define the probability that a worker meets a firm as $\frac{M(u,v)}{u} \equiv p(\theta)$ and the probability that a firm meets a worker as $\frac{M(v,u)}{v} \equiv q(\theta)$. Shimer (2005) reports that while the job-finding rate is strongly pro-cyclical, the separation rate is not as responsive

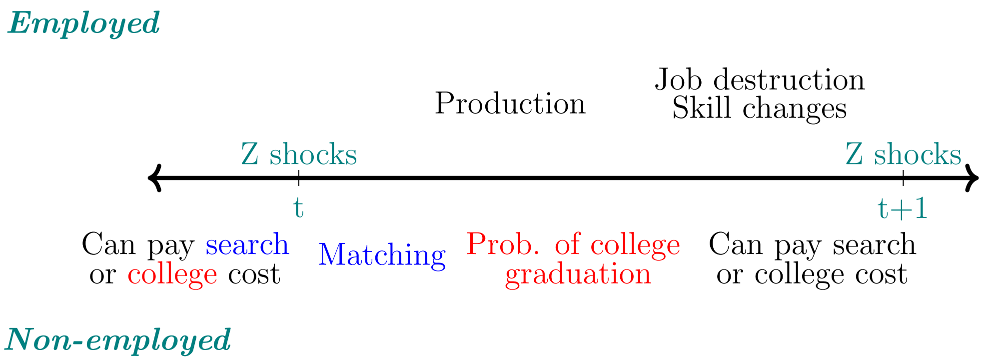
[8] We create a grid of possible z values, where the lower bound of the grid equals 0 and the upper bound equals 4.5. We allow for 150 grid points so that $\Delta_z = \frac{4.5-0}{150-1} = 0.0302$. Note that the parameter estimates π_τ respond to the choice of Δ_z.

to aggregate fluctuations and is just weakly counter-cyclical. Therefore, we assume that once a match is formed, it continues unless destroyed with exogenous probability δ.

3.1.5 Timing

In summary, *Figure 7* illustrates the timing of the model. The top of the timeline lists events as they occur for employed agents, while the bottom lists events as they occur for non-employed agents. Both employed and non-employed agents observe any changes to aggregate productivity Z at the beginning of each period. When non-employed, an agent chooses at the end of the period whether they want to pay κ to attend college in the next period or pay their realization of the random search cost c_w for the chance of being matched with a firm at the beginning of the next period.[9] If the non-employed agent paid the college attendance cost, they graduate from college with probability p_c. Graduating from college gives the agent an immediate skill increase Δ_g and allows them to acquire skill faster on the job. If the agent paid the search cost, they could be matched with a firm and produce in the next period. If the agent chooses not to pay the search or college cost, they remain non-employed and can make this decision again at the end of the next period.

Figure 7: Model Timing



When employed, an agent does not search on the job. However, job-to-job transitions still occur. At the end of each period, agents realize any changes to their skill from learning on the job, and exogenous separations occur with probability δ. An employed agent who loses their job can immediately search and find new employment at the beginning of the next period. If not matched, they follow the same timeline as any other non-employed agent. This assumption that employed agents who lose their job can immediately search

[9]This timing assumption that agents do not bear the search cost c_w and make their directed search decision in the same period greatly simplifies the model. Suppose instead that agents pay c_w and make their search decision in the same period. If they were successfully matched, the utility they receive in the current period would be $u(w - c_w)$, and if they were unsuccessful, they would receive $u(b(z) - c_w)$. The relative value of finding a job then would depend on the amount c_w that the worker must pay so that the agent's directed search decision would depend not only on their skill, age, and the aggregate state, it would also depend on their randomly drawn value of c_w.

and match with a new firm at the start of the next period without paying a search cost simplifies the model in that we only solve for the maximum search cost that an agent is willing to pay when they are non-employed. Recently employed agents may retain more contacts and networking capital from their previous employment and receive a match due to this. This assumption aligns with observations that recently employed workers are more likely to search and remain part of the labor force than those who have been non-employed for more extended periods of time.

3.2 Equilibrium

3.2.1 Value Functions

Before formally defining an equilibrium in this model, we must introduce a few additional pieces of notation. Let $N_a^S(\tau, Z, z)$ denote the value of non-employment when the current aggregate state is Z to an agent of age a, with education τ, and with individual productivity z who is paying cost c_w to be able to search at the start of the next period. The non-employed agent receives leisure/home production benefit $b(z)$, and if after observing the value of c_w drawn independently from log-normal$(\mu_{cw}, \sigma_{cw}^2)$ they decide to pay the cost, they can search for a job at the start of the next period. The value $\widehat{U}_a(\tau, Z, z)$ denotes the value of searching, which is defined by (6).

$$N_a^S(\tau, Z, z) = u(b(z) - c_w) + \beta \mathbb{E}[\widehat{U}_{a+1}(\tau, Z', z')] \tag{2}$$

$N_a^C(\tau, Z, z)$ is the value of non-employment to an agent choosing to pay cost κ to attend college in the next period when current aggregate productivity is Z, and the agent is age a with current education τ and productivity z.[10] The cost κ is constant over time. An agent who attends college graduates with probability g per period. The agent's education type affects how quickly they accumulate skill when employed, and upon graduation, the agent's type becomes $\tau = 2$. Agents also enjoy an immediate increase of Δ_g in their productivity upon graduation. After graduation, the agent is non-employed at the start of the next period, a state where they may choose to search for a job, attend college, or do neither.

$$N_a^C(\tau, Z, z) = u(b(z) - \kappa) + \beta \mathbb{E}\left[(1-g)N_{a+1}(\tau, Z', z) + gN_{a+1}(2, Z', z + \Delta_g)\right] \tag{3}$$

Let $N_a^N(\tau, Z, z)$ denote the value of non-employment for an agent who has decided to neither search for a job nor attend college. The non-employed agent obtains benefit $b(z)$

[10] Notice that all agents may attend college regardless of age or education. An agent who has already attended college enjoys an increase in their skill level of Δ_g upon graduation. However, unlike an agent who has yet to graduate from college, a previous graduate does not experience an increased skill accumulation rate when employed. This makes the value of college attendance significantly higher for agents who have not already graduated from college.

and is generally non-employed at the start of the next period, where they again face the decision to search for a job, attend college, or do neither.

$$N_a^N(\tau, Z, z) = u(b) + \beta\mathbb{E}[N_{a+1}(\tau, Z', z')] \tag{4}$$

Allow $N_a(\tau, Z, z)$ to denote the value of non-employment in aggregate state Z for an agent of age a, education-type τ, and skill-level z. At this stage, the agent chooses, in the same period, between the values presented by equations (2), (3), and (4).

$$N_a(\tau, Z, z) = \max\left\{N_a^N(\tau, Z, z), N_a^S(\tau, Z, z), N_a^C(\tau, Z, z)\right\} \tag{5}$$

The non-employed agent choosing to pay cost c_w is able to search in the next period, giving them value $\widehat{U}_a(\tau, Z, z)$. Based on the aggregate state and the worker's characteristics, a worker searching for wage w will be successfully matched with a job offering that wage with probability $p\left(\theta_a(\tau, Z, z, w)\right)$. If they are not matched with a job, they remain non-employed.

$$\widehat{U}_a(\tau, Z, z) = \max_w \left\{p\left(\theta_a(\tau, Z, z, w)\right) W_a(\tau, Z, z, w) + \left(1 - p\left(\theta_a(\tau, Z, z, w)\right)\right) N_a(\tau, Z, z)\right\} \tag{6}$$

Notice that an unemployed agent choosing the optimal wage to search for faces a trade-off: increasing the wage they search for increases their employment value if they are successful but decreases their probability of success. Because the value of posting a vacancy declines for the firm when the wage increases, firms post fewer vacancies in sub-markets offering higher wages, given the same mass of searching workers and worker characteristics.

Define $W_a(\tau, Z, z, w)$ as the value of employment to an age a worker with education type τ, skill z, and wage w when aggregate productivity is Z. The worker gets utility $u(w)$ from their wage w, and they are separated from their job with probability δ at the end of the period. If they are not separated, they remain employed at the same wage and may experience changes to their skill or to aggregate productivity. If they are separated from their job, they can search at the start of the next period. This assumption that workers can search immediately after employment, without having to pay random cost c_w in the preceding period, assumes that employment gives workers a notion of momentum or connections in the labor market that allows them the ability to search immediately after an employment spell.

$$W_a(\tau, Z, z, w) = u(w) + (1 - \delta)\beta\mathbb{E}\left[W_{a+1}(\tau, Z', z', w)\right] + \delta\beta\mathbb{E}\left[\widehat{U}_{a+1}(\tau, Z', z')\right] \tag{7}$$

The free-entry condition given by (8) summarizes the vacancy posting decision of firms in the model. Firms post vacancies in every sub-market until the cost of posting a vacancy (c_f) equals the expected benefit from posting a vacancy. Here the expected benefit is the value of employing a worker in the sub-market $J_a(\tau, Z, z, w)$ multiplied by the probability of matching with a worker $q\left(\theta_a(\tau, Z, z, w)\right)$. In sub-markets where vacancies are posted, free entry implies that firms will take advantage of any arbitrage opportunities until (8) holds with equality. Firms post no vacancies in sub-markets where the cost of posting a vacancy exceeds the expected benefit.

$$c_f \geq q\left(\theta_a(\tau, Z, z, w)\right) J_a(\tau, Z, z, w) \qquad \forall\ a, \tau, Z, z, w \tag{8}$$

$J_a(\tau, Z, z, w)$ denotes the value to a firm of employing an age a worker of education type τ with skill z earning wage w when aggregate productivity is Z. The worker and firm together produce $f(Z, z)$, and the firm pays wage w to the worker. The job is destroyed with probability δ, leaving the firm with nothing. With probability $(1-\delta)$, the match remains intact into the next period, where the worker is one quarter older, and there may be changes to the aggregate state and the worker's skill.

3.2.2 Definition of Equilibrium

Definition 1: *A Recursive Equilibrium (RE) is given by:*

1. *Value functions* $\left\{N_a^S(\tau, Z, z), N_a^C(\tau, Z, z), N_a^N(\tau, Z, z), N_a(\tau, Z, z), \widehat{U}_a(\tau, Z, z), W_a(\tau, Z, z, w), J_a(\tau, Z, z, w)\right\}$
2. *Equilibrium market tightness function* $\{\theta_a^U(\tau, Z, z)\}$ *solves the workers' search problem*
3. *Optimal search and college attendance rules* $G_a(\tau, Z, z)$ *and* $C_a(\tau, Z, z)$
4. *Aggregate transition probabilities consistent with policy functions and stochastic* Z *process.*

We can also define a Block Recursive Equilibrium in this environment.

Definition 2: *A Block Recursive Equilibrium (BRE) is a RE where value and policy functions are independent of the aggregate distributions of agents across states.*

Menzio, Telyukova, and Visschers (2016) prove that a unique BRE exists, and that there is no other RE for this type of model.[11] This provides great convenience for obtaining the solution of the model when quantifying it. We now turn to our quantification strategy.

[11] See Theorem 1 in Menzio, Telyukova, and Visschers (2016). This theorem extends this result in Menzio and Shi (2011) to an environment where workers are heterogeneous in terms of age and productivity characteristics.

4 Quantification

4.1 Calibration

In solving the model outlined in Section 4, we assume $u(\cdot) = \ln(\cdot)$, $f(Z_t, z_t) = Z_t + z_t$, and $b(z_t) = b_c + z_t$. We approximate the AR(1) process followed by aggregate productivity (Z_t) as an N-state Markov chain following Tauchen (1986).[12] In addition to the calibrated parameter values recorded in *Table 3*, *Table 2* lists other assigned parameter values. The quarterly discount factor (β=0.99) corresponds to a 4% risk-free annual interest rate, while the autocorrelation of aggregate shocks (ρ) coincides with a common value in the literature chosen in den Haan et al. (2000), Hansen and Wright (1992), among others.

Table 2: Assigned Parameters

Parameter	Description	Value
β	Discount factor	0.99
ρ	Autocorrelation of aggregate shocks	0.95

We calibrate the remaining twelve parameters recorded in *Table 3* to match twelve relevant moments. The first[13], third[14], fourth [15], fifth[16], and ninth[17] moments in the table are calculated as averages over 2015 through 2019 using data from the Federal Reserve Bank of St. Louis FRED site.

[12] N=5 unless otherwise specified.

[13] U.S. Bureau of Labor Statistics, Unemployment Rate - 20 Yrs. & over [LNS14000024], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/LNS14000024

[14] Calculated using U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 25 to 54 years [LEU0252887900A] and U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 20 to 24 years [LEU0252887100A]

[15] Calculated using: U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: Bachelor's degree and higher: 25 years and over [LEU0252918500Q] and U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: High School graduates, no college: 25 years and over [LEU0252917300Q]

[16] Calculated using: U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 55 to 64 years [LEU0252890900A] and U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 25 to 54 years [LEU0252887900A]

[17] Calculated using U.S. Bureau of Labor Statistics, Employment-Population Ratio - Bachelor's Degree and Higher, 25 Yrs. & over [LNS12327662] and U.S. Bureau of Labor Statistics, Employment-Population Ratio - High School Graduates, No College, 25 Yrs. & over [LNS12327660]

Table 3: Calibrated Parameter Values and Empirical Targets

Parameter	Estimate	Targeted Moment	Data	Model
ℓ	1.8709	Unemployment rate (%) for age 20+	4.035	3.939
κ	0.5606	Pop. age 25+ with Bachelor's degree (%)	31.920	31.972
Δ_g	0.1501	Wage ratio: prime age to 20-24	1.646	1.360
π_2	0.3689	College wage premium (age 25+)	1.814	1.785
π_1	0.0865	Wage ratio: 55-64 to prime age	1.086	1.344
c_f	0.8735	Vacancy posting rate	4.153	4.248
μ_{cw}	-0.2411	Age 20-64 participation rate (%)	87.140	87.650
σ_{cw}	0.3985	Prime age/60-64 participation rate	1.500	1.581
b_c	0.8057	College to non-college employment rate ratio	1.316	1.112
δ	0.1042	Monthly separation rate	0.036	0.036
g	0.0357	College graduation rate over 4 yrs	0.441	0.441
σ_ϵ	0.0027	Participation change w/ 1% shock: ages 18-24	0.360	0.359

We obtain the percentage of the population aged 25 and older with a Bachelor's degree or more as an average over 2015 through 2019 using data from the American Community Survey (ACS) 1-year estimates. The Job Openings and Labor Turnover Survey (JOLTS) data reports the vacancy posting and monthly separation rates. Specifically, the sixth moment is the total nonfarm, seasonally adjusted average job openings rate reported from 2015 through 2019. The monthly separation rate is the average nonfarm, seasonally adjusted total separation rate over these same years. Notice that the JOLTS total separation rate includes involuntary separations along with quits, retirements, and transfers to other locations. This separation rate most closely matches the separations occurring in the model because the agent can immediately search and become employed at the start of the next period following a separation, mimicking quits and relocations to other jobs.

The National Center for Education Statistics reports the college graduation rate over four years, most recently for 2015-2016.[18]. Finally, we calculate the age 20-64 labor force participation rate and prime-age to age 60-64 participation rate ratio using data from the Current Population Survey (CPS). For these moments, individuals who reported non-participation due to illness, disability, or because they were taking care of their home or family were excluded, as in *Figure 2*. The final moment in the table corresponds with the same-quarter change in the participation rate of 18-24 year-olds associated with 1% higher GDP growth reported in *Figure 4*.

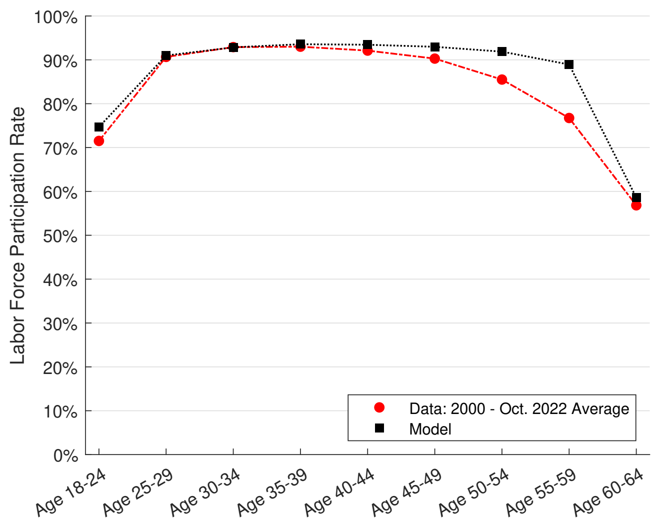
[18]See https://nces.ed.gov/fastfacts/display.asp?id=569

4.2 Model Predictions

To assess whether ours is a reasonable model of labor force participation over the life cycle, we examine what it implies for some key non-targeted moments regarding labor market dynamics and college attainment statistics.

A key characteristic of the data that the model captures is the profile of labor force participation by age. *Figure 8* plots the participation rate for each age group in the data and from the model. The participation rate forms an inverted u-shaped pattern over the life-cycle, with participation at its highest from around age 25-54. At the beginning of the life cycle, the benefit of education is at its greatest. The model only captures the relatively lower participation rate among 18-24 year-olds by including the endogenous college attendance decision. As individuals approach age 65 in the model, the expected benefit of paying a search cost to re-enter employment declines. If a worker can successfully search and find a job, the time they can remain at that job is limited. Therefore, individuals are less likely to re-enter the labor force after a job loss as they approach age 65 in the model. The same effect seems to be present in the data, but it appears to set in sooner, perhaps due to differences in the age that individuals choose to retire or early retirements due to health or family circumstances.

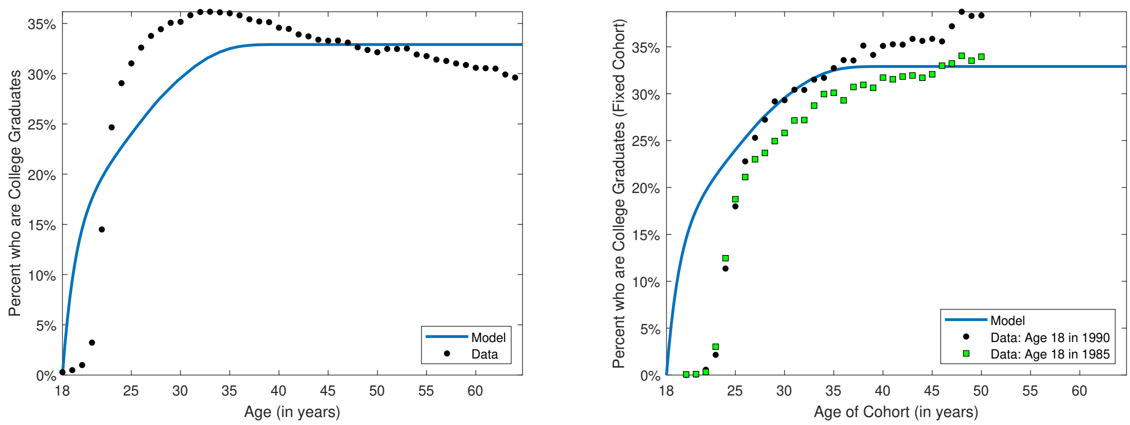
Figure 8: Participation Rate by Age Group: Model vs. Data



Individuals in the model can attend college at any age, but the benefit is greatest when they are young. *Figure 9* displays the fraction of the population who are college graduates by age. Recall that while the total fraction of the 25+ population who are college graduates is a targeted moment, the age profile of college graduates that the figure displays is untargeted. The data used in this figure is the average for each age from 2000 to 2022 from the monthly CPS. College attendance and graduation is most common among those aged 18-30, and after around age 30, the percent of the population who are college graduates levels off. The data also captures changes in college attendance over time. Individuals became more likely to attend and graduate from college as time progressed. This leads to the decline in the percentage of the population who are college graduates after around age 30 in the data.

Figure 9: Percent of Population who are College Graduates by Age

2000-2022 Average over Cohorts | *Following 1985 and 1990 Cohorts*

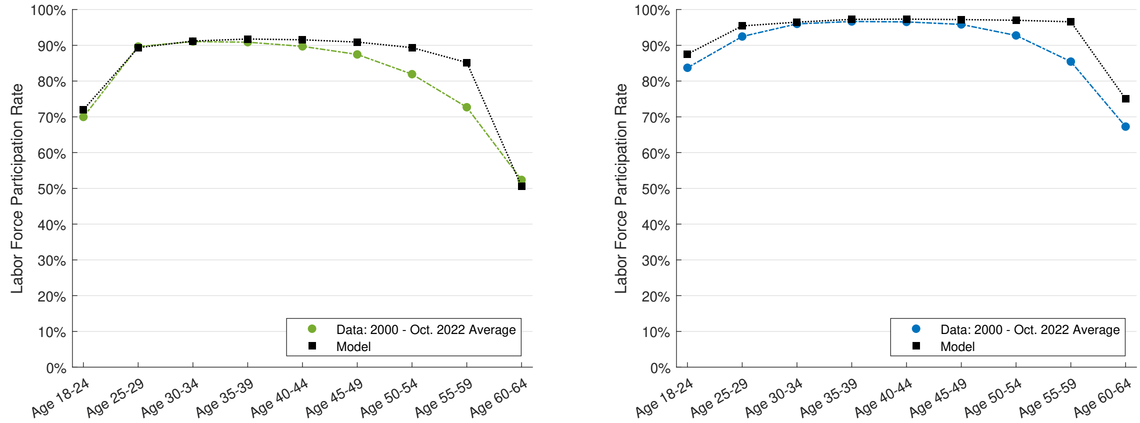


We also compute how the enrollment of agents aged 18-24 responds to a positive aggregate shock lasting for one period, resulting in a 1% increase in model GDP. We find that in the model simulation, the 18-24 enrollment rate decreases by 0.35 percentage points in the period of the positive aggregate shock before moving back toward its steady-state value. This model prediction is close to the impulse response prediction in *Figure 6*, which showed a 0.30 percentage point decrease in the enrollment rate of this age group following a positive shock of the same size. The main difference in the model response compared to the data is the response timing. In the impulse response estimate from the data, the enrollment rate drops about four quarters after the shock, likely due to the time gap between applying and becoming a student.

Previously, the data displayed in *Figure 3* showed that the participation rate among college graduates is higher than that of non-graduates for each age group. *Figure 10* reveals this same feature is also present in the model. Although the total participation rate and prime-age to age 60-64 participation rate ratio were targeted moments, the overall participation by age profile and the differences among college graduates and non-graduates are untargeted. While the model still overestimates participation among 55-59 year-olds when controlling for education, the overall shape of the participation by age profile and the difference in the level of participation between college graduates and non-graduates match the data closely. In particular, it successfully matches the empirical feature that labor participation declines faster for non-college graduates compared to college graduates. This is due to the endogenous human capital channel. In the model, non-college workers experience higher non-employment rates because of lower human capital. Due to a lack of employment opportunities and on-the-job learning, their human capital grows at a slower rate compared to the college graduates (see *Figure 12*, left panel). Hence their participation rate declines at a faster rate as they age.

Figure 10: Participation Rate by Age Group and Education: Model vs. Data

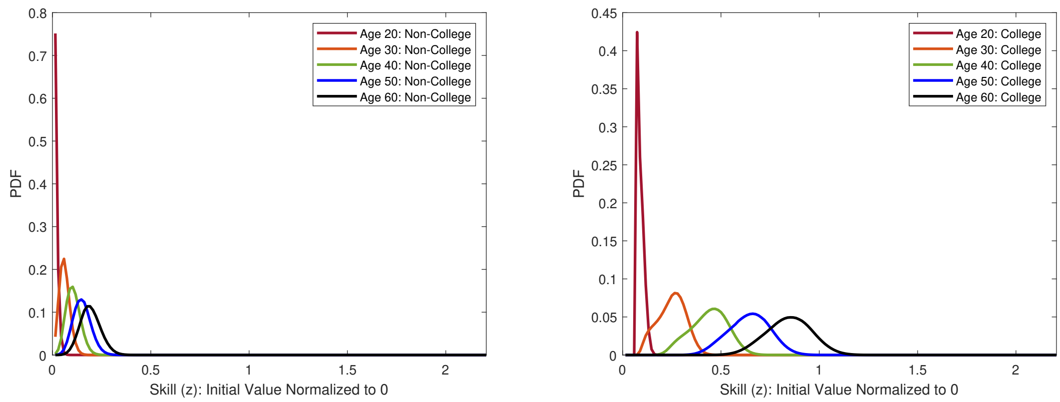
Less than Bachelor's Degree | *Bachelor's Degree or More*



Agents in the model all start with a skill level (z) equal to zero. They then can acquire skill either through learning on the job or through college graduation. Recall that college graduation leads to an immediate increase in skill by the amount Δ_g and allows agents to acquire skills more quickly on the job. *Figure 11* displays the skill distributions across different ages for college graduates and non-graduates. The initial increase in skill gained from college graduation allows even young college graduates to have a higher average z than non-graduates. As both groups age, college graduates acquire skills more quickly on the job leading to larger differences in average skill among agents of the same age.

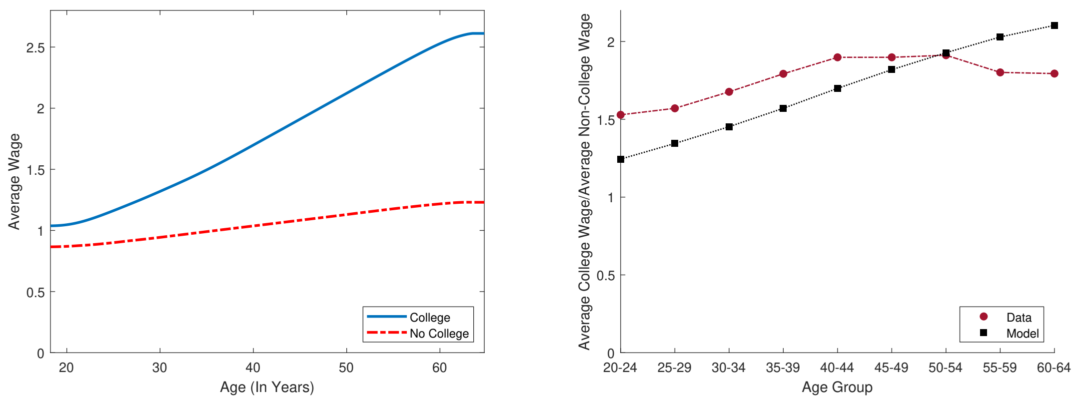
Figure 11: Skill Distributions

Less than Bachelor's Degree | *Bachelor's Degree or More*



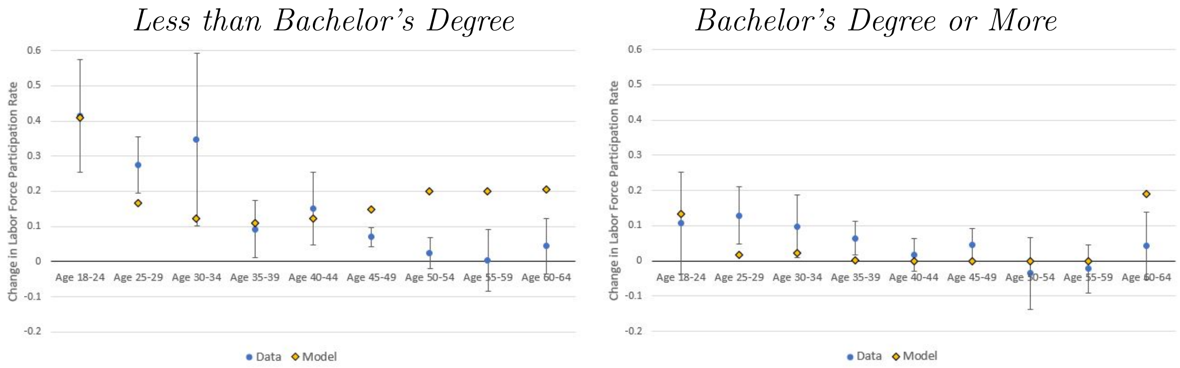
The assumption that college graduation has both an immediate and longer-term impact on worker productivity allows us to more accurately match the wage-age profile for college graduates and non-graduates. The data shows that there is an immediate college wage premium, even among those aged 20-24 who would have just recently graduated. Although the model slightly underestimates the college wage premium for all age groups, as previously indicated in *Table 3*, the initial increase in skill of Δ_g upon college graduation allows the model to capture the immediate college wage premium enjoyed by younger workers. Additionally, in both the data and model, the college wage premium is increasing with age. In other words, on average, those with a bachelor's degree or more experience faster wage growth throughout their lives. The model can capture this feature of the data because the probability of skill increase on the job is allowed to differ by education group (leading to different estimates of π_1 and π_2), and these different probabilities target the average wage growth among each education type.

Figure 12: Wage Profiles by Age and Educational Attainment



Given that the model can capture relevant steady-state features of the data, we turn to look at how well it captures the responsiveness of labor force participation to aggregate shocks. *Figure 13* displays how the labor force participation rates of different age groups in the model respond to a one-period shock to aggregate productivity (Z), resulting in a 1% increase in output produced from employment (GDP). While the overall responsiveness of those aged 18-24 (not conditioning on educational attainment) is targeted in the calibration, all points displayed in *Figure 13* are untargeted. Just as in the data, we see that the participation of those without a Bachelor's degree is more responsive to aggregate productivity changes.

Figure 13: Same-Quarter Change in Participation Rate Associated with 1% Higher GDP Growth: Model vs. Data by Education Group



In both the data and model, the participation of younger and older workers is most responsive to changes in aggregate productivity. Workers beyond age 30 are very unlikely to attend college and effectively choose between two options when non-employed: unemployment and non-participation. However, younger workers get a greater lifetime return from college attendance and choose from three options when non-employed. When the value of searching for a job increases, as in cases where there is a positive aggregate shock, young agents substitute away from the college attendance option into labor force participation. Older workers benefit less from search due to the horizon effect discussed previously, but this effect is lessened when there is a boost to aggregate productivity and thereby their expected return from employment.

4.3 Participation Response to Large Aggregate Shocks: A Comparison with the 2020 Recession

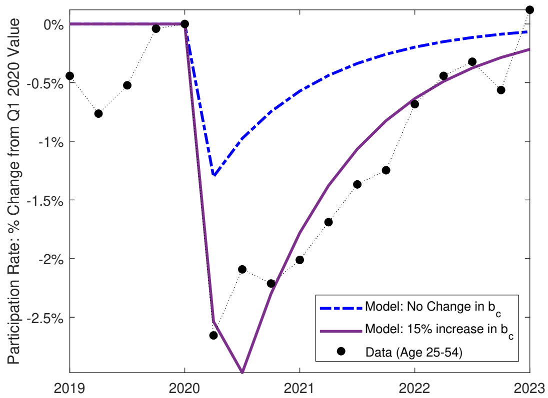
We now examine the ability of the model to replicate salient features of the 2020 pandemic recession, particularly the dynamics of labor participation across different ages. The 2020 recession saw a sharp 8.48% drop in real GDP, which *Figure 1* illustrates had lasting effects on labor force participation. Here we investigate how well the model mimics the relative differences in participation observed across age groups following the shock.[19] First, we consider a negative one-period shock to aggregate productivity large enough to result in an 8.48% decline in model GDP. We plot the participation response of each age group to this shock to aggregate productivity (Z) alone in *Figures 14* and *15*. The figures show that when there is only the shock to aggregate productivity, the decline in participation is not as dramatic as reported following the 2020 recession. To match the approximate magnitude of the change in participation observed in the data, we introduce a simultaneous one-period increase in the leisure value b_c (recall the total leisure value

[19] For these model experiments, we use a 7-state Markov chain approximation of the AR(1) process that aggregate productivity follows, increasing the accuracy of the approximation relative to the steady-state results previously presented.

an agent recieves when non-employed is $b(z) = b_c + z$).[20] We assume that during the period of the shock, agents act as if they believe the change in b_c to be permanent. We find that a one-period 15% increase in b_c along with an aggregate shock resulting in an overall 8.48% reduction in model GDP results in participation responses that match the data reasonably well.

Figure 14 plots the percent deviation of prime-age participation from February 2020 in the data. The figure also displays the model response of prime-age participation when there is an 8.48% drop in GDP due only to an aggregate shock and no change in b_c, and the same response when there is an immediate one-period 15% increase in b_c.

Figure 14: Prime Age (25-54) Participation Response



Both the initial decline in prime-age participation and the persistence of its response match the data well.

Figure 15 displays the participation response of agents aged 20-24 and 55 and older to exactly the same shocks. One could consider that agents of different ages saw different changes in b_c following the 2020 recession and could determine the series of shocks needed to match the exact responses of each age group. For example, one could assume that b_c remained lower for older workers for much longer due to health risks imposed by working during and following the pandemic. Although these considerations could be reasonable, we instead show that even when all agents face exactly the same shocks, individuals of different age groups have different participation responses in the model that match what was observed in the data reasonably well.

[20]The change in the value of b_c can be motivated in different ways. A direct interpretation of the increase in b_c is the increased generosity of unemployment insurance. However, because b_c does not enter into the government budget constraint, we are more inclined to consider it as capturing workers' changing taste for work-life balance or the benefit obtained from avoiding illness at work.

Figure 15: Younger and Older than Prime Age Participation Response

Age 20-24 Response *Age 55+ Response*

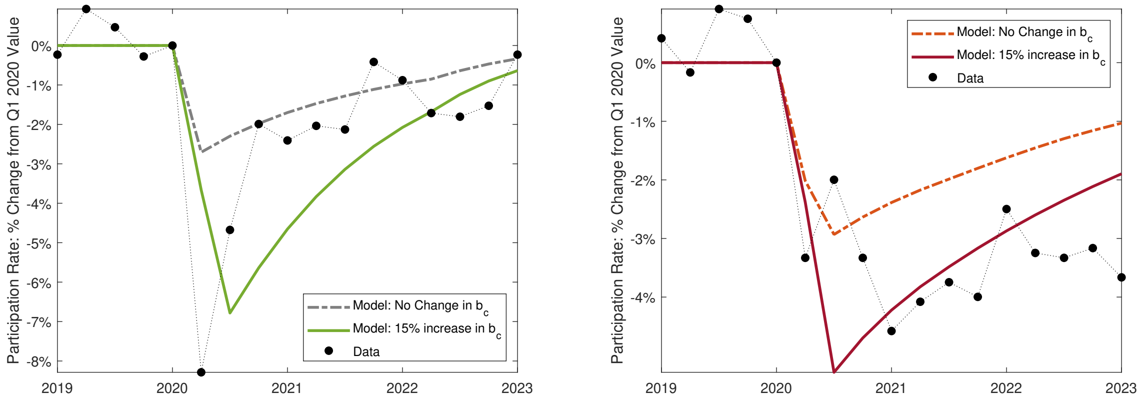
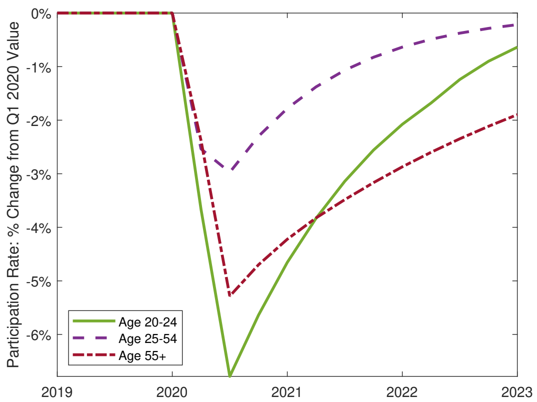


Figure 16 presents the labor participation response for different age groups, comparing the model to the data. The model successfully reproduces the magnitude of the downturn and rebound for each group. Specifically, it aligns with several key aspects of the data: the prime-aged population experienced the least severe decline, while younger and older age groups faced more substantial drops, with the youngest group being hit the hardest on impact. Regarding the pace of recovery, the older age group exhibits the slowest recuperation among all three groups (see right panel of *Figure 1*).

Figure 16: Relative Participation Responses



5 Optimal Subsidy over Life and Business Cycle

Given that our quantitative model successfully replicates various aspects of the labor participation profile and its responses to business cycle shocks, we now use our model as a lab to conduct policy analysis. We examine the efficiency of labor force participation choices by introducing a simple subsidy that incentivizes workers to search for jobs. This subsidy can be seen as capturing various components of workfare programs implemented in different countries, such as the "New Deal" introduced in the UK in 1998. These programs typically provide unemployment insurance, assisted job search, and/or subsidized jobs to encourage workers to join the labor force. Our subsidy represents these elements in a parsimonious manner, allowing us to focus on how such policies should be designed over the life and business cycle.

In this model, the subsidy has the potential to enhance overall welfare by fulfilling two roles: providing public insurance and addressing a fiscal externality associated with job-seeking activities. In the model, with no private insurance market available for non-employment risk, the consumption of non-employed individuals is lower than that of their employed counterparts. This creates an opportunity for the government to intervene and offer social insurance. The subsidy would be especially valuable to the unemployed, given that they bear an out-of-pocket cost of job searching as well as the job-search risk.

Furthermore, when workers engage in job search, they create a positive externality: finding a job increases the government's total tax revenue. As a result, the government can set a lower tax rate to balance its budget, benefiting all other workers. Hence, a worker's job-seeking behavior positively impacts other workers through the government's budget constraint. By subsidizing job search efforts, the government can correct this fiscal externality and enhance overall welfare.

We begin by analyzing these search subsidies in a steady-state context. These experiments can be viewed as capturing the long-term desirability of such policies. Our key finding is that providing subsidies to younger individuals could harm long-term welfare by reducing college enrollment, thereby negatively affecting human capital accumulation. We also find that offering subsidies to older individuals is the most cost-effective way to improve overall long-term welfare.

Next, we study how the search subsidy should vary over the business cycle. We find such a policy responding to business cycles yields negligible welfare gain in comparison to its steady-state counterparts. In the model, household income is subject to two types of risk: an aggregate productivity shock Z and an idiosyncratic separation shock δ. Given the relatively modest volatility of the productivity shock in contrast to the separation shock, it is more important for the government to offer insurance against the latter rather than the former.[21] Our steady-state policy already addresses the idiosyncratic risk, implying a considerably smaller window for welfare gain when adjusting the policy based on the business cycle. Moreover, an economic inefficiency arises when a more generous search subsidy is provided during a recession, as agents are incentivized to search at times when search is least productive. Firms post fewer vacancies during recessions and the

[21] As table 3 illustrates, the idiosyncratic separation risk is substantial in the model: there is a 10 percent probability (δ) the workers may lose their job, resulting in a potential drop in consumption by as much as 20 percent (b_c). The standard deviation of aggregate productivity shock (σ_ϵ), on the other hand, is 0.0027, a rather low number.

job-finding rate is lower, so agents are encouraged to pay a search cost when the benefit of doing so is lower (and during booms, they are not as encouraged to search when the benefit is higher).

5.1 Steady-State Effects of Subsidizing Search: Role of Age Dependence

In this section, we study policy implications at the steady-state, focusing on the effects of age-based subsidies for search. Specifically, suppose that the government pays a lump-sum subsidy s to unemployed individuals in the economy if they search for a job. A proportional income tax on wages offsets the cost of this subsidy and balances the government's budget. With tax rate τ, an employed individual earning wage w receives after-tax income $w(1-\tau)$. We consider the effects of providing this subsidy for search when the subsidy is paid to all individuals, only to those 18-24, only prime age (25-54), and only to age 55+. In each scenario, taxing employed individuals of all ages funds the subsidy. We ensure that the corresponding tax rate balances the government's budget in the risky steady-state.[22]

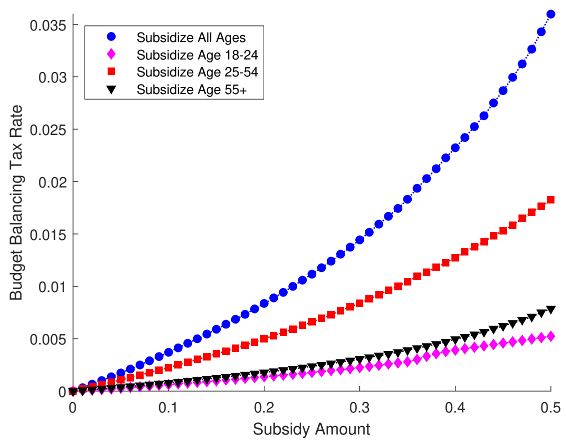
Let $\underline{a_s}$ be the lowest age for which the government subsidizes for search, and let $\overline{a_s}$ be the highest age that receives the subsidy. Let $u(a)$ be the total mass of agents unemployed of age a in the risky steady-state, and let $e(a,w)$ be the mass of agents employed at age a earning wage w. Then, the total cost of the subsidy is equal to the subsidy amount multiplied by the mass of agents receiving the subsidy $\left(s \sum_{a=\underline{a_s}}^{\overline{a_s}} u(a)\right)$. The tax revenue from the proportional tax on wages equals the sum of the tax rate multiplied by the wage over the total mass of agents employed at each wage. Therefore, for any subsidy, the corresponding tax rate τ must satisfy the following government budget constraint.

$$s \sum_{a=\underline{a_s}}^{\overline{a_s}} u(a) = \sum_{a} \sum_{w} \tau w e(a,w) \tag{9}$$

Figure 17 displays the tax rate on wages (τ) needed to offset the cost of the subsidy.

[22]This is the same risky steady-state concept described by Coeurdacier et al. (2011).

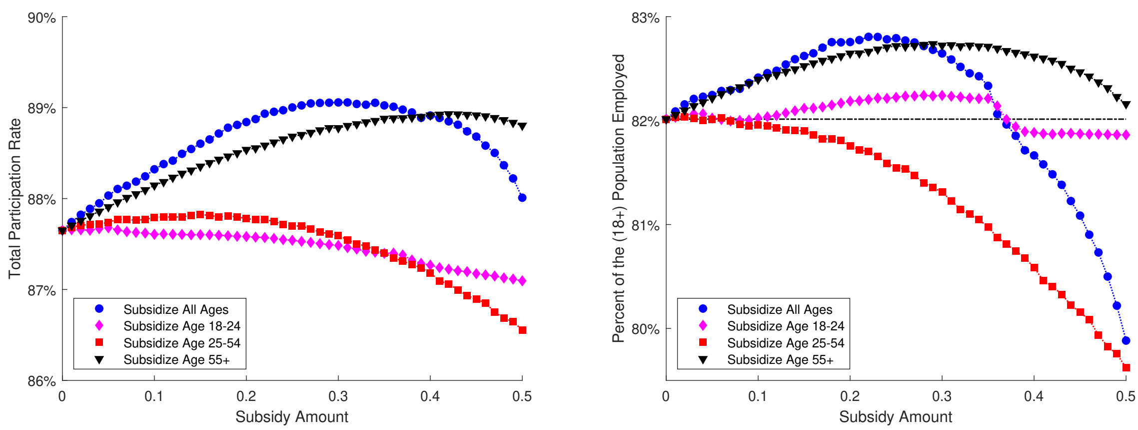
Figure 17: Budget Balancing Tax Rate



Increasing the subsidy on search requires a higher tax rate τ to balance the government's budget and can result in two opposing effects on incentives. First, paying a subsidy to those who search increases the incentive to search. However, because higher subsidies result in higher taxes when employed, this reduces the benefit of finding employment. *Figure 18* displays the effects subsidizing different age groups has on the labor force participation rate and the total percentage of the population who are employed. In both cases, small subsidy amounts can increase participation and employment. However, because higher subsidies must be offset by higher taxes on the employed, eventually very high subsidies can reduce participation and employment.

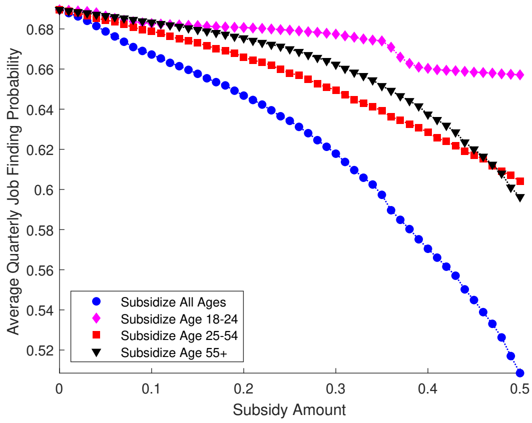
Figure 18

Participation Rate *Employment Rate*



Because agents direct their search, one side-effect of subsidizing search is that agents are willing to spend more time in unemployment. Recall that agents in the model trade-off searching for higher wages with a lower job-finding probability, and higher wage jobs take longer to find, holding all else constant. When unemployment is relatively more attractive, agents optimally spend more time searching.

Figure 19: Average Quarterly Job-Finding Probability



The effect of subsidizing search on total output is determined not only by the mass of agents that enter employment, but also by the productivity of those agents. In the model, non-employed agents choose between search, attending college, and engaging in no activity. Encouraging search by providing a subsidy makes search relatively more attractive than pursuing education. *Figure 20* illustrates how subsidizing search affects both the percentage of agents who choose to attend college and the total skill accumulated by the end of the life cycle. The left-hand side of *Figure 20* shows that generally, subsidizing search makes the option of college attendance less attractive and results in a reduction in educational attainment. However, subsidizing the search only of agents age 55 and older leads to a very slight increase in the percentage of the 25+ population who are college graduates.

Figure 20: Effects of Search Subsidy on Educational Attainment and Life-Time Productivity

Percent College Graduates Among Age 25+ *Percent Change Avg. Age 60-64 Skill*

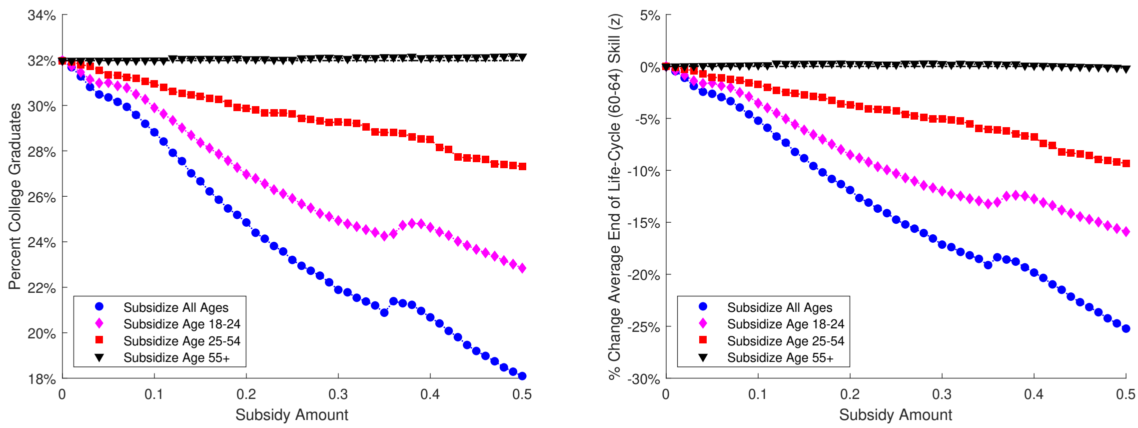
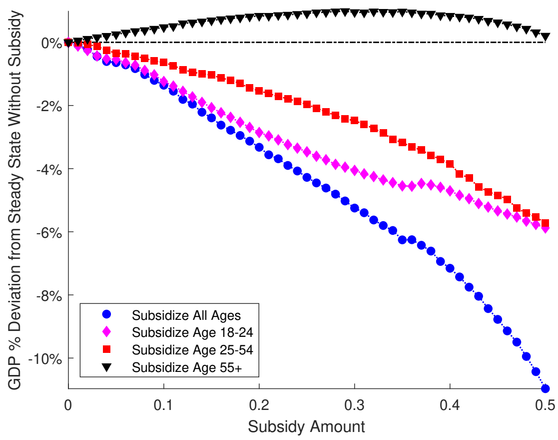


Figure 9 showed that in both the data and model, individuals largely make their college attendance choice when young, and most individuals who graduate college do so by age 30. Subsidizing the search only of those 55 and older does not make search relatively more attractive compared to college attendance during the time when individuals benefit most from college attendance. Subsidizing this older demographic does generally extend employment and participation in expectation to older years and, therefore, slightly increases the expected benefit of college attendance. Each subsidy's impact on college attendance has an almost symmetric influence on average skill near the end of the life-cycle, computed as the average skill (z) among those aged 60-64.

Subsidizing search can generally increase participation and employment at low subsidy levels. However, these subsidies disincentivize college attendance unless provided only to those closer to the end of the life-cycle, and otherwise lead to a decline in individual productivity. These two effects, generally increasing employment but decreasing individual productivity, simultaneously determine how the subsidy influences total GDP in the economy, measured as output produced from employment. *Figure 21* shows that generally subsidizing search reduces production from employment because although smaller subsidy amounts increase employment, they also immediately start to disincentivize college attendance.

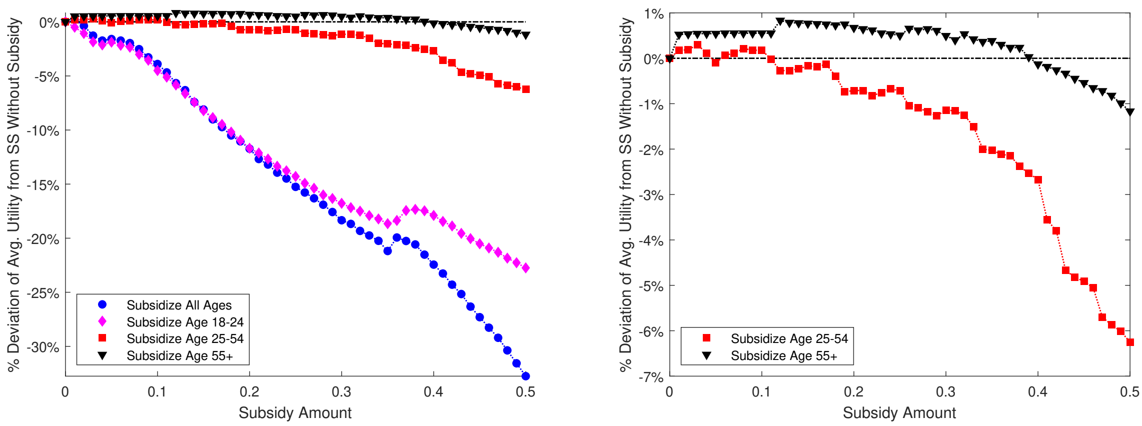
Figure 21: GDP: Production from Employment



However, because subsidizing the search only of agents 55 and older encourages employment while not discouraging and even slightly increasing college attendance, the economy experiences a boost in GDP when subsidizing the search of this oldest age group for modest subsidy amounts.

Finally, we investigate how subsidizing the search of different age groups influences average utility. *Figure 22* shows that only age-targeted subsidies to prime-age and older individuals lead to steady-states with higher average utility.

Figure 22: Average Utility



The left-hand side of *Figure 22* shows that subsidizing search among individuals of all ages and those aged 18-24 reduces average utility for all nonzero subsidy amounts. This reduction is due to the crowding-out effect of subsidizing search on college enrollment. *Figure 20* showed that subsidizing search for all ages and agents 18-24 led to immediate and sharp declines in the percentage of the population who are college graduates and average skill at the end of working life. The right-hand side of *Figure 22* depicts welfare gains when subsidizing the search of prime age and age 55 plus individuals for relatively small subsidy amounts. Increasing subsidies necessitates higher taxes on the employed, and eventually average utility starts to decline at high subsidy amounts.

The result from *Figure 22* that subsidizing the search of all individuals reduces average utility may be initially surprising, especially given that many models where the participation and college decisions are not endogenized find a welfare-increasing role for search subsidies. In these models with directed search, risk-averse agents, and incomplete insurance markets, a search subsidy makes unemployed workers willing to spend more time searching, so they direct their search to decrease their probability of moving into employment. Despite this negative side effect, a reasonable search subsidy can still increase welfare by smoothing agents' consumption as they move between employment and unemployment. However, these models generally do not consider the effect of subsidizing search on participation and schooling decisions. To further investigate this result, we compare the predictions of our full model with a version of our model where participation and college attendance policy functions are fixed to the case where there is no subsidy. This version of the model considers the effects of subsidizing the search of all ages when only the (directed) search policy functions react to the subsidy. *Figure 23* displays the resulting predictions.

We find that when our model only considers how agents' directed search choice is impacted by the subsidy but not their participation and college attendance choices, we underestimate the budget-balancing tax rate, especially for higher subsidy amounts. Additionally, when we do not allow the participation and college attendance policy functions to react to the subsidy, the model predicts that subsidizing the search of all ages can substantially raise average utility. These results demonstrate that considering the subsidy's effects on participation and college attendance not only significantly impacts the model's quantitative predictions, it also results in qualitatively different results and policy recommendations.

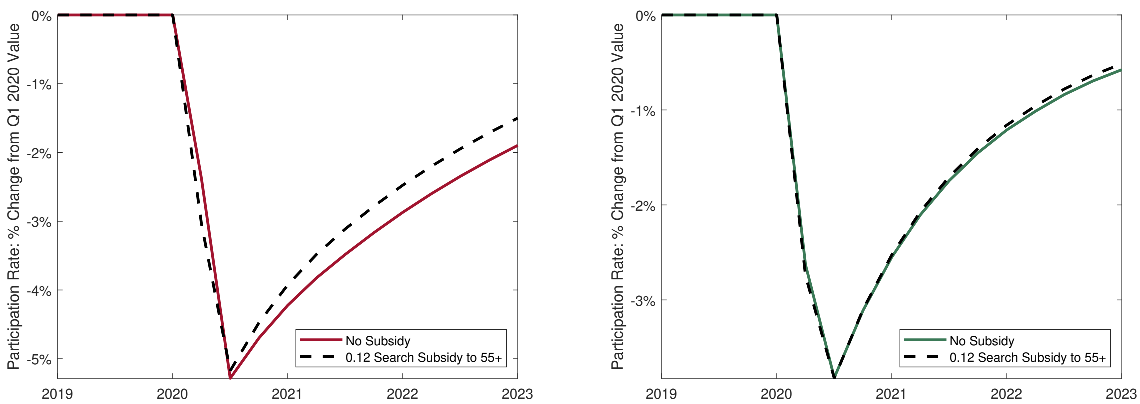
Figure 23: Effects of Search Subsidy given to all Ages: Comparison of Full Model and Model Where Only Directed Search Decision is Endogenized

Although the version of our model where only the directed search decisions of agents react to the subsidy predicts the same qualitative effects on the average job-finding rate and production on-the-job, it significantly underestimates the decline in each of these in response to larger subsidy amounts. As discussed in Section 4, with directed search, agents face a trade-off between job quality and the probability of finding a job. Better jobs take longer to find on average. When unemployed agents are subsidized, they are willing to spend more time searching, so they search for higher wages and their quarterly job-finding probability declines. This is true in both versions of the model, but the effect is significantly stronger in the full model, where the percentage of agents who graduate from college declines with larger subsidy amounts. The same is true regarding the predicted response of GDP. Although GDP declines when only the search decisions of agents are affected by the subsidy, as agents spend more time in unemployment, the decline is much larger in the full model due to the drop in productivity resulting from lower college attendance.

Finally, we consider how the economy responds to the 2020 shock displayed in *Figures 14* and *15* when the utility-maximizing search subsidy is given to agents 55 and older. Recall that in order to roughly match the participation rate responses, we needed to increase b by 15% in the same period as an aggregate shock resulting in a total drop in GDP of 8.48% during the period of the shock (the same percentage drop in real GDP observed during 2020). *Figure 24* displays the change in participation from its steady-state in reaction to this shock when there is no subsidy and when the utility-maximizing search subsidy is given to agents 55 and older. The subsidy unsurprisingly speeds up the recovery of participation among agents 55 and older. However, because the search subsidy is only offered to older agents, the effect on quickening the recovery of overall participation is slight.

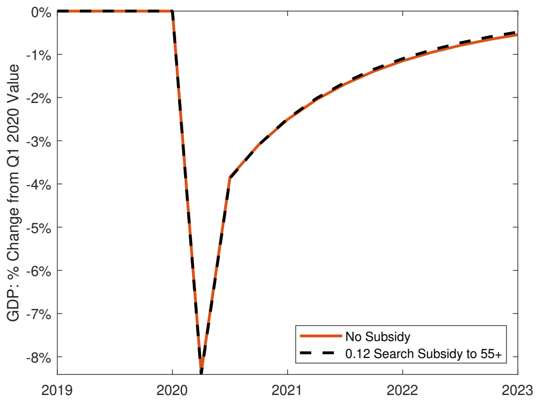
Figure 24: Model Response to 2020 Shock: With and Without Subsidy to Ages 55+

Age 55+ Participation Response | *Total (All Ages) Participation Response*



The search subsidy has a minimal effect on boosting the recovery of GDP in response to the shock. These findings suggest that although the search subsidy can significantly affect the older agents to which it is offered, the overall impact on the economy's recovery from a shock similar to the one experienced in 2020 is negligible. These results motivate us to investigate how varying the subsidy amount in response to changes in aggregate productivity affect the economy's response to series of simulated shocks. We discuss these results in subsection 5.2.[23]

Figure 25: GDP Response to 2020 Shock: With and Without Subsidy to Ages 55+



5.2 Subsidizing Search Over the Business Cycle

We now compare how the economy that offers search subsidies behaves over a long series of simulated shocks with an economy that does not. Recall that aggregate productivity (Z) follows an AR(1) process such that $\ln(Z_t) = \rho \ln(Z_{t-1}) + \epsilon_t$ where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.[24] In 5.2.1, we track the economy as it experiences this simulated series of shocks when there is no search subsidy and when there is the subsidy to agents age 55 and older that maximizes average utility in the risky steady-state. Then, in 5.2.2, we compare these results with the case where the subsidy amount is dependent on aggregate productivity.

5.2.1 Fixed Subsidy

In this section, we study the case where the subsidy does not vary with the business cycle. The left-hand panel of *Figure 26* displays how the per-period subsidy cost and tax revenue vary over the simulated series of shocks. In times with high aggregate productivity, it is common for the per-period tax revenue to exceed the per-period cost of the subsidy, but the opposite is true during recessions. In expectation, however, the budget is closely balanced. Over the 250 years in which we track the model economy, the right-hand panel of *Figure 26* shows the cumulative government net income as a percentage of just one quarter of steady-state GDP. Over the 250 simulated years, the cumulative

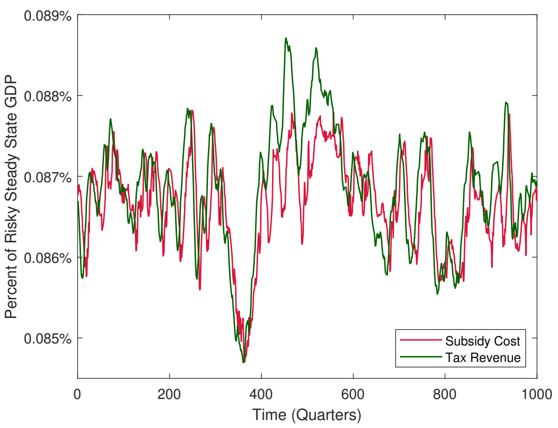
[23]Further results regarding the effects of search subsidies are provided in the Appendices. Appendix B discusses alternative welfare measures of interest and computes the percentage of the population who would vote for different subsidy amounts and schemes if agents were only self-interested. Appendix C shows how the economy transitions to the new steady-state where the utility maximizing search subsidy is given to agents 55 and older.

[24]Computationally, this AR(1) is approximated as an N-state Markov chain following Tauchen (1986). In this section, we simulate a series of shocks over 1,000 quarters (250 years) where the AR(1) process aggregate productivity follows is approximated with a 15-state Markov chain. *Figure 43* in Appendix D displays the simulated AR(1) and its Markov approximation.

government net income is slightly positive, but is just 0.18% of one quarter of steady-state GDP (0.00018% of cumulative steady-state GDP over 1,000 quarters).

Figure 26: Fixed Subsidy Amount: Government Budget

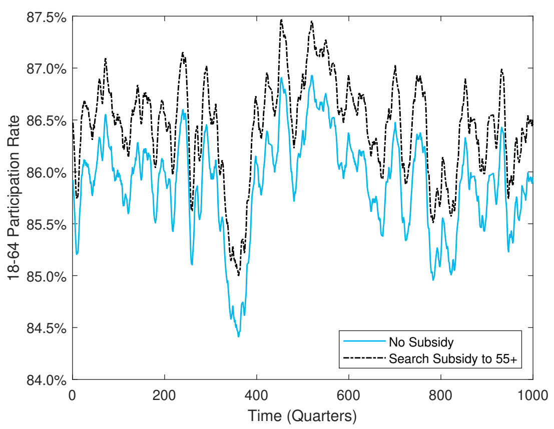
Subsidy Cost and Tax Revenue | *Cumulative Net Government Income*



Consistent with the subsidy results from *Figure 18*, subsidizing search for agents age 55 and older boosts participation. *Figure 27* shows that the economy with the subsidy has higher labor force participation over the series of simulated shocks. The higher total participation rate in the economy with the search subsidy is mostly due to higher participation among those aged 55 and older. Other age groups experience very slightly lower participation rates in the economy where their wages are taxed to provide the search subsidy to the older workers, but the difference over the series of shocks is negligible.

Figure 27: Comparison with Fixed Subsidy Amount: Labor Force Participation

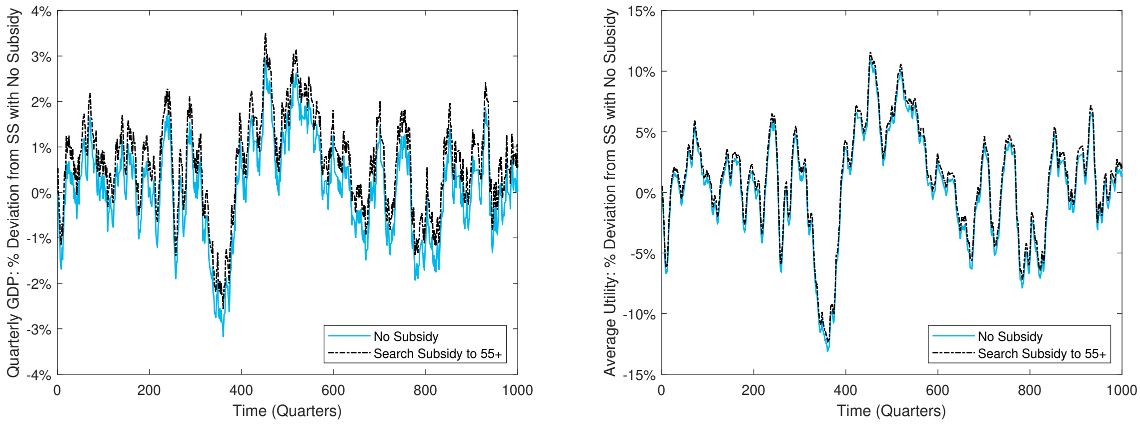
Age 18+ Participation Rate | *Age 55+ Participation Rate*



Over the simulation, GDP (measured as output from firms and employed workers) is consistently higher when the search subsidy is given to those 55 and older. Average utility is also consistently slightly higher in the economy where the subsidy is given. *Figure 28* shows that while there may be differences in the volatility of GDP and average utility when a search subsidy is offered, the differences are slight. Next, we explore how allowing the subsidy amount to vary with aggregate productivity affects both the volatility and average levels of these same variables of interest.

Figure 28: Comparison with Fixed Subsidy Amount: GDP and Average Utility

GDP: % Deviation from No Subsidy SS *Avg Utility:% Deviation from No Subsidy SS*



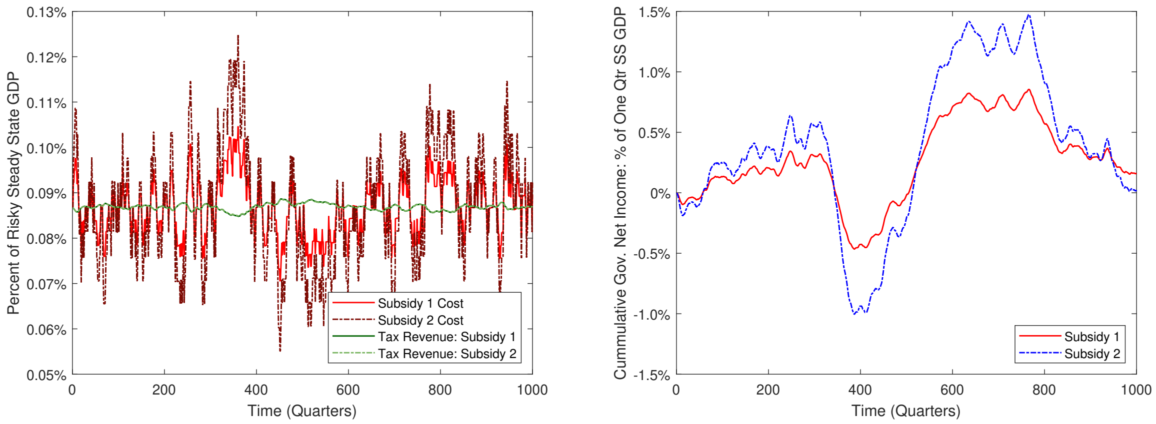
5.2.2 Aggregate State-Dependent Subsidy

Now, consider the same steady-state subsidy amount given only to agents 55 and older that maximizes steady-state average utility. Rather than keeping the subsidy amount fixed over the business cycle, we examine how allowing the subsidy amount to vary with aggregate productivity affects how the economy responds over the simulated series of shocks. In this section, we consider two variations of this subsidy. Additional variations, including subsidizing search for individuals of all ages, which we found lowered average steady-state utility, are discussed in Appendix E. Let s^* be the subsidy amount given only to agents 55 and older that maximizes steady-state average utility, and consider two variations of this policy. "Subsidy 1" pays amount $s(Z) = s^* - \frac{(Z-Z_{SS})}{Z_{SS}}$ to agents 55 and older who search for a job, where Z_{ss} indicates the steady-state value of Z. "Subsidy 2" varies even more over the business cycle and pays amount $s(Z) = s^* - 2\left(\frac{(Z-Z_{SS})}{Z_{SS}}\right)$.

While the subsidy cost is higher during recessions compared to the case where the subsidy did not vary with the aggregate state of the economy, the subsidy cost is also relatively lower following positive shocks to aggregate productivity. *Figure 29* shows that while the per-period government deficit or surplus may at times be larger than in the fixed subsidy case, the government budget is still closely balanced in expectation when the subsidy amount varies over the business cycle.

Figure 29: Aggregate State-Dependent Subsidy: Government Budget

Subsidy Cost and Tax Revenue *Cumulative Net Government Income*

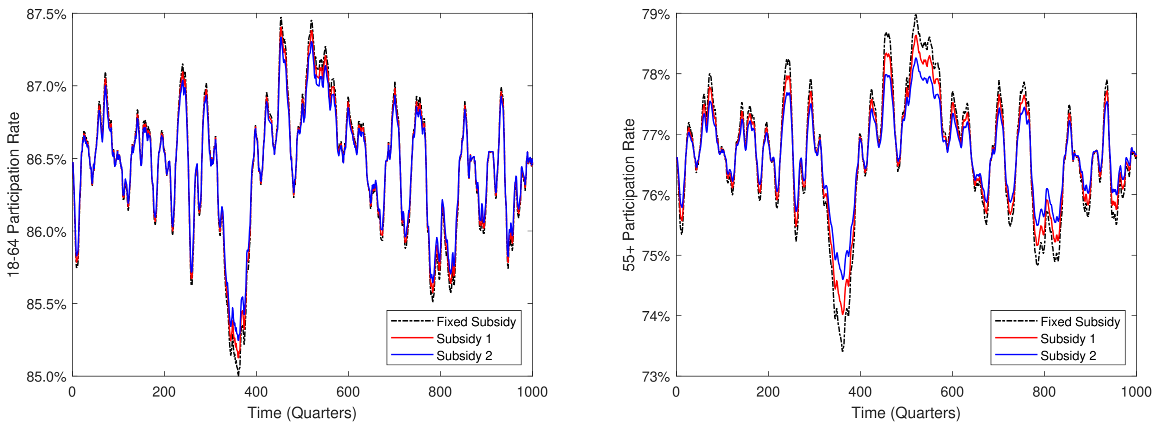


The right panel of *Figure 29* shows that after the simulated 250 years, the government's cumulative net income is close to zero. While the government's budget is balanced in the long-run, it experiences larger deficits and surpluses in certain periods with more volatile subsidies.

While having no notable effect on mean participation, allowing the subsidy amount to vary with the business cycle has evident dampening effects on the volatility of participation. Since the search subsidy is only offered to those aged 55 and older, the effects are concentrated in this age group.

Figure 30: Aggregate State-Dependent Subsidy: Labor Force Participation

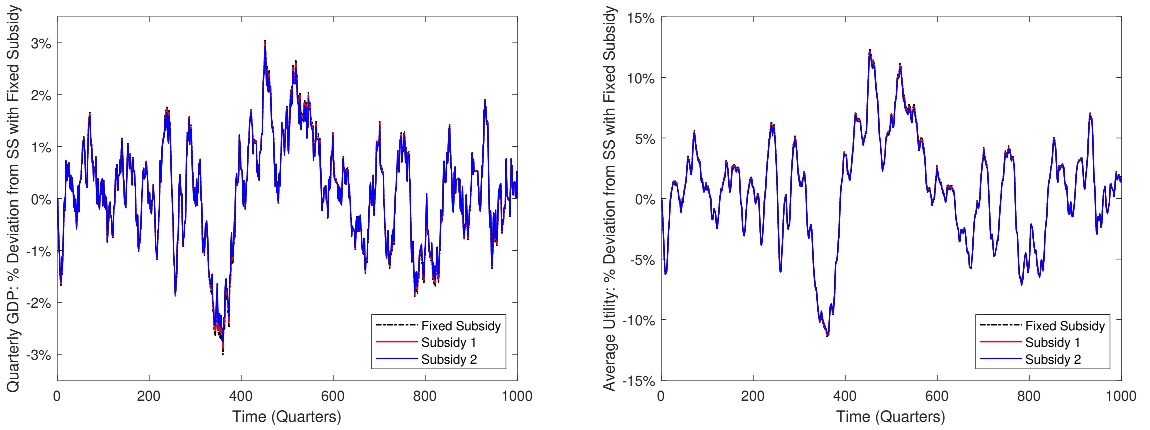
Age 18+ Participation Rate *Age 55+ Participation Rate*



Although having a noticeable impact on the volatility of labor force participation, *Figure 31* shows that allowing the subsidy amount to vary with the aggregate state results in only a slight reduction in the volatility of GDP and average utility.

Figure 31: Aggregate State-Dependent Subsidy: GDP and Average Utility

GDP: % Deviation Fixed Subsidy SS *Avg Utility: % Deviation Fixed Subsidy SS*



The results of these policy experiments are summarized in the following tables. *Table 4* displays how various variables changes under different policies. Consistent with *Figure 22* we find that offering a search subsidy to agents age 55 and older increases average utility, consumption, GDP, and participation over the series of shocks. Allowing this subsidy to vary over the business cycle has no sizeable effects on the long-term means of these variables, however. A noteworthy feature of this experiment is that the average consumption gain declines slightly from .17% to .16% with an increasingly generous subsidy during economic downturns (refer to *table* 4, second row), leading the average utility gain to decline from around .6% to .5% as well (first row). This unveils an inefficiency associated with offering search subsidies during recessions: firms post fewer vacancies during such times resulting in a lower job-finding rate, so agents are encouraged to pay a search cost when the benefit of doing so is lower (and during booms, they are not as encouraged to search when the benefit is higher). Therefore, such a subsidy results in lower consumption and correspondingly, reduced welfare.[25] Therefore, while business cycle policies can mitigate the volatility of several aggregate variables, they predict marginal or even negative welfare gains when compared to their steady-state policy counterparts.[26]

[25]To understand this argument more formally, consider the following Cobb-Douglas matching function, where the total number of matches m is given by:

$$m = Av^{0.5}u^{0.5}.$$

A represents the match efficiency, v denotes the total number of vacancies, and u is the share of unemployed workers. Note that the marginal efficiency of boosting u to increase the total number of matches is $Av^{0.5}$, which decreases in recessions because vacancy rates are low.

[26]Table 4 in Appendix E reports variance statistics over the simulated periods.

Table 4: Comparing Different Subsidy Policies

	Fixed Subsidy to 55+	Subsidy 1	Subsidy 2
Avg. Utility: % Change	.614%	.559%	.503%
Consumption: % Change	.170%	.165%	.159%
GDP: % Change	.551%	.554%	.556%
18-64 Participation : Rate Change	.549%	.551%	.553%
55+ Participation : Rate Change	2.887%	2.895%	2.903%

Note: All changes are relative to the case of no subsidy. Subsidy 1 and 2 refer to policies with different sensitivity to business cycle shocks.

6 Conclusion

The economic downturn caused by the pandemic led to a considerable decline in workforce participation, particularly among the younger and older demographics. This situation prompts us to explore policy measures that encourage workers to return to the job market and how a government should design such measures. This paper employs a heterogeneous-agent search model to shed light on this issue. The model's key elements are endogenous college participation and human capital accumulation over workers' life cycle. The model successfully matches disaggregate evidence on labor participation rates across different age cohorts and their responses to GDP growth shocks.

The policy recommendation from our model is as follows: subsidies should be aimed at older workers rather than younger ones, as subsidizing the youth could unintentionally discourage college enrollment, adversely impacting long-term productivity. Furthermore, these subsidies should be provided consistently, not only during periods of economic downturn. The rationale is that subsidizing job search in recessions is less productive, as good jobs are harder to find during recessions than booms.

Our model provides a foundation that can be expanded in various ways. For instance, this model does not consider asset accumulation, and broadening this aspect would enable the study of the interplay between labor market dynamics and wealth inequality. Such a framework can be further extended with an equilibrium model of colleges, akin to Cai and Heathcote (2022), to examine the role of college education in this interaction. Furthermore, the model does not consider active on-the-job training offered by a wide range of firms in practice. Considering human capital's significant role in driving labor participation, as shown in our model, such training could have crucial implications for labor market dynamics and workers' participation choices. We leave these to future research.

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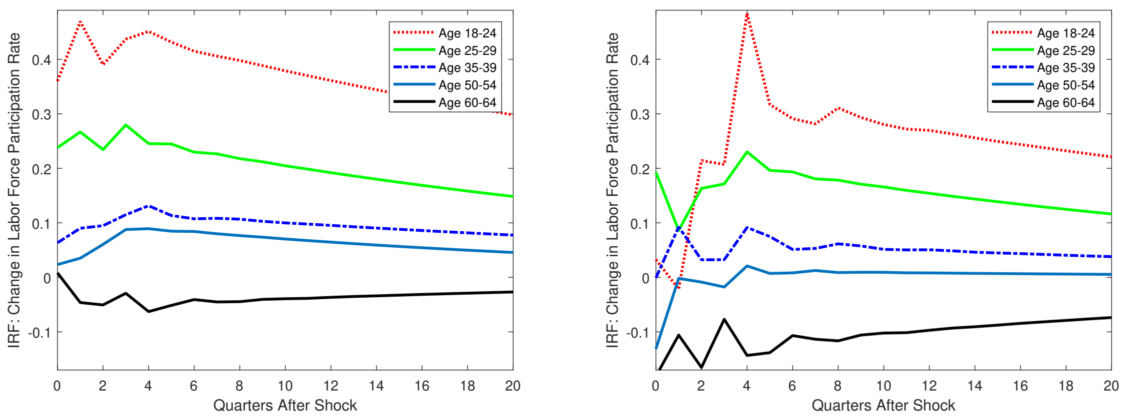
Appendix A

This appendix investigates how estimated participation rate responses to aggregate shocks would differ if only pre-pandemic data were used. *Figure 32* shows the impulse response functions (IRFs) estimating the response of the participation rate of each age group to a positive shock resulting in 1% greater GDP growth. The left panel of the figure shows the IRFs estimated using the full data spanning 2000-2022. (The 95% confidence bands are excluded so as to make the figure readable, but these confidence bands are included in *Figures 34*, *35*, and *36*.) The right panel of *Figure 32* shows the same IRFs estimated using only pre-pandemic data from 2000-2019. We see that including data from the pandemic recession changes the estimated timing of the response of age 18-24 individuals, but not the general size of the response after around four quarters.

Figure 32: Participation Response to a 1% GDP Growth Shock

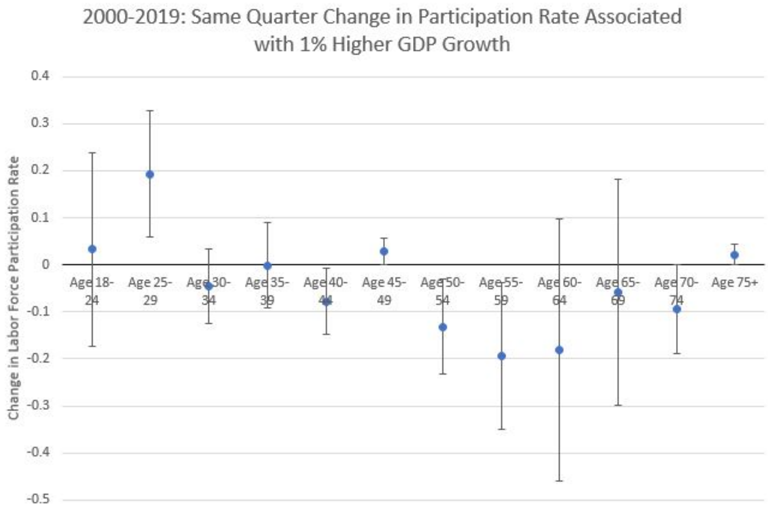
IRFs Using 2000-2022 Data | *IRFs Using 2000-2019 Data*



The estimated responses of age groups 25-29, 35-39, and 50-54 appear similar when only using pre-pandemic data, although the initial same-quarter response to the shock appears slightly lower in each of these cases. Agents aged 60-64 have a surprisingly negative estimated participation response to positive shocks when only pre-pandemic data is used. Once the response of 60-64 year-olds during the pandemic is included in the data set, the estimated IRF shifts upward.

Figure 33 displays how the participation response in the same quarter as the positive GDP shock appears when only pre-pandemic data is used. Compared to *Figure 4*, the response of the younger agents appears smaller. However, *Figure 32* shows that much of the difference in the estimated response of younger agents appears to be related to the timing of the response more than the overall size. When comparing the same-quarter responses using only pre-pandemic data to those estimates with the full data set, we also see that the responses of individuals aged 50 through around 64 have an estimated slightly negative participation response that becomes slightly positive once more recent data is included.

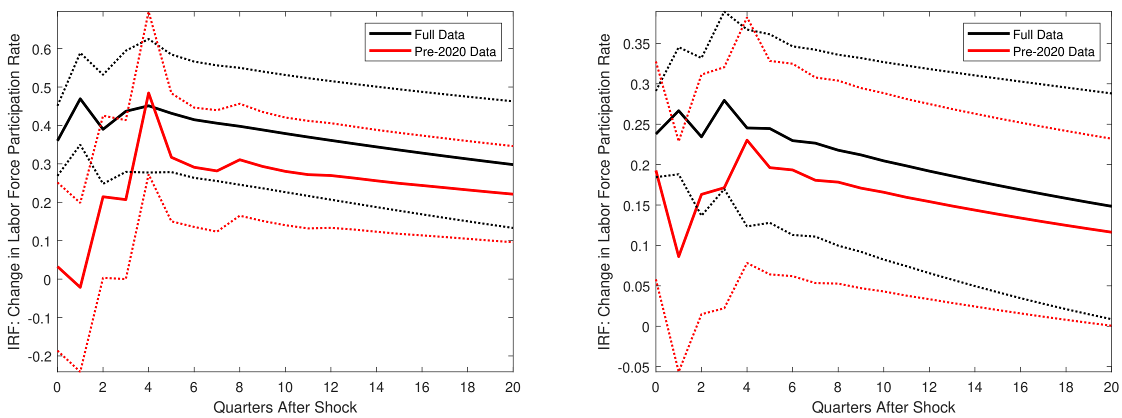
Figure 33: Same-Quarter Change in Participation Rate Associated with 1% Higher GDP Growth: Estimated Using Only 2000-2019 Data



Seeing the estimated responses when using the full data plotted along with those using just the pre-pandemic data is likely most informative in judging the effect that the 2020 recession had on these estimations. *Figure 34* displays these IRFs for the youngest two age categories.

Figure 34: IRF Comparison Using Full vs. Pre-2020 Data: Youngest Age Groups

Age 18-24 IRF *Age 25-29 IRF*



For both of the youngest two age groups, the inclusion of data following the 2020 recession increased the immediate estimated response. After approximately four quarters, however, the magnitude of each response estimate appears very similar.

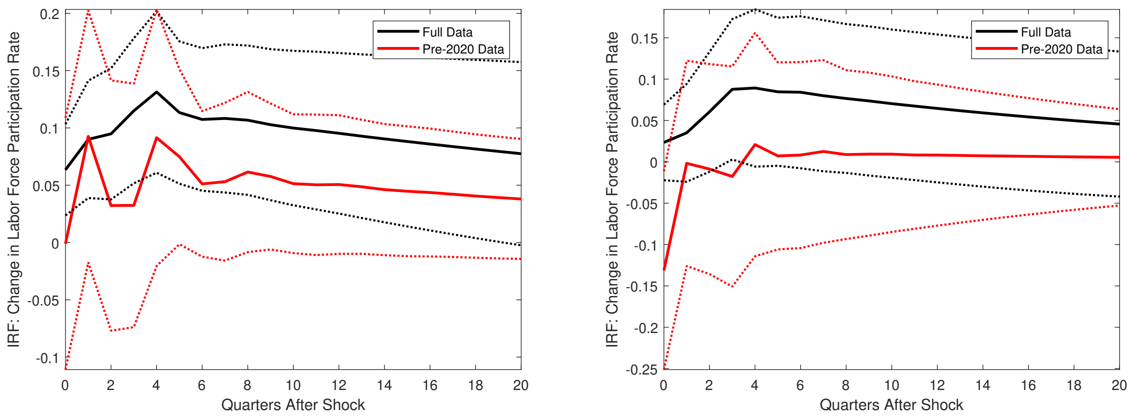
Figure 35 allows us to compare the IRFs for individuals aged 35-39 and 50-54.[27] In both cases, the data from the 2020 recession increased the size of the estimated initial response and had a somewhat lasting impact on the response sizes after four quarters.

[27]Results for other prime-aged groups appeared very similar to the results of the 35-39 age group and were excluded for brevity.

However, each estimated response generally falls within the 95% confidence bands of the other.

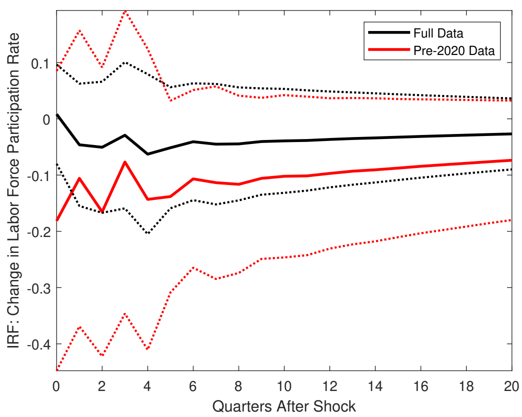
Figure 35: IRF Comparison Using Full vs. Pre-2020 Data: Middle Age Groups

Age 35-39 IRF *Age 50-54 IRF*



Finally, we look at the estimated impulse responses of the oldest age group included in the model in *Figure 36*. We see again that the inclusion of data following the 2020 recession increased the initial estimated participation response of this age group. However, after about two quarters following the shock, the two estimated IRFs appear to converge and fall within the 95% confidence bands of the other.

Figure 36: IRF Comparison Using Full vs. Pre-2020 Data: Age 60-64



Appendix B: Alternative Welfare and Equivalent Variation Calculations

In this appendix, we discuss alternative welfare measures that could be considered to examine the effects of offering different search subsidies. First, we consider focusing on how different subsidies affect new agents entering the economy. We report how search subsidies offered to all agents and to different age groups affect the discounted expected lifetime utility value of new entrants as well as the consumption equivalent variation of new entrants. To compute the latter, we estimate the share of remaining lifetime consumption each agent would be willing to forgo (or must receive) to enter the economy with the policy change.

Let $\left(\{c_{jt}\}_{t=1}^{T}\right)$ be agent j's consumption over future periods and contingencies in the baseline economy, and let $\left(\{\tilde{c}_{jt}\}_{t=1}^{T}\right)$ denote the same series after the change in policy. We first estimate the scaling parameter η_j such that individual j is indifferent between living the remainder of their life under both policies. For each individual j, η_j is such that

$$\sum_{t=1}^{T} \beta^{t-1} \ln(\eta_j c_{jt}) = \sum_{t=1}^{T} \beta^{t-1} \ln(\tilde{c}_{2jt}).$$

Let $\tilde{V}_j \equiv \sum_{t=1}^{T} \beta^{t-1} \ln(\tilde{c}_{jt})$. With the chosen utility function, we can solve for η_j as follows.

$$e^{\sum_{t=1}^{T} \beta^{t-1} \ln(\eta_j c_{jt})} = e^{\tilde{V}_j}$$

$$e^{\ln(\eta_j c_{j1})} e^{\beta \ln(\eta_j c_{j2})} e^{\beta^2 \ln(\eta_j c_{j3})} \ldots = e^{\tilde{V}_j}$$

$$\eta_j c_{j1} (\eta_j c_{j2})^{\beta} (\eta_j c_{j3})^{\beta^2} \ldots = \left(\eta_j^{\left(1+\beta+\beta^2+\ldots\right)}\right) \left(e^{\ln(c_{j1})} e^{\beta \ln(c_{j2})} e^{\beta^2 \ln(c_{j3})} \ldots\right) = e^{\tilde{V}_j}$$

$$\left(\eta_j^{\sum_{t=1}^{T} \beta^{t-1}}\right) \left(e^{\sum_{t=1}^{T} \beta^{t-1} \ln(c_{jt})}\right) = e^{\tilde{V}_j}.$$

Let $V_j \equiv \sum_{t=1}^{T} \beta^{t-1} \ln(c_{jt})$. Then

$$\left(\eta_j^{\sum_{t=1}^{T} \beta^{t-1}}\right) e^{V_j} = e^{\tilde{V}_j}$$

$$\eta_j = \left(e^{\tilde{V}_j - V_j}\right)^{\left(\frac{1}{\sum_{t=1}^{T} \beta^{t-1}}\right)}.$$

So that η_j can be used to determine the share of consumption that agent j is willing to give up (or must receive) to enter the economy with the new policy, we use the following transformation

$$\widehat{\eta}_j = 100(\eta_j - 1).$$

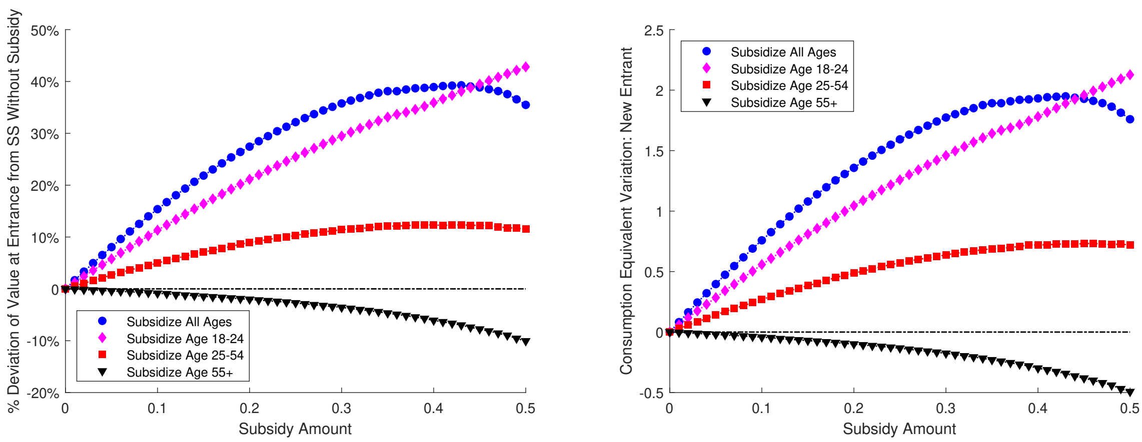
We use the equivalent variation solved for any individual j first to compute the share of consumption a new agent would be willing to forgo (or receive) in all future periods and contingencies to enter into the economy with the policy change as opposed to the baseline economy. The expected discounted series of consumption that the agent would receive upon entering the baseline economy is specified by equation (5) where $a = 1$, $\tau = 1$, $Z = Z_{ss}$, and $z = 0$. Let $\tilde{N}_1(1, Z_{ss}, 0)$ denote this entrance value after the policy change considered. The consumption equivalent variation of a new entrant is

$$EV^{Entrant} = 100 \left(\left(e^{\tilde{N}_1(1,Z_{ss},0) - N_1(1,Z_{ss},0)} \right)^{\left(\frac{1}{\sum_{t=1}^{T} \beta^{t-1}} \right)} - 1 \right).$$

Figure 37 plots the percentage change in the value of expected discounted lifetime utility and the consumption equivalent variation of a new entrant under different policy changes. In both panels of the figure, we see that all subsidies except subsidies to those 55 and older increase the entrance value of new agents. When subsidies are given only to those 55 and older, new entrants who discount the future see a far-away benefit and an immediate cost of having to pay higher taxes when employed. Each panel of the figure shows qualitatively the same results but offers a different way to quantify these results.

Figure 37: Alternative Welfare Measures: Focus on New Entrant

Percent Change in Value of New Entrant $EV^{Entrant}$



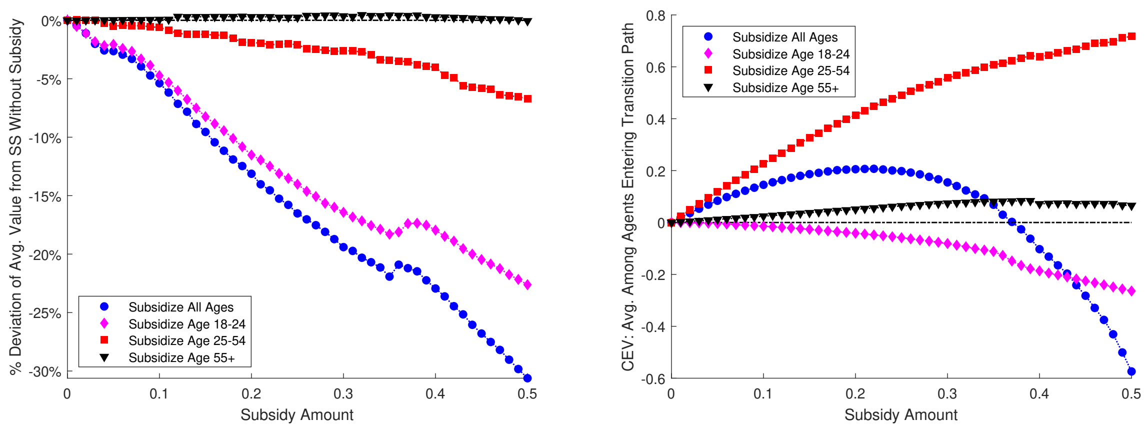
Now we consider how policy changes affect all agents who are in the economy at the time of the policy change. These agents vary in age, skill, education, and employment status. We report the percentage change in the average *remaining* expected discounted lifetime utility of all agents in the economy at the time of the policy change. We also report the average consumption equivalent variation of agents at the time of the policy change. This second metric that we consider is the average share of consumption that existing agents in the steady-state of the baseline economy would be willing to give up (or must receive) for the remainder of their lives before retirement to undergo the policy change. Specifically, we consider that in the steady-state of the baseline economy, there are agents of all ages and employment statuses and consider values of $\widehat{\eta}_j$ for their remaining consumption. Considering a large representative sample of N agents from the baseline steady-state, we compute

$$EV^{Transition} = \frac{1}{N}\sum_{j=1}^{N}\widehat{\eta}_j.$$

The left-hand side of *Figure 38* displays the percentage change in the average remaining expected discounted lifetime utility of all agents in the economy at the time of the policy change. The results look similar in nature to *Figure 22*, as only the subsidy paid to agents age 55 or older increases the average remaining expected discounted lifetime utility. The right-hand side of *Figure 38* plots the average consumption equivalent variation of agents at the time of the policy change ($EV^{Transition}$).

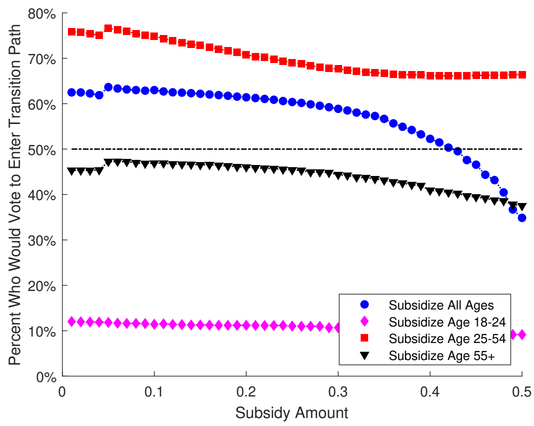
Figure 38: Alternative Welfare Measures: Weighting All Agents Present at Time of Policy Change

% Change in Avg. Value At Policy Change $EV^{Transition}$



Finally, we consider the percentage of agents in the risky steady-state of the baseline economy with no search subsidy who would vote for each policy considered. We assume that agents are only self-interested, and they vote for a policy only if it increases their remaining expected discounted lifetime utility. Unsurprisingly, we find that policies that offer a search subsidy to a larger group of agents, rather than just the very young or old, are more likely to gain the approval of a majority of agents. Policymakers might consider expanding the ages to which the policy is offered to ensure that the policy would meet the approval of a majority of voters. (For example, they might consider the effects of offering the search subsidy to agents 50 and older rather than 55 and older to find a utility-increasing policy that would also likely be approved by voters.)

Figure 39: Percent Who Would Vote For Each Policy Change

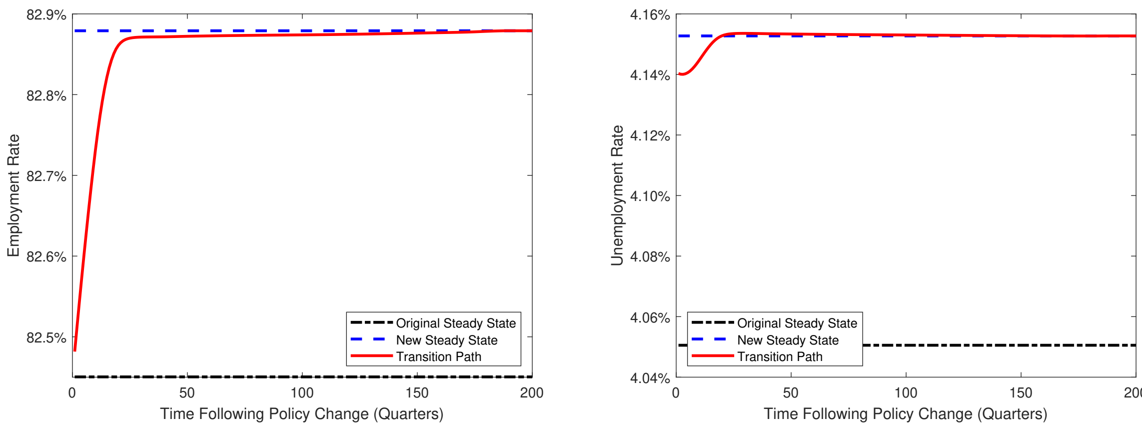


Appendix C: Transition Paths To New Steady-State for Utility Maximizing Subsidy to Ages 55+

This appendix discusses the transition path of the economy from the risky steady-state with no search subsidy to the risky steady-state with the utility-maximizing subsidy paid to unemployed agents age 55 and older. *Figure 40* shows that the employment and unemployment rates transition to their new steady-state values relatively quickly after the policy change. The search subsidy encourages those who receive it to pay the cost to search for employment, so it is unsurprising that the policy change would have a sudden and positive impact on the percentage of the total population that is employed. Because the subsidy is only given to older workers, the overall impact on the total employment rate is modest, resulting in an increase from around 82.5% to 82.9% in the new steady-state.

Figure 40: Transition Paths of Employment and Unemployment

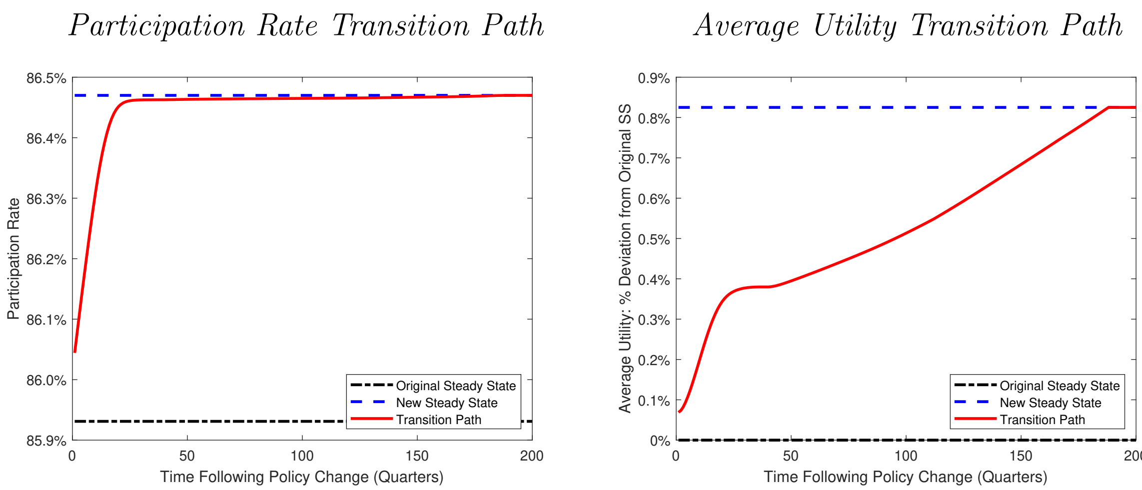
Employment Rate Transition Path *Unemployment Rate Transition Path*



The total unemployment rate in the economy also transitions fairly quickly to its new steady-state value. The search subsidy lessens the benefit of moving from unemployment to employment, so when agents direct their search, they are willing to trade off a lower job-finding rate for a higher wage if matched. Therefore, agents receiving the subsidy spend more time in unemployment on average, and the total unemployment rate increases.

Figure 41 displays the transition paths of the total participation rate and average utility following the policy change. Like employment and unemployment, the participation rate responds relatively quickly to the policy change, while average utility moves more slowly to its new steady-state value. The search subsidy immediately promotes participation, and the participation rate is close to its new steady-state value around 30 quarters following the policy change. We that the increase in average utility is also sharpest in approximately the first 30 quarters after the policy change.

Figure 41: Transition Paths of Average Utility and Total Participation

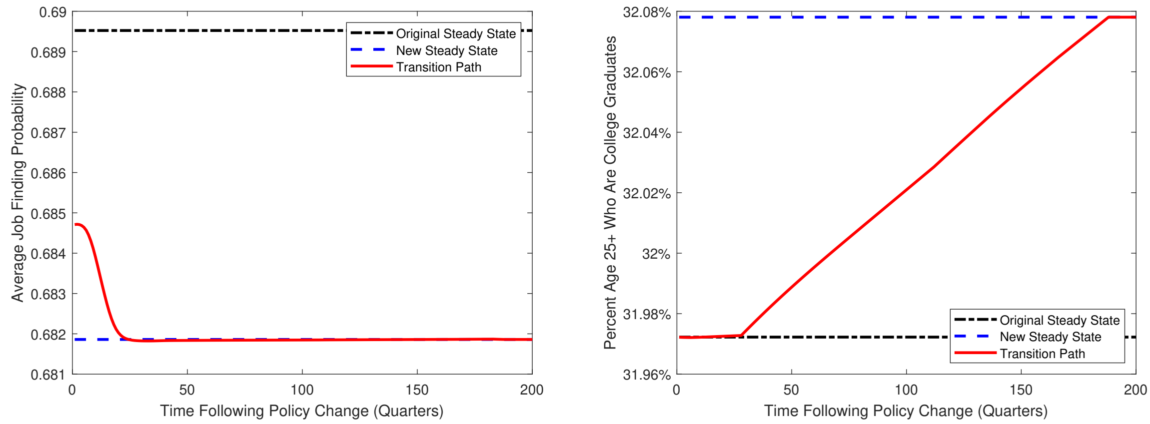


Although participation and employment respond quickly to the policy and influence the transition of average utility, the policy change also influences the percentage of the population who attend and graduate from college. *Figure 42* shows how the share of college graduates among agents age 25 and older responds to the policy change. Although young agents are not initially eligible for the subsidy, the policy change affects their enrollment decision because it impacts the length of time they expect to be employed and, therefore, the length of time they expect to benefit from obtaining a college degree. The policy change does not immediately have much impact on the share who have graduated because although it affects the enrollment decision of younger agents, these agents do not immediately graduate and are only included in the average for educational attainment after they turn 25 (as is common in many data series reporting educational attainment). The gradual transition of the share of agents who are college graduates towards its new steady-state value explains the more gradual increase in average utility after the initial

30 quarters following the policy change. *Figure 42* also shows how the average job-finding probability of those who are unemployed responds to the policy change. The immediate drop in this probability coincides with the immediate increase in the unemployment rate shown in *Figure 40*.

Figure 42: Transition Paths of Job-Finding Rate and Percentage of Agents 25+ Who are College Graduates

Job-Finding Probability Transition Path | *College Graduate Share Transition Path*



Appendix D: Additional Aggregate State-Dependent Subsidy to Agents 55 Plus Results

This section provides additional details and results regarding the aggregate state-dependent subsidies offered only to agents age 55 and older discussed in 5.2.2. For each of the subsidies considered, we simulated the model economy with the subsidy over 1,000 quarters (250 years). So that the results are comparable over different simulations, we generate a series of shocks to aggregate productivity and apply this same series of shocks to the model economy under each simulation. Recall that aggregate productivity is assumed to follow an AR(1) process that is approximated using an N-state Markov chain.[28] *Figure 43* displays the simulated AR(1) and its Markov approximation.

[28] In this case, the AR(1) process is approximated with a 15-state Markov chain.

Figure 43: Simulation of Aggregate Productivity AR(1) Process

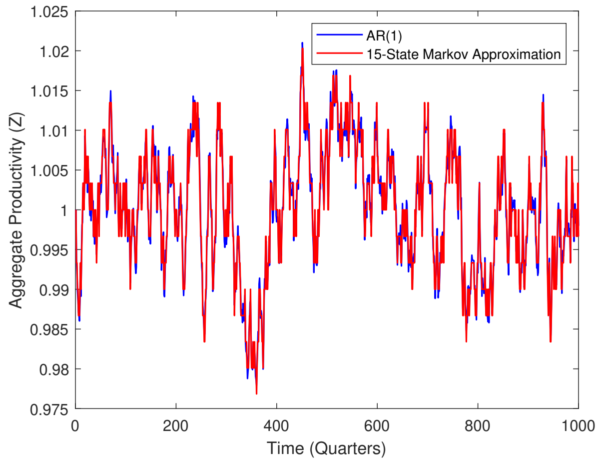


Table 5 reports the coefficient of variation (CV) for each series of interest over the simulated shocks. Consistent with *Figures 30* and *31*, allowing the subsidy amount to increase during recessions and decline during expansions reduces the volatility of utility, GDP, and participation. Consumption, which includes leisure consumption, is notably less volatile than GDP, and its volatility only slightly declines as we consider subsidies more responsive to the aggregate state.

Table 5: Coefficient of Variation over Simulated Shocks

	No Subsidy	Fixed Subsidy to 55+	Subsidy 1	Subsidy 2
Average Utility	0.0441	0.0438	0.0432	0.0426
Consumption	0.0041	0.0041	0.0041	0.0040
GDP	0.0106	0.0105	0.0102	0.0099
Participation Rate: 18-64	0.0056	0.0055	0.0051	0.0046
Participation Rate: 55+	0.0152	0.0139	0.0115	0.0091

Appendix E: Effects of Aggregate State-Dependent Search Subsidies Offered to All Ages

In this appendix, we evaluate how offering a search subsidy that depends on aggregate productivity to agents of all ages affects the economy as it evolves over the same series of shocks displayed in *Figure 43*. This series simulates 250 years in the model economy.

Zero Subsidy in the Steady-State

Recall that subsidizing search for all ages resulted in lower GDP and average utility. This is because subsidizing search makes job search relatively more attractive compared to college attendance. This results in younger workers having a lower college enrollment rate and reduces average worker productivity. Therefore, since subsidizing the search of all ages has negative steady-state implications, we first consider a subsidy that equals zero when aggregate productivity is at its steady-state value.

Specifically, we consider "Subsidy 1" which pays amount $s(Z) = -\frac{(Z-Z_{SS})}{Z_{SS}}$ to all agents who search for a job, where Z_{ss} indicates the steady-state value of Z. "Subsidy 2" varies even more over the business cycle and pays amount $s(Z) = -2\left(\frac{(Z-Z_{SS})}{Z_{SS}}\right)$. *Figure 44* shows the per-period subsidy cost and tax revenue of each policy on the left-hand panel, while the right-hand panel shows the cumulative deficit or surplus over the entire simulation. Just as when the state-dependent subsidy was offered only to agents 55 and older, the government's budget is closely balanced in the long-run. The government does experience relatively larger deficits and surpluses in certain periods as the variability of the subsidy increases and as more individuals (age groups) are eligible to receive the subsidy.

Figure 44: Aggregate State-Dependent Subsidy offered to all Ages: Government Budget

Subsidy Cost and Tax Revenue *Cumulative Net Government Income*

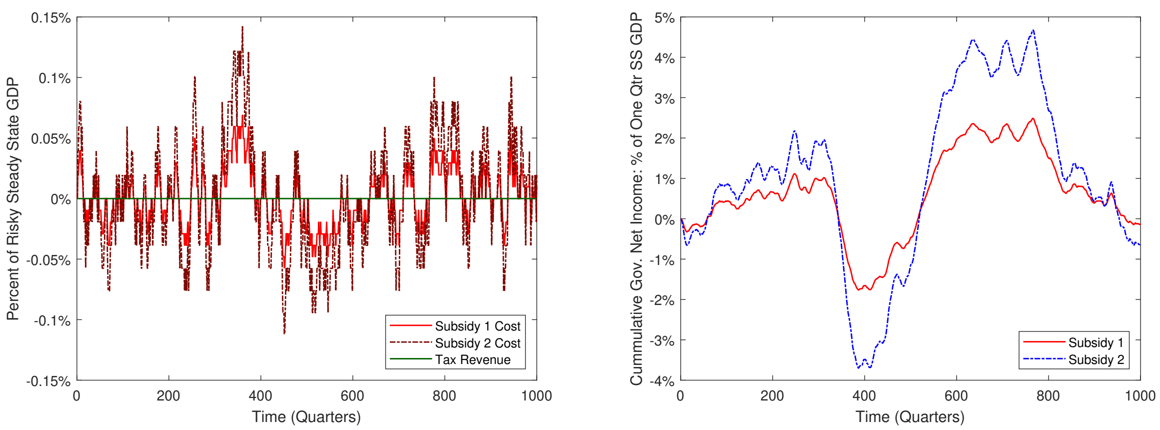
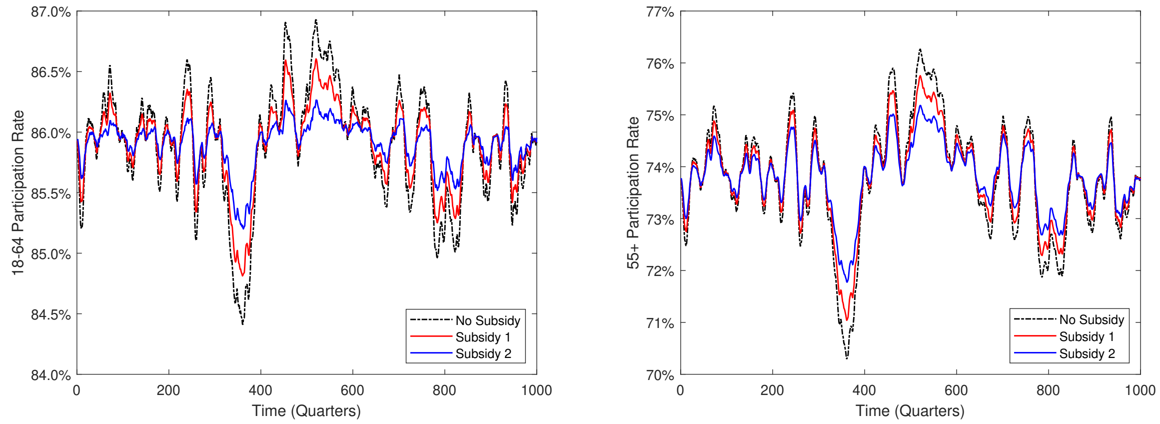


Figure 44 shows how each subsidy affects the total and age 55 plus participation rates. The mean of the participation rates over the series of shocks are very similar, and *Table 6* shows that the means of both the total and 55 plus participation rates increase only very slightly with the more volatile subsidies.

Figure 45: Aggregate State-Dependent Subsidy offered to all Ages: Participation

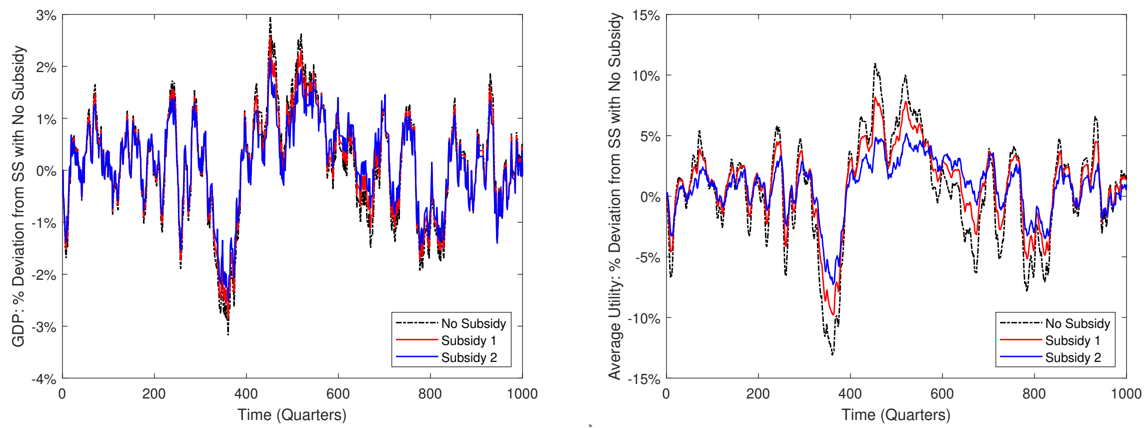
Age 18+ Participation Rate Age 55+ Participation Rate



Although the means are similar, we see that the volatility of the total and 55 plus participation rates are much lower in the case where the subsidy amount varies with the aggregate state. In the case of Subsidy 2, where the subsidy is the most responsive to changes in aggregate productivity, the drop in participation during recessions and the increase during booms is much less than in cases where no subsidy is applied.

Figure 46: Aggregate State-Dependent Subsidy offered to all Ages: Avg. Utility and GDP

GDP: % Deviation from No Subsidy SS Avg Utility: % Deviation from No Subsidy SS



In addition to smoothing participation, the aggregate state-dependent subsidies reduce the volatility of both GDP and average utility. *Figure 46* shows that this effect is greater for average utility than for GDP.

Offering a search subsidy smooths consumption as agents move between employment and unemployment and also encourages participation. However, encouraging participation makes the option of attending college relatively less attractive. With directed search, the subsidy also makes agents willing to spend more time in unemployment, reducing the probability they will move out of unemployment. In the cases considered here, the subsidy equals zero when aggregate productivity is at its steady-state value. Allowing this baseline of zero to become slightly positive during recessions and slightly negative during booms very slightly increases average utility over the simulated shocks.

Table 6: Mean over Simulated Shocks

	No Subsidy	Subsidy 1	Subsidy 2
Avg. Utility: % Change from No Subsidy	0%	0.360%	0.169%
GDP: % Change from No Subsidy	0%	0.019%	0.026
Participation Rate: 18-64	85.889%	85.898%	85.902%
Participation Rate: 55+	73.775%	73.792%	73.798%

This increase in average utility indicates that the benefit of offering a mechanism for consumption smoothing during times when unemployment is more common can outweigh undesirable incentives in terms of search and college enrollment when the baseline subsidy amount is zero. *Table 7* reports the coefficient of variation (CV) for each series of interest over the simulated shocks. As with previous subsidies considered, subsidies that vary more in response to changes in aggregate productivity reduce the CV for utility, GDP, and participation.

Table 7: Coefficient of Variation over Simulated Shocks

	No Subsidy	Subsidy 1	Subsidy 2
Average Utility	0.0441	0.0324	0.0224
GDP	0.0106	0.0093	0.0081
Participation Rate: 18-64	0.0056	0.0039	0.0022
Participation Rate: 55+	0.0152	0.0119	0.0088

In addition to the subsidies discussed in this appendix, we also considered slightly positive subsidies offered to all ages which varied with the aggregate state. The results regarding volatility were similar to what we found in this section; the volatility of all variables over the simulated shocks was reduced with more aggregate state-dependent subsidies. However, offering the subsidy to all ages was utility reducing over the simulated shocks (as in *Figure 22*.)