Sovereign Spreads and the Effects of Fiscal Austerity*

Diego Anzoategui†

Rutgers University

November, 2019

Abstract

I analyze the impact of austerity on sovereign spreads. To do so I propose a model with strategic sovereign default and nominal rigidities where the government follows fiscal rules, which are estimated from data. I first analyze the theoretical implications of the model and find that austerity can be self-defeating only when austerity packages are persistent and the economy is expected to be in a recession with high fiscal multipliers. I then calibrate the model using data from Spain and estimate the size and impact of fiscal policy shocks associated with austerity policies. I use the model to predict what would have happened to spreads and economic activity if Spain had continued to follow the pre-2010 fiscal rule instead of switching to the austerity track. I find that, relative to the counter-factual, austerity decreased sovereign spreads and debt-to-GDP ratios even when fiscal multipliers where higher than one during 2010-2013. Overall, the results indicate that the likelihood of facing self-fulfilling austerity episodes depends on the magnitude of fiscal multipliers but is generally low. In fact, this probability is below 3% considering calibrations with overall fiscal multipliers lower or equal than 3.

*I am grateful to Ricardo Lagos and Mark Gertler for their guidance and support throughout this project. I also thank Alessandro Dovis, Diego Perez and all participants to presentations at the SED 2019, 2018 7th Atlanta Fed Workshop on International Economics, Fall 2017 Midwest Macro, McMaster University, Rutgers University, George Washington University.

†Department of Economics, Rutgers University, 75 Hamilton St., New Brunswick, NJ, 08901. Email: diego.anzoategui@rutgers.edu

1 Introduction

The European debt crisis of 2010 triggered a debate on the potential effects of fiscal austerity on sovereign spreads. On the one hand, some economists claim that austerity is beneficial because it can increase creditworthiness and reduce the cost of credit.[1] On the other, those against fiscal consolidation argue that it can worsen recessions and even become self-defeating or ineffective in reducing debt to GDP ratios and default risk.[2]

This is an important debate for any policy maker with the task of designing a plan to reduce the stock of external debt. Yet, there is no model with endogenous strategic sovereign default analyzing the impact of austerity on sovereign spreads. This paper aims to fill this gap. It proposes a small open economy general equilibrium model that incorporates the trade-off behind the debate. In this model, fiscal austerity can reduce debt and sovereign spreads, but it can also cause a deeper recession with higher unemployment that might in turn increase default risk and spreads. The model has three salient characteristics. First, in order to display a realistic fiscal policy it incorporates fiscal rules estimated using historical data. These rules enter the model in a similar way to how a Taylor rule is incorporated into a nominal DSGE model. These rules are a key element to identify discretionary changes in fiscal policy aiming to reduce government spending. In other words, I use these rules to define fiscal austerity.

Second, motivated by high unemployment rates in some European countries, the model features downward nominal wage rigidity as in Schmitt-Grohé and Uribe (2016a). This nominal rigidity, coupled with a fixed exchange rate, generates a real rigidity that can cause unemployment in equilibrium. Third, it assumes that the government can strategically default on its debt as in Eaton and Gersovitz (1981) and Arellano (2008). To the best of my knowledge, this is the first paper to propose a model with these three characteristics jointly.

I analyze the theoretical implications of the model and derive conditions under which austerity is more likely to be self-defeating. I find that austerity can be self-defeating only when austerity packages are persistent and the economy is expected to be in a recession with high

[1] "Sharp corrections are needed in countries that already face high and increasing risk premia on their debt. Failure to consolidate would not only raise the cost of borrowing for the government; it would also undermine macroeconomic stability with widespread economic costs." Corsetti (2010)

[2] See Corsetti (2012) for a summary of the discussion. Also see Cafiso and Cellini (2012), Cottarelli (2012), Gros (2011), Krugman (2015) and Wren-Lewis (2016).

fiscal multipliers. The reason is that under those conditions a decrease in government spending can increase the likelihood of a deeper recession in the future and reduce expected taxes in a potential future default scenario and, as a consequence, increase default risk and sovereign spreads even when there is a drop in the stock of external debt.

I also use the model to analyze the case of Spain during the last sovereign debt crisis. Spain was one of the countries that implemented important austerity packages after 2010. I assess the effects of an important decrease in government spending through a counterfactual exercise. I ask what would have happened to sovereign spreads, debt to GDP ratios and economic activity if, instead of implementing fiscal austerity after the second quarter of 2010, Spain had followed historical fiscal rules. I find that austerity was effective in decreasing sovereign spreads and debt to GDP ratios even in a context of relatively high fiscal multipliers. Nevertheless, austerity had meaningful costs in terms of economic activity.

Overall, I find that self-defeating austerities are generally unlikely at least for economies that are similar to Spain. After simulating different calibrations of the model implying different fiscal multipliers I find that the likelihood of facing self-defeating episodes is less than 3% when fiscal multipliers are lower or equal to 3.

Literature Review. This paper is related to three lines of research: (i) fiscal austerity, (ii) models with strategic sovereign default, and (iii) analysis of the Eurozone Crisis.

Regarding fiscal austerity analysis, this paper is most closely related to papers following a structural approach. In that line, House et al. (2017) pursue a similar exercise but without incorporating sovereign debt in their analysis. They find significant output costs as a consequence of austerity. Arellano and Bai (2016) and de Córdoba et al. (2017) use sovereign default models to analyze the Greek crisis. However, these models do not display unemployment in equilibrium. Moreover, de Córdoba et al. (2017) does not analyze the impact of austerity on sovereign spreads and Arellano and Bai (2016) does not use data on fiscal variables to discipline their calibration. Bianchi et al. (2019) is also directly related to this paper. They use a similar model to perform a normative analysis of fiscal policy. They show that wasteful expansionary fiscal policy might be desirable during recessions. Mendoza et al. (2014) analyzes how austerity through tax increases can restore fiscal solvency. However, the authors use a model without unemployment or default risk. Finally, Bi et al. (2013) study the macroeconomic impact of fiscal consolidations when the starting date of an austerity plan is unknown. They find that the composition, monetary policy stance and debt levels are important to determine the final impact.

The fiscal austerity literature has an older empirical branch starting with Giavazzi and Pagano (1990) and Alesina and Ardagna (2010). In these papers the authors use panels of austerity events to identify fiscal austerity shocks. Subsequent developments in this line of research are Alesina et al. (2012) who analyze medium term fiscal plans instead of fiscal policy shocks, and Guajardo et al. (2014) and Alesina et al. (2015) who follow a narrative approach as in Romer and Romer (2010). These papers generally find significant costs in terms of GDP growth, but the size of costs varies depending on the composition of austerity packages. In a similar spirit, Easterly et al. (2008) and Easterly and Servén (2003) remark that fiscal consolidation might have important supply side effects when it is highly focused on public investment. Following a different approach, this paper contributes to this branch of literature by computing the effects of fiscal consolidation on real GDP. Moreover, it also assesses the impact on default risk and sovereign spreads. More recently, Born et al. (2018) use a panel of developing and developed countries and show that the impact of austerity on spreads depends on the fiscal stress of the country. In particular, they show that when spreads and debt to GDP ratio are relatively high fiscal austerity leads to higher spreads. On the opposite side, and consistent with the results in this paper, David et al. (2019) find that austerity announcements reduce sovereign spreads after analyzing a panel of developed and developing countries.

The strategic default part of my model is taken from the sovereign default literature. In particular, I model default as in Eaton and Gersovitz (1981), Arellano (2008) and Mendoza and Yue (2012). I also adopt the technique for modeling long term debt from Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). Hatchondo et al. (2015) incorporate fiscal rules into a sovereign default model, but in the form of debt to GDP and sovereign spreads ceilings. They use their model to perform a different exercise. They assess the impact of debt and spreads ceilings on equilibrium sovereign spreads.

There are several papers that analyze the recent European crisis. This paper contributes to this literature by analyzing how fiscal policy after 2010 affected the severity of the crisis. Lane

(2012) and Shambaugh (2012) provide a detailed description. Martin and Philippon (2014) use a structural DSGE model to assess the different factors that might have caused the crisis. They find that pre-crisis fiscal policy was an important ingredient. Like this paper, Schmitt-Grohé and Uribe (2016a) highlight the importance of downward nominal wage rigidity as a propagation factor in Europe.

Outline. The rest of the paper is organized as follows. Section 2 to 5 describe the model, section 6 analyze the theoretical implications of the model, section 7 applies the model to the case of Spain. Section 8 concludes.

2 Model

The model is in discrete time and describes an economy populated with four different agents: Households, Firms, International Creditors and a Government.

2.1 Household

There is a representative household that chooses consumption (C_t) and labor (H_t) plans to maximize a time separable utility,

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right\} \tag{1}$$

where σ is the risk aversion coefficient, χ is a labor disutility parameter and θ denotes the Frisch elasticity. Consumption (C_t) is a composite of nontradable and tradable goods defined by the following Armington aggregator with elasticity of substitution μ and nontradables weight ω,

$$C_t = \mathcal{C}(C_{Nt}, C_{Tt}) = \left[\omega \left(C_{Nt}\right)^{\frac{\mu-1}{\mu}} + (1-\omega)\left(C_{Tt}\right)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \tag{2}$$

C_{Nt} and C_{Tt} denote nontradable and tradable consumption, respectively. The budget constraint that this agent faces is given by,

$$p_{Ct}C_t = w_t H_t + e_t p_{Tt}\Pi_t - e_t p_{Tt} T_t$$

Where w_t is the nominal wage, e_t is the nomninal exchange rate, p_{Tt} and p_{Nt} represent tradable and nontradable prices. Further, Π_t are firm profits and T_t denotes lump sum taxes, both expressed in units of tradable goods. p_{Ct} is the consumer price level and is defined by,

$$p_{Ct} = \left[\omega^{\mu} p_{Nt}^{1-\mu} + (1-\omega)^{\mu} (e_t p_{Tt})^{1-\mu}\right]^{\frac{1}{1-\mu}}$$

The tradable good has a constant price that I normalized to one, hence, $p_{Tt} = p^* = 1$. Moreover, since the empirical application focuses on a country in the Eurozone, I set $e_t = 1$. Therefore, the budget constraint and consumer price definition can be re-expressed as,

$$p_{Nt}C_{Nt} + C_{Tt} = w_t H_t + \Pi_t - T_t \tag{3}$$

$$p_{Ct} = \left[\omega^{\mu} p_{Nt}^{1-\mu} + (1-\omega)^{\mu}\right]^{\frac{1}{1-\mu}} \tag{4}$$

2.2 Firms

There are two firms that employ labor to produce nontradable and tradable goods. They use the following production technologies,

$$Y_{Nt} = A_{Nt}(H_{Nt}^d)^{\alpha_N} \tag{5}$$

$$Y_{Tt} = A_{Tt}(H_{Tt}^d)^{\alpha_T} \tag{6}$$

Y_{Nt} and Y_{Tt} are the production levels in every sector. Productivity levels are denoted by A_{Nt} and A_{Tt}, and labor demand in each sector is represented by H_{Nt}^d and H_{Tt}^d. I assume that productivity follow AR(1) processes,

$$log(A_{Nt}) = \rho_N log(A_{Nt-1}) + \sigma_N \epsilon^A_{Nt} \tag{7}$$

$$log(A_{Tt}) = \rho_T log(A_{Tt-1}) + \sigma_T \epsilon^A_{Tt} \tag{8}$$

Firms hire labor and maximize the following profit functions,

$$\Pi_{Tt} = A_{Tt}(H^d_{Tt})^{\alpha_T} - w_t H^d_{Tt} \tag{9}$$

$$\Pi_{Nt} = p_{Nt} A_{Nt}(H^d_{Nt})^{\alpha_N} - w_t H^d_{Nt} \tag{10}$$

2.3 Labor Market

As in Schmitt-Grohé and Uribe (2016a), I introduce downward nominal wage rigidity in the labor market. In particular, I assume that nominal wages at time t cannot be lower than a proportion, γ, of the wage level at $t-1$.

$$w_t \geq \gamma w_{t-1} \tag{11}$$

Constraint (11) implies that the labor market might not clear in equilibrium and, therefore, this model can display unemployment. As will be explained in detail in section 6, this rigidity is key determining the size of fiscal multipliers.

2.4 Government

I assume the government is benevolent and has different policy instruments: government spending G_t, the stock of public external debt at the end of period t B_{t+1}, and the decision of defaulting on its debt d_t.

I assume government spending is allocated to tradables and nontradables in a similar way to how the household does it. In other words, I assume G_t is given by,

$$G_t = \left[\omega \left(G_{Nt}\right)^{\frac{\mu-1}{\mu}} + (1-\omega)\left(G_{Tt}\right)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{12}$$

where ω and μ are the private non-tradable weight and elasticity, respectively. As a result, government spending of tradables and nontradables are given by,

$$G_{Nt} = \omega^{\mu}\left(\frac{p_{Nt}}{p_{Ct}}\right)^{-\mu} G_t \qquad G_{Tt} = (1-\omega)^{\mu}\left(\frac{1}{p_{Ct}}\right)^{-\mu} G_t$$

The government issues long-term debt to finance deficits. As in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), the government can issue bonds with a geometrically decaying coupon δ. In particular, a bond issued at time t promises a stream of coupon payments $\delta(1-\delta)^{i-1}$ in periods $t+i$ for $i \geq 1$. Hence, the government budget constraint is given by,

$$p_{Gt}G_t + (1-d_t)\delta B_t = T_t + (1-d_t)q_t\left[B_{t+1} - (1-\delta)B_t\right] \tag{13}$$

where B_t is the face value of external debt, q_t is the price of debt that international creditors are willing to pay, δB_t represents the coupon payments the government needs to pay at time t, and $B_{t+1} - (1-\delta)B_t$ is the amount of debt units issued (if $B_{t+1} \geq (1-\delta)B_t$) or purchased (if $B_{t+1} < (1-\delta)B_t$). Further, d_t represents the default decision, $d_t = 1$ if the country decides to default and $d_t = 0$ otherwise. Debt issuance and coupon payments are multiplied by $(1-d_t)$ because they only happen when the country is not in default.

2.5 International Creditors

As it is usual in the sovereign default literature, international creditors are deep pocket. They have a time separable linear utility function given by,

$$\sum_{t=0}^{\infty} \tilde{\beta}^t \left[C_t^* - \nu_t q_t B_{t+1}\right]$$

where C_t^* is consumption of international creditors and $\nu_t q_t B_{t+1}$ represents a utility cost of holding illiquid assets that is increasing in the total value of bond holdings $q_t B_{t+1}$. Moreover, ν_t is a liquidity shock that acts as an exogenous shock to the price of debt as in Krishnamurthy and Vissing-Jorgensen (2012). This shock is incorporated so that spreads in equilibrium can be affected by factors that are exogenous from the government's point of view. Typical narratives of the European debt crisis put an important weight on these types of shocks.[3]

No arbitrage implies the following pricing equation,

$$q_t = \frac{\tilde{\beta}}{1+\nu_t} E_t \left\{ (1-d_{t+1}) \underbrace{(\delta + (1-\delta) q_{t+1})}_{\text{payoff if repay}} + d_{t+1} \underbrace{q_{t+1}^D}_{\text{price in default}} \right\} \tag{14}$$

As expressed in equation (14), q_t is the discounted expected payoff flow at $t+1$. Future payments depend on the default decision d_{t+1}. Hence, if government does not default ($d_{t+1} = 0$) creditors receive a coupon payment δ and the market value of outstanding debt $(1-\delta)q_{t+1}$. On the other hand, if government defaults creditors receive a defaulted bond with price q_{t+1}^D. The price of a defaulted bond is given by,

$$q_t^D = \frac{\tilde{\beta} R^*}{1+\nu_t} E_t \left\{ \phi \left[(1-\tilde{d}_{t+1}) \underbrace{(\delta + (1-\delta) q_{t+1})(1-\psi)}_{\substack{\text{payoff if return} \\ \text{and repay}}} + \tilde{d}_{t+1} \underbrace{q_{t+1}^D}_{\substack{\text{payoff if return} \\ \text{and default}}} \right] + (1-\phi) \underbrace{q_{t+1}^D}_{\text{no return}} \right\} \tag{15}$$

A country in default faces an exogenous probability ϕ to have the chance of regaining access to international capital markets. If it happens to have that chance the government has to decide whether to start repaying the outstanding debt or not. The repaying decision is represented by the endogenous variable $\tilde{d}_t$ that equals zero if there is repayment and one otherwise. As noted in

[3]For example, Mario Draghi's "whatever it takes" speech in July 2012. See here.

equation (15), if the government has access to capital markets and decides to start repaying, the government receives the usual repayment with a face value haircut ψ, $(\delta+(1-\delta)q_{t+1})(1-\psi)R^*$.[4] Moreover, the international creditor keeps the same defaulted bond if the country can not regain access to capital markets or is able to return but decides not to start repaying. In other words, the repayment in those cases is given by $q^D_{t+1}R^*$. Note that both repayments in equation (15) incorporate accrued interests on defaulted debt. In particular, we assume the the stock of defaulted debt is updated by the risk free interest rate R^*.

In both price equations (14) and (15) the liquidity premium shock is given by $\nu_t = e^{m_t+\frac{\sigma_m^2}{2(1-\rho_m^2)}} - 1$, where m_t is a random variable that follows the AR(1) process,

$$m_t = \rho_m m_{t-1} + \sigma_m \epsilon_t^m$$

where ϵ_t^m is a standard normally distributed shock.

3 Implementable Equilibrium

Given $B_0, A_{N0}, A_{T0}, m_0, G_0$, a sequence of fiscal policy variables $\{G_t, T_t, B_{t+1}, d_t, \tilde{d}_t\}_{t=0}^{\infty}$ and a shocks sequence $\{A_{N,t+1}, A_{T,t+1}, m_{t+1}\}_{t=0}^{\infty}$, an equilibrium is a set of prices $\{w_t, p_{Nt}, p_{Ct}, q_t\}_{t=0}^{\infty}$ and allocations $\{H_{Nt}, H_{Tt}, H_t^s, C_{Nt}, C_{Tt}\}$ such that,

1. Household maximizes utility (1) subject to (3)

2. Firms maximize profits (9) and (10)

[4] The exogenous haircut assumption is made for simplicity but it is common practice in quantitative sovereign default models. For models with endogenous haircuts see: Benjamin and Wright (2013) and Yue (2010)

3. Markets clear

$$G_{Nt} + C_{Nt} = Y_{Nt} \tag{16}$$

$$C_{Tt} + G_{Tt} + \delta B_t = Y_{Tt} + q_t \left[B_{t+1} - (1-\delta)B_t\right] \tag{17}$$

$$(w_t - \gamma w_{t-1})(H_t^s - H_{Nt}^d - H_{Tt}^d) = 0 \tag{18}$$

$$w_t \geq \gamma w_{t-1} \tag{19}$$

4. q_t satisfies (14)

Note that market clearing in labor markets (equations (18) and (19)) is not standard because of downward nominal wage rigidity. In particular, labor demand $H_{Nt}^d + H_{Tt}^d$ determines the equilibrium total working hours. If the nominal wage rigidity constraint is slack $w_t > \gamma w_{t-1}$ labor demand equals supply (H_t^s) and there is no unemployment. In turn, if the constraint binds total hours equal $H_{Nt}^d + H_{Tt}^d$ and there is positive unemployment as $H_t^s > H_{Nt}^d + H_{Tt}^d$.

4 Government Policy

The government has access to three different instruments when it has access to capital markets: a sovereign default decision, government spending G_t, and the face value of external debt B_{t+1}. For a given set of prices in equilibrium, the interaction of these three instruments determines the evolution of the net taxes T_t through the government budget constraint given by,

$$p_{Ct} G_t + \delta B_t - q_t \left[B_{t+1} - (1-\delta)B_t\right] = T_t \tag{20}$$

I assume that the government follows fiscal rules to determine G_t and B_{t+1}. These fiscal rules describe fiscal policy in normal times and will be calibrated using fiscal data. I will provide more details on these rules below.

I also assume that the government can strategically choose to default even when the government is able to get the necessary funds to repay the debt. This default decision is modeled as in typical sovereign default models à la Eaton and Gersovitz (1981). Hence, every period the government decides whether to repay the debt and keep having access to international debt markets, or to default and stay in autarky for a random number of periods bearing productivity costs.

4.1 Fiscal Rules

Following Blanchard and Perotti (2002), I assume that government spending G_t evolves according to the following fiscal rule,

$$\log\left(\frac{G_t}{\bar{G}}\right) = \rho_G \log\left(\frac{G_{t-1}}{\bar{G}}\right) + \rho_{GY}\log\left(\frac{Y_{t-1}}{\bar{Y}}\right) + \rho_{GB}\log\left(\frac{B_{t-1}}{\bar{B}}\right) + \sigma_G \epsilon_t^G \tag{21}$$

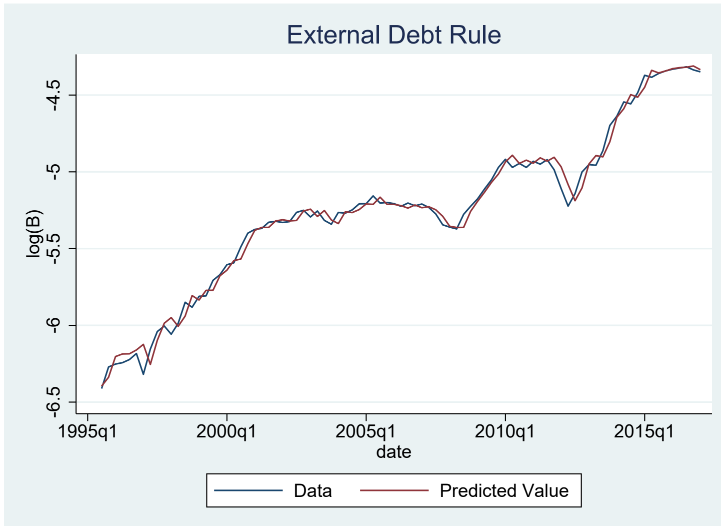
where ϵ_t^G is a standard normally distributed fiscal shock, Y_{t-1} is lagged output and B_{t-1} is the lagged debt level. Further, $\bar{B}$, $\bar{G}$ and $\bar{Y}$ are debt, government spending and GDP in a non-stochastic steady state without default risk. This fiscal rule includes lagged GDP and debt in order to allow for systematic changes in fiscal policy as a respond to changes in fundamentals. Hence, for instance, the government might systematically react to low GDP by increasing government spending. The implicit assumption in this rule is that the government can only react to changes in GDP or debt to GDP ratios with a lag of one quarter. This lag is supposed to be related with the time needed to pass new legislation or a lag between the time macroeconomic shocks arrive and the release of new economic data measuring the consequences of these shocks.

Moreover, the face value of external debt follows the fiscal rule given by,

$$\log\left(\frac{B_{t+1}}{\bar{B}}\right) = \gamma_B \log\left(\frac{B_t}{\bar{B}}\right) + \gamma_G \log\left(\frac{p_{Gt}G_t}{\bar{G}}\right) + \gamma_Y \log\left(\frac{Y_t^{nom}}{\bar{Y}}\right) \tag{22}$$

where Y_t^{nom} is nominal GDP and B_t is the stock of external public debt. The rule (22) matches the law of motion of external debt in Spain quite well. As shown in figure 1, the rule is a good approximation of how the stock of external debt evolves. The figure shows that the rule predicted values are close to the actual data. In fact, the R^2 value is 98.9%.

Figure 1: Debt Rule. Actual and Predicted Values



4.2 Default Decision

The Government decides whether to repay or default in order to maximize Household's welfare. This welfare maximization is constrained by the implementable equilibrium conditions defined in section 3 and the fiscal rules defined in section 4.1. As it is usual in the sovereign default literature I focus on a Markov Perfect Equilibrium. This equilibrium definition implies that the default decision depends on a set of states $\mathbf{S}$, ie. $d = \Phi(\mathbf{S})$. The relevant set of states is given by $\mathbf{S} \equiv \{\mathbf{S_1}, \mathbf{S_2}\}$, where $\mathbf{S_1} \equiv \{B, w_{-1}\}$ and $\mathbf{S_2} \equiv \{A_N, A_T, G, m\}$. $\mathbf{S_1}$ is composed by the two endogenous states in the economy, the face value of debt and the lagged wage level. Further, $\mathbf{S_2}$ groups the exogenous variables that hit the economy and follow stochastic processes described in section 2.

Let $\mathcal{V}(\mathbf{S}, \kappa)$ be the government's value function before making the repayment/default decision. Also, let $\mathcal{V}^R(\mathbf{S})$ and $\mathcal{V}^D(\mathbf{S})$ be the value functions after deciding to repay and default, respectively. Define $C(\mathbf{S}), H(\mathbf{S}), Y(\mathbf{S}), p_G(\mathbf{S}), w(\mathbf{S})$ as consumption, hours, total output, price of government spending and wage rate consistent with the implementable equilibrium. Hence, for a given price function $q(\mathbf{S_1'}, \mathbf{S_2})$ where $\mathbf{S_1'} \equiv \left\{B', w_{-1}'\right\}$ the value functions satisfy,

$$\mathcal{V}(\mathbf{S},\kappa)=\max_{d\in\{0,1\}}\Big\{(1-d)\,\mathcal{V}^R(\mathbf{S})+d\Big[\mathcal{V}^D\Big(\mathbf{S}\Big)-\kappa\Big]\Big\} \tag{23}$$

$\mathcal{V}(\mathbf{S},\kappa)$ summarizes the default/repayment decision. Every period the government compares the value of repaying $\mathcal{V}^R$ with the value of defaulting $\mathcal{V}^D$ and acts accordingly. The variable κ is an iid utility cost of default with mean zero and standards deviation σ_κ. This variable is incorporated with the aim of improving the convergence properties of the quantitative solution of the model.[5]

The value of repayment $\mathcal{V}^R(\mathbf{S})$ satisfies,

$$\begin{aligned}
\mathcal{V}^R(\mathbf{S}) &= u\left(C(\mathbf{S}),H(\mathbf{S})\right)+\beta E\left\{\mathcal{V}(\mathbf{S}',\kappa')|\mathbf{S}\right\} \qquad (24)\\
&st.\\
\log\left(\frac{B'}{\bar{B}}\right) &= \gamma_B\log\left(\frac{B}{\bar{B}}\right)+\gamma_G\log\left(\frac{p_G(\mathbf{S})G}{\bar{G}}\right)+\gamma_Y\log\left(\frac{Y^{nom}(\mathbf{S})}{\bar{Y}}\right)\\
w'_{-1} &= w(\mathbf{S})\\
\log\left(\frac{G'}{\bar{G}}\right) &= \rho_G\log\left(\frac{G}{\bar{G}}\right)+\rho_{GY}\log\left(\frac{Y(\mathbf{S})}{\bar{Y}}\right)+\rho_{GB}\log\left(\frac{B}{\bar{B}}\right)+\sigma_G\epsilon^G\\
&+\text{ Equilibrium Conditions}
\end{aligned}$$

The value of repayment consists of an instantaneous utility $u\left(C(\mathbf{S}),H(\mathbf{S})\right)$ and a discounted continuation value $E\left\{\mathcal{V}(\mathbf{S}')|\mathbf{S}\right\}$. The first two constraints in (24) are the laws of motion of the endogenous state variables of the problem. The first one is the fiscal rule for the stock of debt, whereas the second describes the law of motion for lagged wages w_{-1}, it simply states that future lagged wages are equal to wages today. The third constraint shows the fiscal rule for government spending G, and the fifth line highlights the fact that private allocations are consistent with the implementable equilibrium.

The value in default $\mathcal{V}^D(\mathbf{S})$ is given by,

[5] Default models with long-run debt have poor convergence properties. See Chatterjee and Eyigungor (2012).

$$\mathcal{V}^D(\mathbf{S}) = u\left(C(\mathbf{S_1}, \tilde{\mathbf{S}}_\mathbf{2}), H(\mathbf{S_1}, \tilde{\mathbf{S}}_\mathbf{2})\right) + \beta E\left\{\phi \max\left\{\mathcal{V}^D(\mathbf{S}'), \mathcal{V}^R((1-\psi)B', w'_{-1}, \mathbf{S}'_\mathbf{2})\right\} + (1-\phi)\,\mathcal{V}^D(\mathbf{S}')|\mathbf{S}\right\} \tag{25}$$

$st.$

$T(\mathbf{S_1}, \tilde{\mathbf{S}}_\mathbf{2}) = p_G(\mathbf{S_1}, \tilde{\mathbf{S}}_\mathbf{2})G$

$B' = BR^*$

$w'_{-1} = w(\mathbf{S_1}, \tilde{\mathbf{S}}_\mathbf{2})$

$\log\left(\frac{G'}{\bar{G}}\right) = \rho_G \log\left(\frac{G}{\bar{G}}\right) + \rho_{GY}\log\left(\frac{Y(\mathbf{S})}{\bar{Y}}\right) + \rho_{GB}\log\left(\frac{B}{\bar{B}}\right) + \sigma_G\epsilon^G$

+ Equilibrium Conditions

$\mathcal{V}^D(\mathbf{S})$ consists of an instantaneous utility level plus a continuation value. In this case, the continuation value includes an exogenous probability of coming back to international capital markets, ϕ. In particular, the max operator in the expectation term in (25) represents decision of starting to repay the debt once the country has the chance to regain access to markets with probability ϕ. In this case, if the country decides to start repaying the debt, the stock of debt gets a haircut of ψ percent incorporated in the argument of $\mathcal{V}^R$. In turn, if the government decides to delay the repayment, the haircut is not applied and the country continues as if it had not had the chance of regaining access to international capital markets.

The first constraint in (25) is the government budget constraint, where the value of government spending equals net taxes. The second highlights the assumption that, once in default, the stock of debt that the government owes is updated by the international risk free rate $R^* = 1/\tilde{\beta}$. The third constraint is the law of motion for wages, and the forth represents the government spending fiscal rule. Hence, this model assumes that once in default, the government follows the government spending fiscal rules and adjusts net taxes in order to keep a balanced budget.

Moreover, $\tilde{\mathbf{S}}_2 \equiv \left\{\tilde{A}_N, \tilde{A}_T, G, m\right\}$ in (25) is a modified state vector that includes exogenous productivity costs of default.[6] In particular, I define $D(A)$ as a productivity cost of default and

[6] This is a typical assumption in sovereign default models. Mendoza and Yue (2012) provide a microfoundation for this assumption. They claim that during financial autarky firms are not able to import certain intermediate

set,

$$\tilde{A}_N = A_N - D(A_N)$$
$$\tilde{A}_T = A_T - D(A_T)$$

As it is usual in the sovereign default literature productivity costs of default $D(A)$ are convex. This is key assumption to generate more incentives to default when the economy is facing low productivity levels, and therefore low GDP. As Chatterjee and Eyigungor (2012) claim, this specific functional form allows sovereign default models to better match the spreads data. I follow Chatterjee and Eyigungor (2012) and adopt the following functional form that is useful to match the spread volatility that we observe in the data,

$$D(A) \equiv \max\left\{0, d_0 A_N + d_1 A_N^2\right\}$$

where d_0 and d_1 are parameters.

5 Recursive Equilibrium

The recursive Markov Perfect Equilibrium incorporates the optimal default decision of the government subject to the implementable equilibrium defined in section 3.

A Markov Perfect Equilibrium is a set of value functions $\mathcal{V}(\mathbf{S}), \mathcal{V}^R(\mathbf{S}), \mathcal{V}^D(\mathbf{S})$, a default decision $d = \Phi(\mathbf{S})$, a decision of defaulting when the country is in default and has the chance to regaining access $\tilde{d} = \tilde{\Phi}(\mathbf{S})$ and price schedules $q(\mathbf{S_1'}, \mathbf{S_2})$ and $q^D(\mathbf{S_1'}, \mathbf{S_2})$ such that,

1. Given the price schedules $q(\mathbf{S_1'}, \mathbf{S_2})$ and $q^D(\mathbf{S_1'}, \mathbf{S_2})$, the value functions solve problems (23), (24) and (25)

goods, generating lower levels of productivity.

2. Given the default decision, the price schedules satisfy the following equation

$$q(\mathbf{S_1'}, \mathbf{S_2}) = \frac{\tilde{\beta}}{1+\nu_t} E\left\{m' \left\{(1-\Phi(\mathbf{S}'))\left(\delta + (1-\delta) q(\mathbf{S_1''}, \mathbf{S_2'})(1-\psi)\right) + \Phi(\mathbf{S}') q^D(\mathbf{S_1''}, \mathbf{S_2'})\right\} | \mathbf{S_1'}, \mathbf{S_2}\right\} \tag{26}$$

$$\begin{aligned} q^D(\mathbf{S_1'}, \mathbf{S_2}) = & \frac{\tilde{\beta}}{1+\nu_t} E_t\{\phi \left[(1-\tilde{\Phi}(\mathbf{S}'))\left(\delta + (1-\delta) q(\mathbf{S_1''}, \mathbf{S_2'})\right) + \tilde{\Phi}(\mathbf{S}') q^D(\mathbf{S_1''}, \mathbf{S_2'}) R^*\right] \\ & + (1-\phi) q^D(\mathbf{S_1''}, \mathbf{S_2'}) R^* | \mathbf{S_1'}, \mathbf{S_2}\} \end{aligned} \tag{27}$$

3. The default decisions solve problems (23) and (25)

6 Analysis

The model described in the previous sections is rich enough to display non-linear responses to fiscal policy. As a consequence, the effect of changes in government expenditure depends on the model parameters but also on what part of the state space the economy is located. In this section I describe the mechanisms behind the fiscal policy impact. In particular, I first explain the main mechanism that determines the size and sign of fiscal multipliers. Second, I analyze when fiscal austerity can become become self-defeating, that is, when a reduction in government spending can actually increase the default probability and sovereign spreads.

6.1 Fiscal Multipliers

What is the impact of a decrease in government spending ? To provide some intuition consider the following equation that comes from the household's first order conditions with respect to consumption of nontradables C_{Nt} and tradables C_{Tt},

$$
\begin{aligned}
p_{Nt} &= \frac{\omega}{1-\omega}\left(\frac{C_{Tt}}{C_{Nt}}\right)^{1/\mu} \\
p_{Nt} &= \frac{\omega}{1-\omega}\left(\frac{Y_{Tt}-G_{Tt}+\Delta\tilde{B}_{t+1}}{Y_{Nt}-G_{Nt}}\right)^{1/\mu}
\end{aligned} \tag{28}
$$

where $\Delta\tilde{B}_{t+1}$ represents debt issuance minus coupon payments $\Delta\tilde{B}_{t+1} \equiv q_t\left[B_{t+1}-(1-\delta)B_t\right]-\delta B_t$. The second line in (28) replaces C_{Nt} for the market clearing condition for nontradable goods, and C_{Tt} for the balance of payments equation. This equation implies that, for a given level of Y_{Tt} and nontradables output Y_{Nt}, the impact of a drop in government spending on p_{Nt} has three different channels: (i) there is a reduction in government spending of nontradables G_{Nt} that tends to reduce the relative price of nontradables, (ii) there is a drop in tradables spending G_{Tt} that tends to generate an increase in p_N, and (iii) if austerity manages to reduce the fiscal deficit, there is also a reduction in debt issuance $\Delta\tilde{B}_{t+1}$ which reduces the supply of tradables and, hence, makes p_N decrease. These three different forces imply that the impact of austerity on p_N is unknown a priori. However, as we show in the appendix, under our assumption that the weights on non-tradable goods in the CES aggregators defining C_t and G_t are the same (they equal ω in both cases), the main impact of austerity on p_N is through debt issuance minus coupon payments $\Delta\tilde{B}_{t+1}$. Therefore, if austerity reduces the amount of external debt issued it will then reduce the price of nontradables.

The drop in p_{Nt} has an effect on the labor market. In particular, consider the labor demand for tradables and nontradables that come from maximizing (9) and (10), respectively.

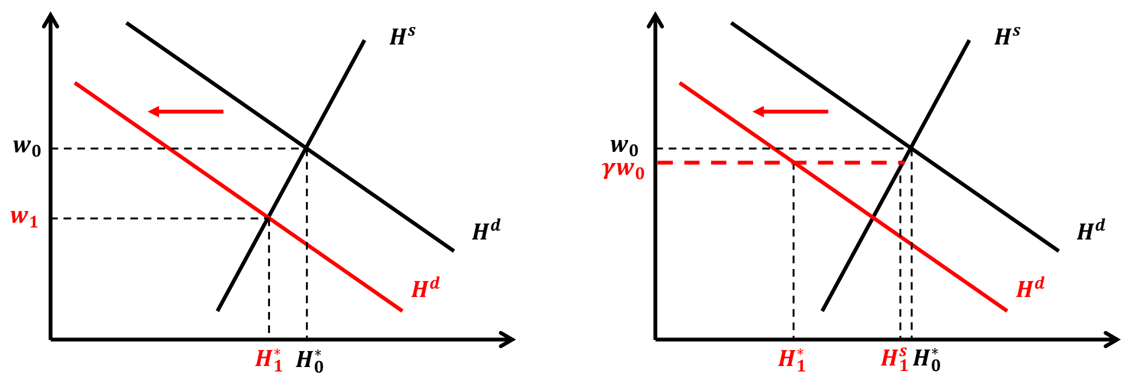
$$
H_{Tt}^d = \left(\frac{\alpha_T A_{Tt}}{w_t}\right)^{\frac{1}{1-\alpha_T}} \tag{29}
$$

$$
\downarrow H_{Nt}^d = \left(\frac{\downarrow p_{Nt}\alpha_N A_{Nt}}{w_t}\right)^{\frac{1}{1-\alpha_N}} \tag{30}
$$

Equation (30) shows that a drop in p_N generates a fall in the nontradable sector labor demand. This fall in labor demand generates a drop in total hours worked and GDP. Hence, the more

p_N drops the higher the fiscal multiplier. However, the magnitude of the final impact on GDP also depends on how much wages decrease as a consequence of austerity. Here is where the downward nominal wage rigidity constraint is important. If the constraint does not bind and wages can freely decrease, the total effect of austerity is mitigated by the drop in wages. Lower wages increase labor demand and reduce the initial drop in H_{Nt} in equation (30) reducing fiscal multipliers. In turn, if wages are sticky downwards the total effect on GDP is more important.

Figure 2: Occasionally Binding Constraint and Fiscal Multipliers



(a) Non-binding constraint

(b) Binding constraint

The two different cases are depicted in figure 2. Panel (a) shows the case in which wages fall reducing the total impact on quantities. On the other hand, panel (b) illustrates the case in which the downward nominal wage constraint binds and wages can fall only up to γw_0. In this second case the impact of the fiscal shock is much more important. As wages can not adjust, labor demand ends up being lower than supply resulting in positive unemployment levels ($H_1^s > H_1^*$).

The following proposition provides an expression summarizing our discussion.

Proposition 1. *Assuming that there is no default risk, the elasticity of real GDP with respect to government spending around the steady state is given by,*

$$\frac{d\log(GDP)}{d\log(G)} = \frac{\kappa\gamma_G}{\frac{\Theta_{cT}}{\Theta_{qB}} - \kappa\gamma_Y} - \iota_w \frac{d\log(w)}{d\log(G)} \tag{31}$$

where,

$$\kappa \equiv \frac{\Theta_{YN}\frac{\alpha}{1-\alpha}}{\mu\frac{\alpha}{1-\alpha}\frac{\Theta_{YN}}{\Theta_{CN}} - \frac{\Theta_{qB}}{\Theta_{CT}}\left(\gamma_G\frac{\Theta_{GN}}{\Theta_G} + \gamma_Y\Theta_{YN}\right)}$$

$$\iota_w \equiv \frac{\alpha}{1-\alpha}\left[\frac{\frac{\Theta_{CT}}{\Theta_{qB}} + r^*\kappa}{\frac{\Theta_{CT}}{\Theta_{qB}} - \kappa\gamma_Y}\right]$$

Where Θ_X represents the GDP share of variable X ($\Theta_X \equiv \frac{p_X X}{GDP}$) and r^* is the risk free net international interest rate.

Proof. See Appendix D. □

The first term in the right side of (31) represents the impact of austerity under the assumption that wages do not change and sovereign spreads are not affected by austerity. As noted before, under our assumptions, the main channel through which government spending affects economic activity is its effect on debt issuance $q_t\left[B_{t+1} - (1-\delta)B_t\right]$. Lower government spending might imply lower debt issuance, which reduces the supply of tradable goods and the price of nontradables p_N. Hence, as shown in the first term of (31) the parameters of the fiscal rule for B_{t+1}, γ_G and γ_Y, seem to be key for the fiscal austerity elasticity. In particular, higher γ_G and γ_Y imply a more important impact of austerity when wages are sticky.

The second term in the right side of (31) describes the role of nominal wages during austerity. As mentioned before, if nominal wages fall during the austerity period then $\frac{d\log(w)}{d\log(G)} > 0$ and, therefore, the total impact on GDP is attenuated. The magnitude of this second term determines the nonlinearity of fiscal multipliers. The higher this term is, the more state-dependent fiscal multiplier are. The magnitude of this term will depend on the value of ι_w that captures the positive effect on production of a decrease in nominal wages. Note that ι_w is an increasing function of γ_G and γ_Y. Higher γ_G and γ_Y imply more important drops in p_N during austerity and, hence, lower wages. This implies that higher values of γ_G and γ_Y increase fiscal multipliers when wages are sticky, but they have an ambiguous impact on flexible-wage multipliers.

The previous analysis shows that fiscal multipliers can be highly non linear and state dependent. But how big can they be with reasonable parameter values ? Table 1 shows the impact multipliers for a version of the model model without sovereign risk, using parameter values in table 3, and considering the effect of fiscal rules parameters and the downward nominal wage rigidity constraint. The Impact multiplier is defined as,

$$\mu_G = \frac{\Delta GDP_t}{\Delta G_t} = \frac{\bar{p}_N \Delta Y_{Nt} + \Delta Y_{Tt}}{\Delta G_t} \tag{32}$$

where $\bar{p}_N$ is the steady-state relative price of nontradables to tradables goods, ΔGDP_t and ΔG_t represent the change in real GDP and government spending.

Table 1: Impact Multipliers, Parameters and Wage Rigidity

Parameters	Flexible Wages	Fixed Wages
Spain calibration		
$\gamma_G = 0.15$ and $\gamma_Y = -0.26$	0.74	1.94
Debt issuance G parameter		
$\gamma_G = 0.1$ and $\gamma_Y = -0.26$	0.97	1.26
$\gamma_G = 0.25$ and $\gamma_Y = -0.26$	0.18	3.45
Debt issuance GDP parameter		
$\gamma_G = 0.15$ and $\gamma_Y = -0.4$	0.85	1.55
$\gamma_G = 0.15$ and $\gamma_Y = 0$	0.48	3.67

Notes. The table shows impact multipliers when government spending decreases by 1% relative to steady state values. I consider an economy without default risk in this case. Values for different key parameters are shown in different rows. The cases of flexible and completely fixed wages are separated in columns.

The columns in Table 1 show μ_G assuming flexible and fixed wages using as a starting point a non-stochastic steady state without default risk[7]. The rows compute the multiplier for different

[7] I assume no default risk in this calculation.

fiscal rule parameters. Particularly, I analyze different values of γ_G and γ_Y calibrating the rest of parameters according to table 3. The estimated fiscal rules for Spain imply parameters $\gamma_G = 0.15$ and $\gamma_Y = -0.26$. These parameters imply fiscal multipliers of 0.74 when wages are flexible and 1.94 when they fixed. In general, table 1 shows that multipliers can be highly non-linear for high values of γ_G and γ_Y. Multipliers tend to be below one with flexible wages and above one with fixed wages. Note that, the fixed wage multipliers are high but in line with fiscal multipliers in recessions that are found using regime switching VARs.[8]

6.2 Fiscal Policy and Sovereign Spreads

What is the effect of fiscal austerity on sovereign spreads ? The model presented in this paper allows us to identify under what conditions fiscal austerity might not be effective in reducing sovereign spreads. In particular, we can describe when it is more likely to face a self-defeating austerity situation.

In order to get intuition let's consider a simplified version of the model. Let's assume a government that starts with a government spending policy such that $G_t = \bar{G}$ for $t \geq 0$, and decides to carry out austerity and reduces spending by ΔG at $t = 0$ and by $\rho \Delta G$ at $t = 1$. The parameter $\rho \geq 0$ measures how persistent the austerity program is expected to be. To simplify the exposition let the value of defaulting be equal to $\mathcal{V}^D(G)$ where $\mathcal{V}^{D'}(G) \leq 0$. The assumption of a negative effect of government spending on $\mathcal{V}^D(G)$ is consistent with the quantitative results. This result is due to the fact that the government budget constraint is balanced in default and, therefore, higher government spending implies higher taxes and lower utility levels when the country decides to default. In addition, we focus on a situation in which wages are fixed for all t. You can interpret this last simplification as the assumption that the country is in a recession with positive unemployment. Moreover, to focus on sovereign spreads in one given period, assume that the government lacks commitment to repay its debt at $t = 1$ only.

Under this scenario, what is the effect of fiscal austerity on the default probability and price of bonds that need to be repaid at $t = 1$? As we mentioned before, in our model the default decision is carried out by a benevolent planner that compares the expected welfare associated to

[8]See for example Auerbach and Gorodnichenko (2012), Nicoletta Batini and Melina (2012) and Owyang et al. (2013).

repay $\mathbb{E}_0\mathcal{V}^R(B_1, G_1)$ with that of defaulting $\mathbb{E}_0\{\mathcal{V}^D(G_1) - \kappa\}$, where κ is a normally distributed utility cost of default. Note that I am dropping the the state variables w_{-1}, A_N, A_T, m from the arguments of the value functions to ease notation. The default probability is therefore,

$$\begin{aligned} P\left(d_1 = 1\right) &= P\left(\mathcal{V}^R(B_1, G_1) < \mathcal{V}^D(G_1) - \kappa\right) \\ &= \Phi\left(\frac{\mathbb{E}_0\mathcal{V}^D(G_1) - \mathbb{E}_0\mathcal{V}^R(B_1, G_1)}{\sigma_\kappa}\right) \end{aligned}$$

The last expression just states that the probability of defaulting at the beginning of $t = 1$ depends upon the difference between the expected welfare associated to defaulting and that of repaying the debt. The second line uses the fact that κ is normally distributed and independent from other exogenous states in the model. Φ represents the normal cdf and σ_κ its standard deviation. From this equation we can infer that if a decrease in government spending is increasing spreads, then it must be that it is increasing the difference of expected value functions $\mathbb{E}_0\mathcal{V}^D(G_1) - \mathbb{E}_0\mathcal{V}^R(B_1, G_1)$. The impact of austerity on this difference can be represented by the following expression where we are assuming differentiability to simplify the exposition,

$$\Delta\mathbb{E}_0\mathcal{V}^D(G_1) - \Delta\mathbb{E}_0\mathcal{V}^R(B_1, G_1) = \underbrace{-\frac{\partial\mathbb{E}_0\mathcal{V}^R(B_1, G_1)}{\partial B_1}\frac{\partial B_1}{\partial G_0}\Delta G_0}_{B_1 \text{ Effect } (<0)} + \underbrace{\left[\frac{\partial\mathbb{E}_0\mathcal{V}^D(G_1)}{\partial G_1} - \frac{\partial\mathbb{E}_0\mathcal{V}^R(B_1, G_1)}{\partial G_1}\right]\rho\Delta G_0}_{G_0 \text{ Persistence Effect } (?)} \tag{33}$$

Note that equation (33) clearly defines two relevant channels through which austerity can affect the expected value functions difference and, hence, spreads. I called the first channel "B_1 effect". This effect summarizes the impact on spreads through a debt reduction. If If we assume the usual result in sovereign default models that the derivative of $\mathbb{E}_0\mathcal{V}^R$ with respect to external debt is negative and that austerity reduces the stock of debt $\frac{\partial B_1}{\partial G_0}\Delta G_0 < 0$ this term has a negative sign. A negative sign means that austerity increases the expected welfare for repaying more than that of defaulting. Thus, expected default risk and sovereign spreads decrease through this channel. The intuition is simple, a reduction in the stock of debt makes repayment more likely because it is less costly to repay that stock in terms of welfare.

The second term in (33) describes what I call "G_0 Persistence Effect". This channel is associated with persistent fiscal austerities only. It represents the impact of a change in expectations about future government spending associated with the reduction in G at $t = 0$. Note that the parameter $\rho \geq 0$ is in that term . This is because, under our simplifying assumptions $\frac{\partial G_1}{\partial G_0} = \rho$. This term can take positive or negative values depending on the state of the economy. We can divide the analysis of this effect in two important cases. First, note that if austerity is not persistent ($\rho = 0$) this effect is null. As a consequence, the only effect of austerity on spreads is the "B_1 Effect" represented by the first term. As discussed above, this implies that sovereign spreads must decrease. Therefore, our first conclusion from this analysis is that a necessary (although not sufficient) condition for a self-defeating austerity is to have a persistent austerity effort ($\rho > 0$). In other words, austerity at time $t = 0$ has to affect the expectations of future levels of government spending. Mathematically, the reason is that the second term in (33) can be positive only when $\rho > 0$. The second important case is when austerity is actually persistent. We just stated that persistence is necessary but not sufficient. Hence, a natural question to ask is, when is austerity more likely to be self-defeating assuming a persistent austerity effort? Mathematically, we can rephrase the question as: when is the G_0 persistence effect more likely to be positive and large? Looking at the second term and bearing in mind that $\Delta G_0 < 0$ we can infer that a self-defeating austerity is more likely when the derivatives difference $\partial \mathbb{E}_0 \mathcal{V}_1^D / \partial G_1 - \partial \mathbb{E}_0 \mathcal{V}_1^R / \partial G_1$ is strongly negative. This difference represents the impact on the relative expected welfare between the default and repayment options of an additional future decrease in government spending at $t = 1$. The sign of this term is unknown and depends on the parameter values and state of the economy.

Intuitively, the G_0 persistence effect has an unknown sign because the expectations of a lower government spending has different opposite effects. First, remember that taxes equal government spending when the country is in default $T_t = p_{Gt} G_t$. Therefore, the expectation of a reduction in the size of the government at $t = 1$ implies that taxes (T_t) would be lower in a potential default scenario and, therefore, expected welfare for defaulting increases (implying a negative derivative $\mathbb{E}_0 \mathcal{V}_1^D / \partial G_1 < 0$). Second, the effect of a future decrease in government spending has an unknown effect on the value for repaying the debt, in other words, the derivative $\mathbb{E}_0 \frac{\partial \mathcal{V}_1^R}{\partial G_1}$ has an unknown sign a-priori. This is because a future reduction in government spending has two opposite effects in expected welfare under repayment. On one hand, a future decrease in government spending decreases the expected stock of debt to be repaid in the future which tends to increase welfare (implying a negative derivative $\mathbb{E}_0 \partial \mathcal{V}_1^R / \partial G_1$). On the other, future austerity can generate a deeper recession at $t = 1$ reducing consumption and welfare determining a positive derivative $\mathbb{E}_0 \partial \mathcal{V}_1^R / \partial G_1$.

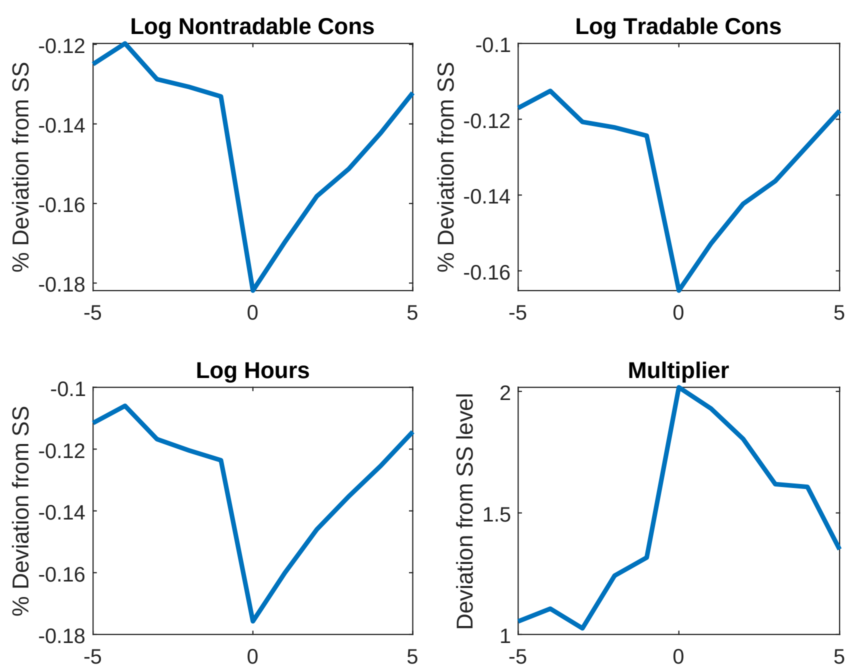
We could conclude that we need to assess the model quantitatively to understand the relative importance of each component and determine the sign of the G_0 persistence effect. However, there is one conclusion we can get before moving to an empirical application. We can state that a self-defeating austerity is more likely when the derivative $\partial \mathbb{E}_0 \mathcal{V}_1^R / \partial G_1$ is high (not necessarily positive, but high). In other words, the recessionary impact of a future low government spending has to be important. When is $\partial \mathbb{E}_0 \mathcal{V}_1^R / \partial G_1$ high ? We can get an answer by just taking derivatives of this value function,

$$\frac{\mathbb{E}_0 \partial \mathcal{V}^R(B_1, G_1)}{\partial G_1} = \mathbb{E}_0 \left\{ C_1^{-\sigma} \frac{\partial C_1}{\partial G_1} - H_1^{\frac{1}{\theta}} \frac{\partial H_1}{\partial G_1} \right\} + \beta \mathbb{E}_0 \left\{ \frac{\partial \mathcal{V}^R(B_2, \bar{G})}{\partial B_2} \frac{\partial B_2}{\partial G_1} \right\} \tag{34}$$

Where the first right side expectation in (34) represents the impact of a change in G_1 on the instantaneous utility function. The last term is the impact on welfare of an additional decrease in the stock of debt to be repaid from $t = 2$ on. By taking a look at the first term we can get some intuition on when it would be more likely to have a self-defeating austerity. Actually, assuming that $\frac{\partial H_1}{\partial G_1} > 0$, we can see that $\partial \mathbb{E}_0 \mathcal{V}_1^R / \partial G_1$ is higher when expected consumption and hours worked at $t = 1$, C_1 and H_1, are low. Moreover, $\partial \mathbb{E}_0 \mathcal{V}_1^R / \partial G_1$ tends to be higher the more important the response of consumption to government spending is: a high $\partial C_1 / \partial G_1$. A relatively high $\partial C_1 / \partial G_1$ happens when fiscal multipliers tend to be high. Hence, the second implication in this analysis is that austerity at $t = 0$ is more likely to end up being self-defeating if the country is expected to be in a recession (low consumption and employment) with high fiscal multipliers.

To sum up, the analysis above indicates that necessary (but not sufficient) conditions for a self-defeating austerity are: (i) a persistent austerity effort, (ii) a situation when agents expect a recession in a context of high fiscal multipliers. I derived these implications using a simplified version of the model. However, it is possible to verify our conclusions using the full model through a simulations exercise. With that aim, I simulated the model for 100,000 quarters and performed the following exercise. First, I computed the derivative of sovereign spreads spr_t with respect to government spending $\frac{\partial spr}{\partial G}$ every simulation period. A negative derivative of spr with respect to G is associated with a potential self-defeating austerity, that is, a situation when a a decrease in G increases spreads. Second, I identify the periods in which $\frac{\partial spr}{\partial G}$ switches from a positive to a negative sign, which are periods when the economy moves to a situation when austerity would be self-defeating if implemented. Third, I plot variables of interest in average time windows around these episodes. These windows are shown in figure 3.

Figure 3: Transition to Self-Defeating Scenarios



Notes. The figure shows consumption (of tradables and nontradables), hours worked and debt in time windows around episodes when the economy switches (at time 0) to a situation when a decrease in government spending increases spreads (a self-defeating austerity). The windows are constructed from model simulated data (100,000 simulations) with an arbitrary calibration such that self-defeating austerities are possible.

Figure 3 shows windows around the quarter, labeled $t = 0$, in which the derivative of spreads with respect to government spending switches from a positive to a negative sign. The figure shows log consumption and hours worked and data on fiscal multipliers. All variables are expressed in deviations with respect to a non-stochastic steady state without sovereign risk. We can see that typical situations when austerity can increase sovereign spreads are recessions with low levels of consumption and employment that become worse rapidly. Consumption levels and hours are close to 10% below the non-stochastic steady-state levels but they further decrease to 15% below steady state at t=0. Note also that these recessions are coupled with a situation with high fiscal multipliers that reach a peak at $t = 0$.

The last European debt crisis seems to be a situation in which all these necessary conditions were met. Europe was in a deep recession with low levels of employment and consumption. Moreover, fiscal multipliers were likely high if we consider the findings in the literature that fiscal multipliers tend to be higher in recessions and are overall higher in currency unions. [9] In the next section we calibrate the model to a country that implemented an austerity package with an important reduction in nominal government spending: a 20% decrease from 2010q1 to 2014q4.

7 Application: Spanish Austerity

7.1 Calibration

I calibrate the model using data from Spain, one of the countries with most important austerity programs in Europe. Standard parameters are set using common values in the literature. The rest of the parameters are chosen to match a set of moments. Table 3 shows the chosen values.

Risk aversion σ, the Frisch elasticity θ, and output labor elasticities α_N and α_T are calibrated to standard values. Household's discount factor β is 0.95, a low value but in line with the sovereign default literature. The labor disutility parameter χ is set to get a steady state labor equal to one. In addition, the elasticity of substitution between tradables and nontradables μ comes from Mendoza (1995) and corresponds to a sample of industrialized countries. The weight

[9] See Nakamura and Steinsson (2014)

of nontradables (ω) is used to match the share of nontradables value added on total GDP in steady state.[10]

Regarding the government, fiscal rules are estimated using data from Eurostat and the Bank of Spain from 1995q1 to 2017q1. Table 2 shows the estimation results. Columns (1) presents the coefficients for the government spending rule. it includes lagged GDP and external debt as controls. Results show that government spending is persistent with an AR(1) coefficient of 0.96. Moreover, lagged GDP and debt are statistically and economically insignificant: a quarterly drop in real GDP of 1% generates a fall in government spending of 0.04% and a 1% increase in external debt is associated with an increase in government spending of 0.02%. For that reason, I assume that G_t follows a simple AR(1) process when I solve the model.

Table 2 column (2) shows the estimation of external debt rule. The coefficient that relates B_{t+1} with B_t is 0.98 describing a stationary but persistent process for external debt. The coefficient relating nominal government spending with external debt, γ_G in equation (22) is significant and equal to 0.15. This indicates that overall a 1% increase in government spending generates an increase of 0.15% in the face value of external debt. Moreover, the coefficient relating external debt with nominal GDP, γ_Y, is also significant and equal to -0.26. This provides evidence of a counter-cyclical fiscal policy as external debt tends to increase in recessions.

The coupon rate is set to match a 6 year average debt maturity [11]. International creditor's discount factor is such that the annual risk free rate is 4%. The default haircut ψ is taken from Edwards (2015) and refers to the median haircut in European countries.[12] The probability of returning to markets ϕ matches the average exclusion time computed by Cruces and Trebesch (2013).

I estimate the downward nominal wage rigidity parameter γ, the productivity costs of default parameters d_0 and d_1 and the shock processes by fitting the moments listed in table 4. The Simulated Method of Moments estimation aims to match the standards deviation of wages, average spreads, and average and standard deviation of debt to GDP ratio. In addition I match

[10] Data from Instituto Nacional de Estadìsticas (INAE) for the period 1997Q1-2014Q2. The tradable sector refers to Agriculture and Industry, whereas the nontradable part of the economy includes Construction and Services.

[11] Data from the Spanish Treasury, see http://www.tesoro.es/sites/default/files/estadisticas/02I.pdf

[12] These are default episodes from 1978 to 2010, corresponding mainly to Eastern European countries.

the standard deviation and serial correlation of spreads, tradable and non-tradable output. In order to get the model implied estimates I simulate the model 10,000 times and compute the moments using data for periods in which the government is not in default.[13] The distance between the data and model implied moments is computed in relative terms.

Model Fit. The model does a decent job fitting the selected moments as table 4 shows. I check the model fit by comparing the relationship between fundamentals and spreads. Figure 4 shows the model implied relationships and the data points for Spain in the period 1998q1-2017q1. The black solid line is a fitted fourth degree polynomial using model simulated data. The blue data points correspond to data. Figure 4 shows a reasonable fit to the data. However, there are some data points that correspond to higher spreads relative to what fundamentals would predict. These are points that correspond to the periods 2010q3-2013q3, when spreads increased and then decreased rapidly. These deviations justify the importance of incorporating a liquidity premium shock to explain such differences. In fact, the fast decrease after an initial increase in

Table 2: Estimated Fiscal Rules

Variables	$log(G_t)$ (1)	$log(B_{t+1})$ (2)
$log(G_{t-1})$	0.96***	
	(0.04)	
$log(GDP_{t-1})$	0.04	
	(0.03)	
$log(B_t)$	0.02	0.98***
	(0.02)	(0.04)
$log(G_t^{nom})$		0.15*
		(0.09)
$log(Y_t^{nom})$		-0.26***
		(0.08)
Observations	87	87
R-squared	0.982	0.989

Notes. Robust standard errors in parentheses, $***p < 0.01$, $**p < 0.05$, $*p < 0.1$.
Data from Eurostat and Bank of Spain. Regressions include a linear trend, not reported.

[13] I exclude periods when the government is in default. I also disregard data on the first 100 quarters after the government regains access to markets.

Table 3: Calibration

Description	**Parameter**	**Value**	**Target**
Panel A: Household, Firms and Labor Market			
Discount factor	β	0.95	Standard
Risk aversion	σ	2	Standard
Frisch elasticity	θ	1	Standard
Labor disutility	χ	1.52	Steady state labor
Elast. of subsitution	μ	0.74	Mendoza (1995)
Nontradables weight	ω	0.83	SS nontradables share
Labor elasticity	α_N,α_T	2/3	Standard
Downward wage rigidity	γ	0.995	Moments
Panel B: Government			
G rule AR(1)	ρ_G	0.96	Fiscal rule estimation
G rule SD	σ_G	1.7%	Fiscal rule estimation
Debt rule parameter	γ_B	0.98	Fiscal rule estimation
Debt rule parameter	γ_G	0.15	Fiscal rule estimation
Debt rule parameter	γ_Y	-0.26	Fiscal rule estimation
Bond coupon rate	δ	1/24	Avg maturity = 6 yrs
Nontradables weight G	ω_g	0.83	$= \omega$
Panel C: Int'l creditors and default			
Int creditors disc factor	$\tilde{\beta}$	0.99	Annual risk free rate = 4%
Default haircut	ψ	19.7%	Edwards(2015)
Prob. of reentry	ϕ	3.85%	Cruces and Trebesch (2013)
Default prod Cost	d_0	-0.20	Moments
Default prod Cost	d_1	0.25	Moments
Panel D: Shock Processes			
Tradables prod AR(1)	ρ_{AT}	0.89	Moments
Nonradables prod AR(1)	ρ_{AN}	0.95	Moments
Risk premium AR(1)	ρ_m	0.46	Moments
Tradables prod SD	σ_{AT}	1.98%	Moments
Nonradables prod SD	σ_{AN}	2.88%	Moments
Risk premium SD	σ_m	1.1%	Moments

spreads in this time window may be related to the European Central Bank implementation of Outright Monetary Transactions (OMT) program. This program was launched by the European Central Bank (ECB) in 2012, and basically allowed the ECB to buy sovereign bonds in order to keep sovereign spreads low. The program has not been used so far, but its announcement might have had important effects on international creditors risk aversion. This is incorporated in the model as an external factor affecting sovereign spreads and is captured by the liquidity premium shock.

Following the sovereign default literature, I am using the productivity cost of default parameters (d_0 and d_1) mainly to match the average standards deviation of sovereign spreads. However, this parameters determine how much output would drop if Spain decided to default on its debt. Unfortunately, there is no data of recent default episodes for Spain to put more discipline to these parameter. Nevertheless, we can perform a sanity check using data from past default episodes in other countries. This exercise is shown in 5 that shows the evolution of government spending before, during and after default. The paths are expressed in percentage deviations with respect to real GDP four years before the default episode. The blue line is computed

Table 4: Model Fit

Target	**Model**	**Data**	**Data Description**
Average Spread	1.15%	1.2%	10 yr bond spread 1995q1-2017q1
Serial Corr spread	0.91	0.96	10 yr bond spread 1995q1-2017q1
SD spread	1.48%	1.36%	10 yr bond spread 1995q1-2017q1
Average Debt/GDP	33.5%	23%	External Debt to GDP 1995q1-2017q1
Std Dev Debt/GDP	12%	9.6%	External Debt to GDP 1995q1-2017q1
Serial Corr $\log(Y_N)$	0.90	0.99	Const and Serv V. Added 1995q1-2017q1
SD $\log(Y_N)$	4.9%	10.3%	Const and Serv V. Added 1995q1-2017q1
AR(1) coefficient $\log(Y_T)$	0.84	0.96	Agr and Ind V. Added 1995q1-2017q1
SD $\log(Y_T)$	9.4%	8%	Agr and Ind V. Added 1995q1-2017q1
SD $\Delta\log(W)$	0.9%	1.2%	Annual Nominal Wages 2008-2016

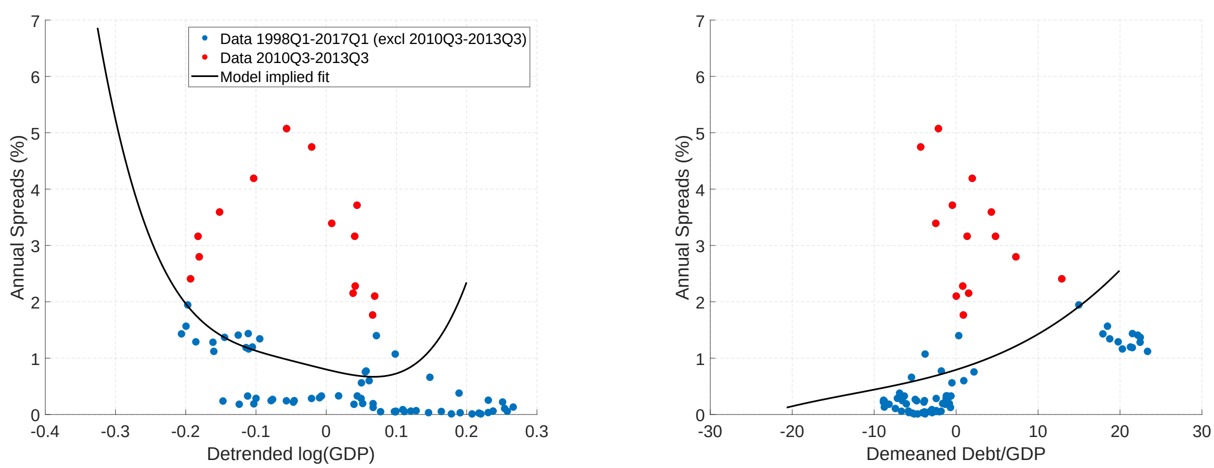
Notes. Spreads and Debt to GDP ratios come from the OECD. Data on tradables and nontradables value added are from the Spanish National Statistics Institute (INE). See Appendix B for a detailed description of data.

using model simulated data, the black line corresponds to the average path computed using real data from historical default episodes, whereas the grey shaded area shows a data-implied two standard deviation confidence interval. The data and model implied series are similar, and the simulation average path is always in the data confidence interval indicating that the costs of default seem reasonable.

7.2 Results

In this section I use the model to quantify the effects of fiscal austerity in Spain. I define austerity as the negative government spending shocks ϵ_t^G that hit the economy after the second quarter of 2010. The exercise simply consists of running a counterfactual and test what would have happened if these negative shocks on G were absent after 2010-Q2. I follow two steps to perform the exercise. First, I recover the realization of shocks such that model implied paths for tradable output (Y_T), nontradable output (Y_N), sovereign spreads, debt to GDP ratios and government spending (G) best fit the empirical paths. Second, I run a counterfactual without

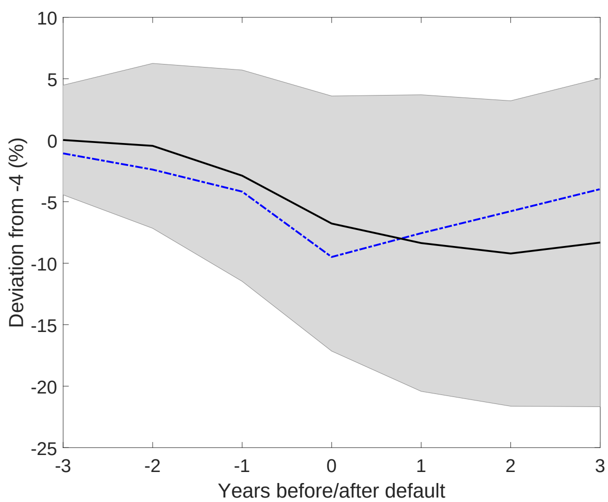
Figure 4: Spreads and Fundamentals



(a) GDP **(b)** Debt/GDP

Notes. The figures show the relationship between spreads and (a) GDP, and (b) demeaned debt to GDP ratio. The solid line is estimated by fitting a fourth degree polynomial to model implied data (simulation of 10,000 periods). The dots highlight true data points for Spain (1998Q1-2017Q1). The red dots highlight data points in the period 2010q3-2013q3, blue dots correspond to the rest of data.

Figure 5: GDP in default events: model vs data



Notes. Figures show the evolution of GDP in typical default episodes. Units are percentage deviation with respect to four years before the default event. Black lines represent data averages, the grey shaded area correspond to 2 standard deviations interval around the data implied average, and blue lines come from model simulations. Data series are average paths for recent default episodes taken from Schmitt-Grohé and Uribe (2016b). Data on GDP and government consumption come from World Bank's World Development Indicators. Model implied series come from 10,000 simulations from the calibrated model.

austerity shocks, and compare implied paths of endogenous variables.

Getting Filtered Shocks. As noted in section 7.1, the model does a good job matching relevant moments for Spain. As it is usually done in the DSGE literature, I can use this structure to extract the model implied shocks that hit the economy during and after the last financial crisis. To do so I employ a Particle Filter as in Fernandez-Villaverde and Rubio-Ramirez (2007). Consider the following state space representation,

$$\boldsymbol{S_t} = f\left(\boldsymbol{S_{t-1}}, \boldsymbol{\epsilon_t}\right) \tag{35}$$

$$\boldsymbol{Y_t} = g\left(\boldsymbol{S_t}\right) + \boldsymbol{\eta_t} \tag{36}$$

Equation (35) is the state equation, where $f(.)$ is a nonlinear function of the state variables in the previous period $\boldsymbol{S_t}$, and $\boldsymbol{\epsilon_t}$ is a vector of structural shocks. Moreover, equation (36) represents the measurement equation where $g(.)$ is a nonlinear function of the states, and $\boldsymbol{\eta_t}$ is a vector of measurement errors.

The observables I use to perform the filtering exercise are: tradable output (Y_T), nontradable output (Y_N), government consumption (G), debt to GDP ratio and sovereign spreads (s). The data correspond to the period 1998Q1-2014Q4. I calibrate the measurement error variances setting them equal to 20% of the variance of each data series.[14] Three of the four structural shock processes are unobserved: the liquidity premium shock and both productivity shocks. Government expenditure shocks are directly observed from data.

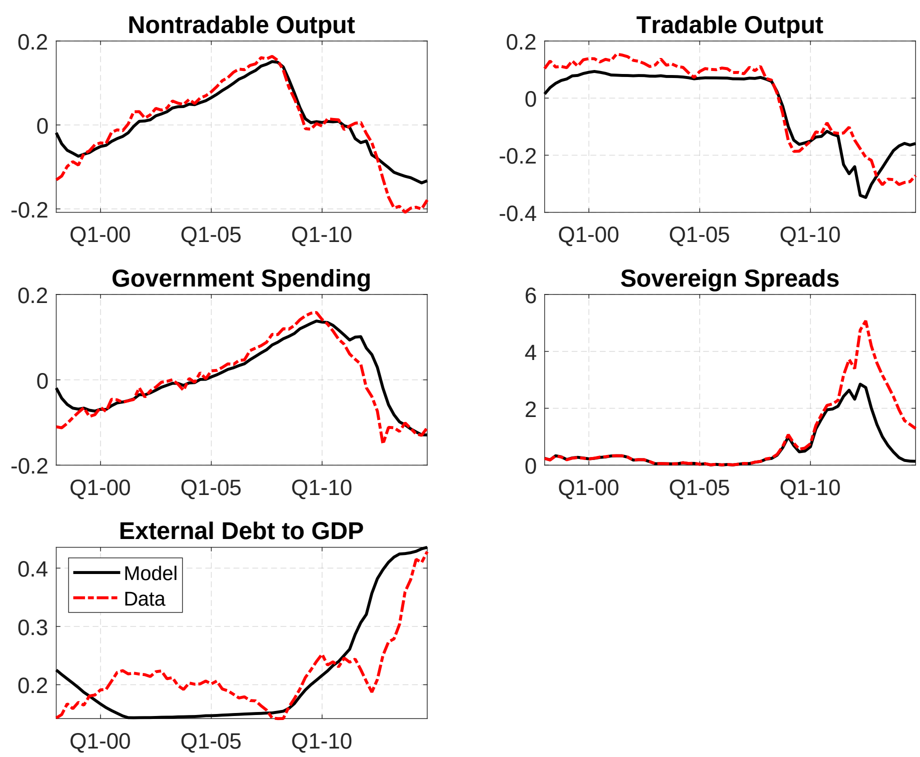
Note that the productivity shocks computed in this exercise should be interpreted as any private sector shock affecting firms marginal costs. The list of factors that might affect marginal costs includes productivity levels, but also financial costs and any shock affecting capital stock levels. I will refer to them in this section as “fundamental shocks”.

Figure 6 presents the data series and the filtered variables implied by the model. The figure shows that the model can actually replicate the recent evolution of macro variables in Spain. Indeed, even though I am including measurement errors, the true series are close to the model

[14] I use 100,000 particles for the results.

implied ones. Note, however, that the model can only explain a part of the increase in sovereign spreads after 2010. This might look surprising given that the model incorporates a liquidity premium shock that plays like an exogenous shock to default. However, notice that in this exercise we are trying to match five observables with only four shocks. Therefore, it is reasonable to see a difference between the model-implied paths and the data.

Figure 6: Filtered Series and Data



Notes. This figure shows the data employed in the filtering exercise. Red dashed lines are data series, solid black lines are filtered series from the model. Government consumption, tradable and nontradable output are log detrended values. Spreads are annual and expressed in percentages.

Figure 7 shows the filtered exogenous states computed by the filter: nontradable and tradable productivity, government spending and liquidity premium. The model suggests that the most important driver of the financial crisis has to do with fundamentals in the nontradable sector.

In fact, productivity in this sector decreased more than 20%. Moreover, the tradable sector productivity also suffered an important drop of around 15%.[15] The government reduced government spending quickly after 2010. From 2010Q1 to 2014Q4 detrended government spending dropped by more than 20%. Liquidity premium shows an upward trend at the beginning of the debt crisis. Interestingly, it starts falling from 2012, year in which the Outright Monetary Transactions (OMT) program was announced. This evolution of the liquidity premium shock shows that spreads were also affected by external factors. [16]

Quantifying the Effects of Austerity. Having a calibrated model allows me to run a counterfactual exercise to isolate the impact of fiscal austerity measures. I define austerity using the government spending fiscal rule (21) in the model. In particular, I interpret the shocks ϵ_t^g as unexpected discretionary fiscal austerity measures. One way to check whether this interpretation is reasonable is to compare the fiscal rule shocks with other measures of fiscal austerity. In this line, figure 8 compares the fiscal austerity shocks with narrative measures of austerity from Alesina et al. (2015). The correlation between these two measures of austerity is 0.37 indicating a positive association between deviations from the rule and austerity announcements in the period of analysis. One interesting finding is that the rule indicates that there was no austerity in 2013 whereas the narrative measure points to a decrease in government spending of more than one percent of GDP. This might be related to the fact that although Spain announced austerity measures for 2013, it decided to slow the pace of consolidation that year mainly because the European Central Bank gave Spain more time to meet austerity goals.[17]

The counterfactual exercise to quantify the impact of austerity consists of simulating the model using the filtered shocks but setting the austerity shocks to zero after the second quarter of 2010, time in which the main austerity measures were announced.[18] In particular, if $\boldsymbol{\epsilon^g} \equiv \left\{\epsilon^g_{98Q1}, \epsilon^g_{97Q2}, ..., \epsilon^g_{14Q4}\right\}$ is the vector of filtered G shocks from 1997Q1 to 2014Q2, the

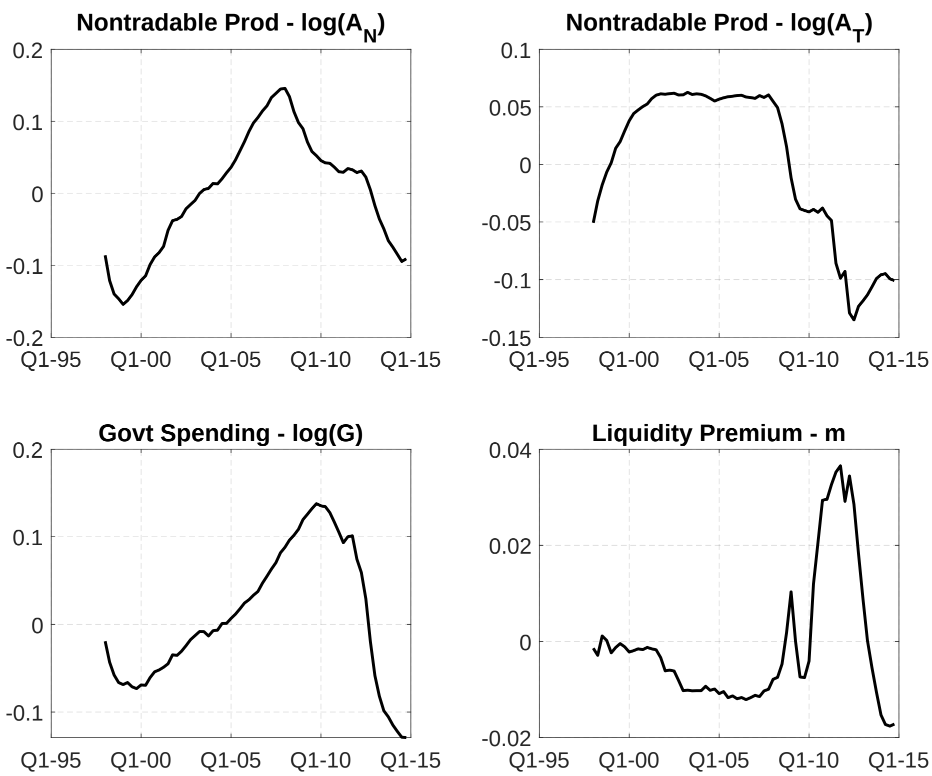
[15] One key driver of the Spanish crisis has to do with the banking sector and credit availability. These factors are not explicitly modeled, but their effects on firms marginal costs are included in the private sector fundamentals A_N and A_T.

[16] Bocola and Dovis (2016) assess the importance of fundamentals in the last European debt crisis.

[17] See https://www.ft.com/content/fd104cc4-ae8a-11e2-bdfd-00144feabdc0

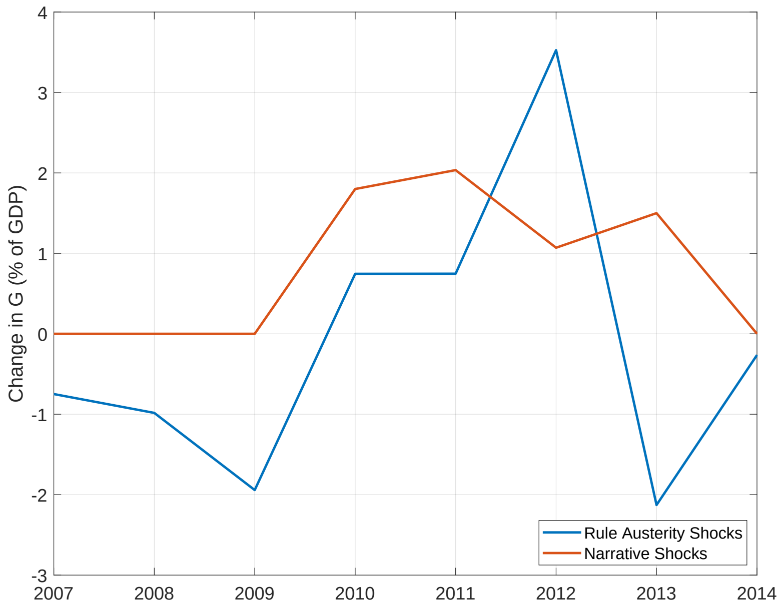
[18] The announced measures included cuts in public wages and public investment, reductions in public health related expenses. See https://www.ft.com/content/91ca42de-5d9e-11df-b4fc-00144feab49a

Figure 7: Filtered Shocks



Notes. This figure shows the filtered shock processes computed using the Particle Filter. The solid black line is the mean value, the grey bars highlight Eurozone recessions.

Figure 8: Austerity Shocks



Notes. Narrative shocks are government spending unexpected shocks. Model based austerity shocks are computed taking annual averages of $[exp\left(-\sigma_G \epsilon_t^g\right) - 1]\, \frac{G_{t-1}}{Y_t}$ *for every* t.

counterfactual “no austerity” sequence of shocks is $\tilde{\boldsymbol{\epsilon}}^{\boldsymbol{g}} \equiv \left\{\epsilon^g_{98Q1}, \epsilon^g_{98Q2}, ...\epsilon^g_{10Q2}, 0, 0, .., 0\right\}$. Hence, instead of causing a sharp contraction in G, the government follows the fiscal rule with a slow convergence toward the steady state levels of government spending.

Figure 9 presents the counterfactual paths for government spending, GDP, consumption and hours worked. Under the counterfactual, G slowly converges to steady state levels instead of dropping steeply from the second half of 2010. Relative to this counterfactual, figure 9 shows that there is an important drop in real GDP. In particular, as a consequence of austerity measures real GDP is around 2.4% lower by the end of 2014. This implies an annual growth rate 0.13% lower from 2010Q2. There is a negative impact on consumption but, as expected, it is lower than that on GDP. By the end of 2014 consumption is lower than the no-austerity counterfactual by only 0.5%.

Figure 10 shows the impact of austerity on sovereign spreads and debt to GDP levels. The results indicate that austerity was effective in reducing debt to GDP levels. By the end of the period analyzed, the external debt to GDP dropped 4 percentage points as a consequence of austerity. Moreover, sovereign spreads decreased by more than 4 percentage points relative to the counterfactual. Notice that this result has to do with the fact that austerity increased the value for repaying the debt relatively more than the increase in the value of defaulting.

We can actually analyze the different mechanisms through which austerity affected spreads. Remember from section 6.2 that the impact on spreads depends upon the relative change of the expected welfare associated with defaulting $\mathbb{E}_t\mathcal{V}^D$ relative to the welfare for repaying the debt $\mathbb{E}_t\mathcal{V}^R$. These two expectations are a function of the vector $(B_{t+1}, w_t, G_t, A_{Tt}, A_{NTt}, m_t)$ and, as a consequence, spreads are determined by these variables too. Three of the six state variables listed change with austerity: sovereign external debt to be repaid at $t+1$ (B_{t+1}), wages at time t (w_t) and government spending (G_t). Therefore, there are three different channels through which austerity impacts the expected value functions and, hence, sovereign spreads.

We can perform the following decomposition to get the relative importance of each channel. Using the superscript a and na to denote austerity and no-austerity cases, we can relate the spreads under austerity with spreads without austerity using the following exact decomposition.

$$
\begin{aligned}
spr_t^a = spr_t^{na} &+ \underbrace{spr\left(B_{t+1}^a, w_t^a, G_t^a\right) - spr\left(B_{t+1}^a, w_t^a, G_t^{na}\right)}_{G_t \text{ persistence effect}} \\
&+ \underbrace{spr\left(B_{t+1}^a, w_t^a, G_t^{na}\right) - spr\left(B_{t+1}^a, w_t^{na}, G_t^{na}\right)}_{w_t \text{ Effect}} \\
&+ \underbrace{spr\left(B_{t+1}^a, w_t^{na}, G_t^{na}\right) - spr\left(B_{t+1}^{na}, w_t^{na}, G_t^{na}\right)}_{B_{t+1} \text{ Effect}}
\end{aligned}
$$

In the last expression we are dropping A_{Tt}, A_{NTt}, m_t from the arguments to simplify the notation. The decomposition implies that spreads under austerity equal spreads without austerity, plus three different terms that summarize the impact of austerity. The first term labeled "G_t persistence effect" has to do with the effect of austerity at time t on the expectation of future government spending. This term corresponds to the persistence term in equation (33). As discussed in section 6.2, this term can have a positive or negative impact of spreads depending on the state of the economy. The main reason is that a lower future government spending is associated with (i) a higher chance of a deeper recession in the future and lower taxes in case of default, which tends to increase spreads, and (ii) a lower future external debt which reduces spreads. In other words, the impact of austerity on the expected welfare associated with repaying is uncertain.

The second term in the decomposition is the "w_t effect" and represents the impact of austerity on wages. Austerity can affect nominal wages and, as a consequence, affect the likelihood of having a binding downward nominal wage rigidity constraint. We excluded this channel in our analysis in section 6.2 because in that section we assumed fixed wages to gain simplicity. However, the main reason for that exclusion is that this channel is not important quantitatively in our empirical application. The reason is that when the austerity package started the downward nominal wage constraint was already binding and, therefore, $w_t = \gamma w_{t-1}$. Hence, austerity did not have any impact on wages as it could not further decrease them.

The third term represents the "B_{t+1} effect" and is related to the effect on spreads through a reduction in the stock of debt to be repaid from $t+1$. This term summarizes the channel represented by the first right hand side term in equation (33). As discussed in section 6.2 this term has an unambiguous effect on spreads: it actually decreases spreads because it increases the expected welfare for repaying the debt increasing the conutry's creditworthiness.

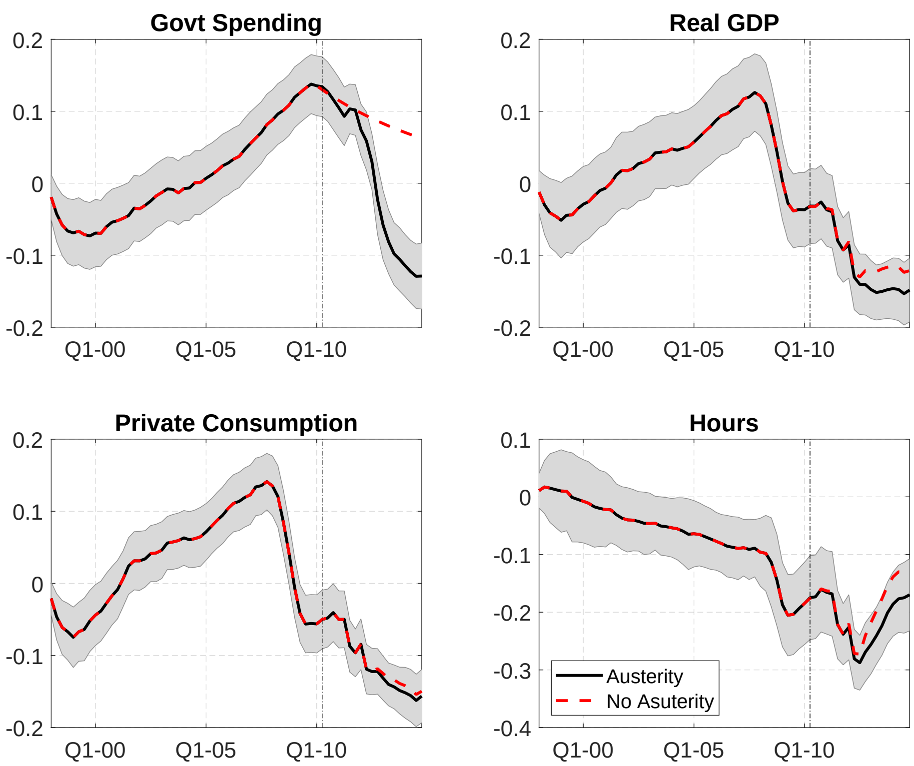
Figure 11 shows the impact of each one of these three channels. The black solid line represents sovereign spreads without austerity spr_t^{na} for all t. The black dashed shows sovereign spreads with austerity, spr_t^{a}. The colored areas between these two series represent the importance of the different channels. First notice that the "wage effect" has no importance explaining the impact of fiscal austerity in Spain. Second, and not surprisingly, the "B_{t+1} effect" tends to reduce sovereign spreads. It explains roughly half the difference between spreads with and without austerity at the end of 2014. Lastly, note that the "G_t persistence effect" tends to reduce the spreads even further, explaining the other half of the difference. This indicates that the persistent austerity implemented in Spain generated the expectation of a persistent decrease in external debt that generated a drop in sovereign spreads even when agents incorporated the negative impact in economic activity.

Multipliers and Self Defeating Austerity. The results in this section seems to imply that the austerity costs in terms of economic activity were not high enough to generate a self defeating austerity. One possible concern would be that multipliers are too low in our calibration and, therefore, the results are not realistic. However, the calibration shown in table 3 implies an overall fiscal multiplier of 1.6 a number relatively high if we consider the empirical literature on fiscal multipliers.[19] In fact, the filtering exercise predicts a fiscal multiplier slightly higher than ther average (around 1.8) during the austerity period.

If an economy with an average multiplier of 1.6 did not have a self-defeating austerity, how high the multiplier needs to be so that these type of events have a serious chance of happening ? I perform the following exercise to answer that question. I first solve the model assuming different parameters γ_G and γ_Y but keeping the rest of them calibrated according to table 3. Modifying these fiscal rule parameters is a way of changing fiscal multipliers keeping the rest of the calibration untouched. I then simulate the model for every calibration and compute

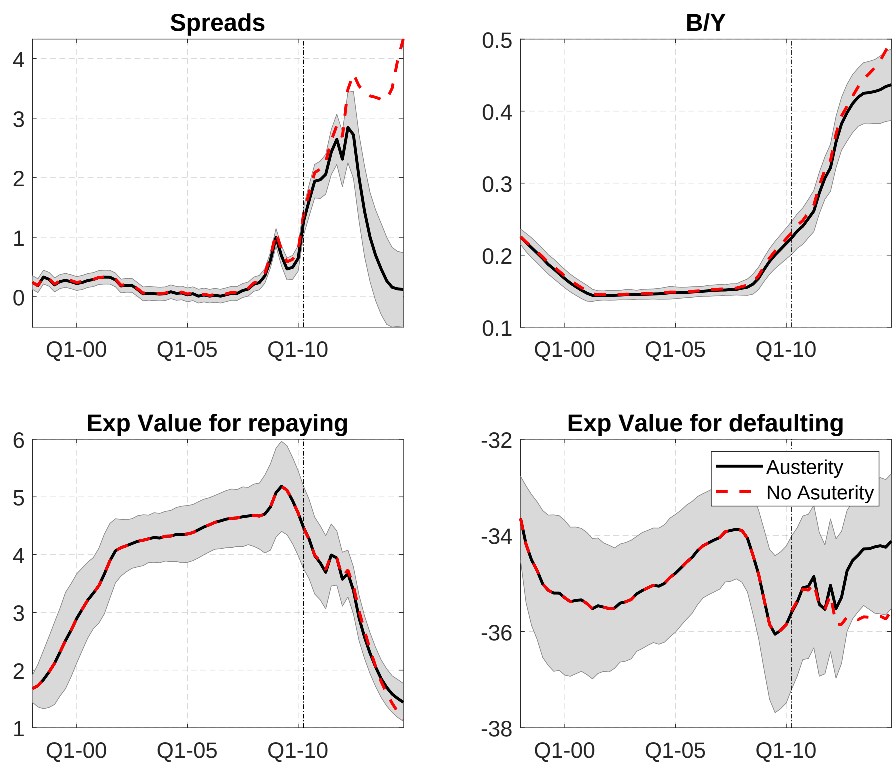
[19] I calculate the overall fiscal multiplier taking the average multiplier from a simulation of 10,000 periods, without considering periods where the country in in default.

Figure 9: Effects of Fiscal Austerity



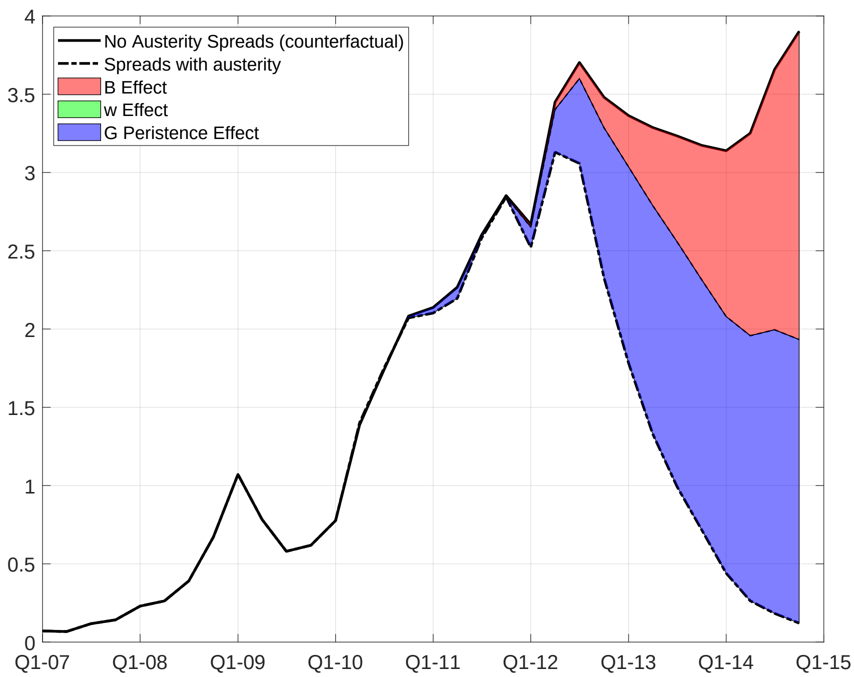
Notes. Black line shows filtered variables with austerity shocks after 2010Q2. The shaded are highlights two standard deviation intervals around the austerity case. Red line depicts a counterfactual exercise where austerity shocks are set to zero after 2010Q2.

Figure 10: Effects of Fiscal Austerity



Notes. Black line shows filtered variables with austerity shocks after 2010Q2. The shaded are highlights two standard deviation intervals around the austerity case. Red line depicts a counterfactual exercise where austerity shocks are set to zero after 2010Q2.

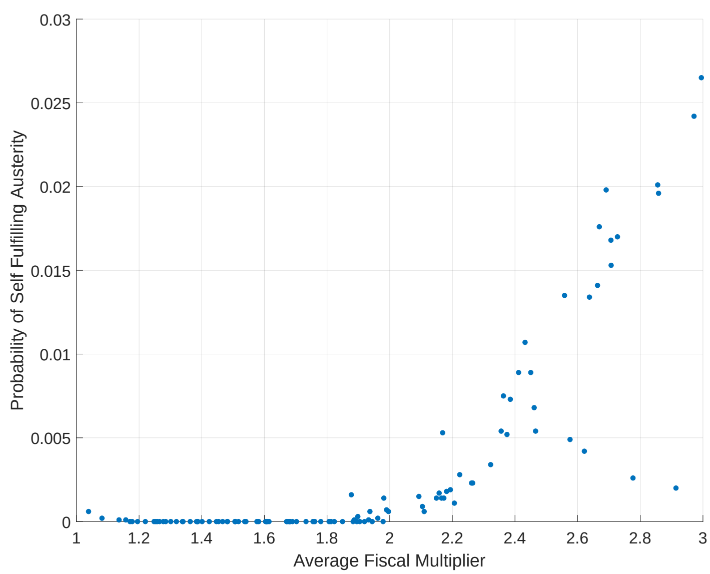
Figure 11: Decomposing Impact on Spreads



Notes. Solid black line represents the counterfactual scenario without austerity measures after 2010-Q2. The dashed black line shows the spread with austerity. The area between the two curves is divided to measure the relative importance of each effect of austerity.

the average fiscal multiplier and the fraction of periods in which the derivative of spreads with respect to government spending is negative (implying that austerity would be a self-defeating if implemented). Figure 12 shows the result of this exercise. There are several things we can learn from this figure. First, notice that self-defeating austerity episodes seem unlikely at least for economies similar to Spain. The likelihood is below 3%. Second, notice that the probability of self-defeating austerities is basically zero if overall multipliers are below 1.8. Third, even for calibrations showing multipliers much higher than the empirical literature the probability of self-defeating episodes is still relatively low.

Figure 12: Fiscal Mutipliers vs Self-Defeating Austerity Probability



Wages and Employment. One concern regarding the results from this section might be that I am understating the importance of downward nominal wage rigidity and, as a result, the model generates a low drop in hours worked after 2010. This is a reasonable concern given the fact that I do not use employment to calibrate the model or extract model implied structural shocks. Figure 13 compares data and model implied series for hours and nominal wages. As shown in panel (a), the fall in nominal wages in the data and the model is similar. This suggests that the downward nominal wage rigidity parameter is appropriate. Panel (b) shows log hours in the model and log total hours worked data (demeaned and normalized by population). Here, again, we can see that the drop in hours durgin the austerity period from 2010 to 2013 is similar.

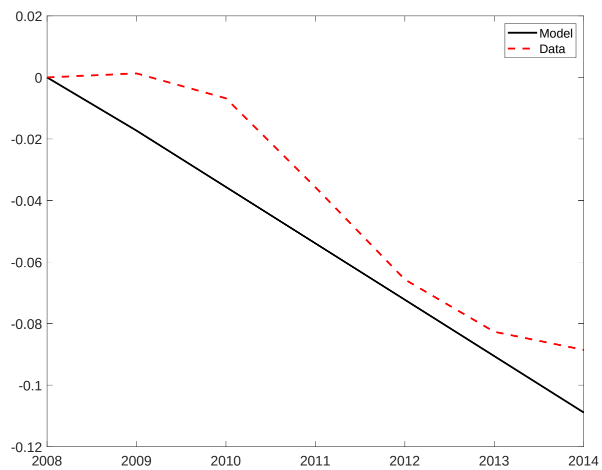
8 Conclusions

I proposed a small open economy model with strategic sovereign default to analyze the effects of fiscal austerity on sovereign spreads. I derived "self-defeating" conditions under which austerity might increase sovereign spreads. These conditions are: (i) austerity needs to be persistent so that it affects expectations about future government spending, (ii) austerity is implemented when a recession is expected by the agents in the economy and fiscal multipliers are high.

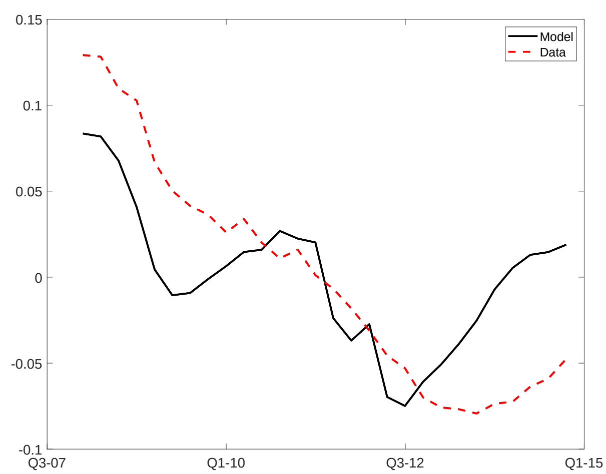
I calibrated the model to a country that might have been satisfying these conditions: Spain during the last European debt crisis. The results indicate that austerity was important to decrease sovereign spreads and debt to GDP ratios. However, this gain generated some costs in terms of economic activity.

This paper contributes to the general discussion about the effects of austerity measures in different ways. First, it proposes a model with a novel feature: the combination of realistic fiscal policy with strategic sovereign default. This model is useful to get theoretical implications given fiscal policy rules and run counterfactuals. Second, it shows that self-defeating austerity is possible but unlikely under reasonable calibrations. Third, it highlights the importance of fiscal rule parameters to determine the size of fiscal multipliers and the likelihood of self-defeating austerity efforts.

Figure 13: Wages and Hours Worked



(a) Nominal Wages



(b) Hours

Notes. Black lines are model implied series, dashed red lines indicate data. Panel (a) shows nominal wages series. Data implied nominal wages are detrended using the average growth rate in real GDP per capita to be consistent with the fact that the series used in the particle smoother are also similarly detrended. Panel (b) shows data and model implied log hours worked (both seraies are demeaned and the data normalized by population). Data on hours come from OECD economic outlook Number 99.

Appendices

A Implementable Equilibrium Equations

$$(H_t^s)^\theta (C_t)^\sigma = \frac{w_t}{p_{Ct}} \tag{37}$$

$$p_{Ct} = \left[\omega^\mu p_{Nt}^{1-\mu} + (1-\omega)^\mu\right]^{\frac{1}{1-\mu}} \tag{38}$$

$$p_{Nt} = \frac{\omega}{1-\omega}\left(\frac{C_{Tt}}{C_{Nt}}\right)^{1/\mu} \tag{39}$$

$$\alpha_T A_{Tt}(H_{Tt}^d)^{\alpha_T - 1} = w_t \tag{40}$$

$$\alpha_N p_{Nt} A_{Nt}(H_{Nt}^d)^{\alpha_N - 1} = w_t \tag{41}$$

$$Y_{Nt} = A_{Nt}(H_{Nt}^d)^{\alpha_N} \tag{42}$$

$$Y_{Tt} = A_{Tt}(H_{Tt}^d)^{\alpha_T} \tag{43}$$

$$G_{Nt} + C_{Nt} = Y_{Nt} \tag{44}$$

$$C_{Tt} + G_{Tt} + \delta B_t = Y_{Tt} + q_t\left[B_{t+1} - (1-\delta)B_t\right] \tag{45}$$

$$(w_t - \gamma w_{t-1})(H_t^s - H_{Nt}^d - H_{Tt}^d) = 0 \tag{46}$$

$$w_t \geq \gamma w_{t-1}$$

B Data

The mapping between data and observables used in the particle filter is defined below. $EXTDEBT_t$ is the face value of external debt from Bank of Spain Statistical Bulletin. $10YRYLDSPAIN_t$ and $10YRYLDGER_t$ are the yields of 10 year sovereign bonds of the Spanish and German governments, respectively (data from $OECD$ economic outlook). $NGOVC_t$ and $NGOVINV_t$ are nominal government consumption and investment from Eurostat. $NPCON_t$ and $NPINV_t$ are nominal private consumption and investment. POP_t is working age population. $GDPDEF_t$ stands for GDP deflator (data from Eurostat) and $NTSHARE_t$ is the output nontradable share (data from the Spanish Statistics Institute (INE)). Y_T, Y_N and G_t were detrended by the average growth rate of $Y_t/GDPDEF_t/POP_t$ over the sample.

$$B_t = EXTDEBT_t \tag{47}$$

$$spr_t = 10YRYLDSPAIN_t - 10YRYLDGER_t \tag{48}$$

$$q_t = \frac{\delta}{\delta + r^* + spr_t/400} \tag{49}$$

$$\Delta B_t = q_t \left(B_{t+1} - (1-\delta)B_t\right) - \delta B_t \tag{50}$$

$$G_t^{nom} = (NGOVC_t + NGOVINV_t)/POP_t \tag{51}$$

$$Y_t^{nom} = (NPCON_t + NPINV_t + \Delta B_t)/POP_t + G_t^{nom} \tag{52}$$

$$G_t = G_t^{nom}/GDPDEF_t \tag{53}$$

$$Y_{Tt} = (Y^{nom}/GDPDEF_t/POP_t) * (1 - NTSHARE_t) \tag{54}$$

$$Y_{Nt} = (Y^{nom}/GDPDEF_t/POP_t) * (NTSHARE_t) \tag{55}$$

C Solution Method

I solve the model using global numerical methods. I use linear interpolation and Gauss Hermite to compute expectations. The steps for the numerical solution are the following

Step 0 (a). Define grids for endogenous and exogenous states $\{B, w_{-1}, G, A_N, A_T, m\}$. I define equally spaced grid points for each variable. I use 101 points for debt B with a lower bound of $0.7\bar{B}$ and upperbound of $2.5\bar{B}$ where $\bar{B}$ is the no default risk and nonstochastic steady state level of debt. I use 15 equally spaced grid points for w_1 with lower and upper bounds defined as $(w_{min}, w_{max}) = (0.8\bar{w}, 1.2\bar{w})$ where $\bar{w}$ is the steady state value of nominal wages. Exogenous shocks have 5 grid points with upper and lower bounds given by +/- 3 standard deviations from their means.

Step 0 (b). Compute private equilibrium allocations for each one of the grid points defined in the previous step. There are two different cases, one when the economy has access to markets and another when the economy is in default. In this second case, incorporate the productivity costs associated with default. In particular compute consumption $C(\mathbf{S}, d)$,hours $H(\mathbf{S}, d)$, price of government spending $p_G(\mathbf{S}, d)$ and output $Y(\mathbf{S}, d)$. These allocations are a function of the states $\mathbf{S}$ and the default decision d ($d = 1$ if default, $d = 0$ otherwise).

Step 1. Update the value functions using the private sector allocations, price schedule and value functions from previous step. Hence, for iteration i the updated value function are computed by,

1. Define the value before deciding whether to default or not as,

$$\mathcal{V}_{(i-1)}(\mathbf{S}) = \left\{\mathcal{V}^R_{(i-1)}(\mathbf{S}); \mathcal{V}^D_{(i-1)}\Big(B, w_{-1}, A_N, A_T, (1-d_g)G, m\Big)\right\} \tag{56}$$

2. Update value of repaying

$$\mathcal{V}^R_{(i)}(\mathbf{S}) = u\left(C(\mathbf{S}, 0), H(\mathbf{S}, 0)\right) + \beta E\left\{\mathcal{V}_{(i-1)}(\mathbf{S}')|\mathbf{S}\right\} \tag{57}$$

where the low of motion of the endogenous states is given by,

$$
\begin{aligned}
&p_G(\mathbf{S},0)G + \delta B = t^* Y(\mathbf{S},0) + \frac{\gamma_T}{4} B + q_{(i-1)}(\mathbf{S_1'}, \mathbf{S_2}) \left[B' - (1-\delta)B\right] \\
&w'_{-1} = w(\mathbf{S},0)
\end{aligned}
$$

where $\mathbf{S_1} = \{B, w_{-1}\}$ and $\mathbf{S_2} = \{A_N, A_T, G, m\}$. When getting B' from its law of motion, get the value of B' is on the left side of the debt Laffer curve. This is done to rule out one possible source of multiplicity.

3. Value of defaulting

$$
\mathcal{V}^D_{(i)}(\mathbf{S}) = u\left(C(\mathbf{S},1), H(\mathbf{S},1)\right) + \beta E\left\{\phi max\left\{\mathcal{V}_{(i-1)}(\mathbf{S}'), \mathcal{V}^D_{(i-1)}(\mathbf{S}')\right\} + (1-\phi)\, \mathcal{V}^D_{(i-1)}(\mathbf{S}') | \mathbf{S}\right\} \tag{58}
$$

Where the budget constraint of the government and law of motion of endogenous states are,

$$
\begin{aligned}
&T(\mathbf{S},1) = p_G(\mathbf{S},1)G \\
&B' = BR^* \\
&w'_{-1} = w(\mathbf{S},1)
\end{aligned}
$$

Step 2. With the updated value functions from the previous step redefine the default decisions (while in markets or right after reentering from default),

$$
d(\mathbf{S}) = \begin{cases} 1 & \mathcal{V}^D_{(i)}(\mathbf{S}) \geq \mathcal{V}^R_{(i)}(\mathbf{S}) \\ 0 & \mathcal{V}^D_{(i)}(\mathbf{S}) < \mathcal{V}^R_{(i)}(\mathbf{S}) \end{cases}
$$

$$
\tilde{d}(\mathbf{S}) = \begin{cases} 1 & \mathcal{V}^D_{(i)}(\mathbf{S}) \geq \mathcal{V}^R_{(i)}(\tilde{\mathbf{S}}) \\ 0 & \mathcal{V}^D_{(i)}(\mathbf{S}) < \mathcal{V}^R_{(i)}(\tilde{\mathbf{S}}) \end{cases}
$$

where $\tilde{\mathbf{S}}$ incorporates the face value haircut. That is, it replaces B for $(1-\psi)B$. Then update the prices of debt using the following expression,

$$\tilde{q}(\mathbf{S}) = \frac{\tilde{\beta}}{1+\nu} E\left\{(1-d(\mathbf{S}'))\left(\delta + (1-\delta)q(\mathbf{S}')_{(i-1)}\right) + d(\mathbf{S}')q^D(\mathbf{S}')_{(i-1)}|\mathbf{S}\right\}$$

$$\tilde{q}^D(\mathbf{S}) = \frac{\tilde{\beta}}{1+\nu} E\left\{\left\{(1-\tilde{d}(\mathbf{S}'))\left(\delta + (1-\delta)q(\mathbf{S}')_{(i-1)}\right) + d(\mathbf{S}')q^D(\mathbf{S}')_{(i-1)}\right\}|\mathbf{S}\right\}$$

$$\tilde{q}^D(\mathbf{S}) = \frac{\tilde{\beta}}{1+\nu} E\{\phi\left[(1-\tilde{d}(\mathbf{S}'))\left(\delta + (1-\delta)q(\mathbf{S}')_{(i-1)}\right) + \tilde{d}(\mathbf{S}')q^D(\mathbf{S}')_{(i-1)}R^*\right] + (1-\phi)q^D(\mathbf{S}')_{(i-1)}R^*|\mathbf{S}\}$$

Step 3. compute distances $r^q = \left|\left|\tilde{q}(\mathbf{S}) - q(\mathbf{S})_{(i-1)}\right|\right|$, $r^{qD} = \left|\left|\tilde{q}^D(\mathbf{S}) - q^D(\mathbf{S})_{(i-1)}\right|\right|$, $r^R = \left|\left|\mathcal{V}^R_{(i)}(\mathbf{S}) - \mathcal{V}^R_{(i-1)}(\mathbf{S})\right|\right|$ and $r^D = \left|\left|\mathcal{V}^D_{(i)}(\mathbf{S}) - \mathcal{V}^D_{(i-1)}(\mathbf{S})\right|\right|$. Check if they are lower than a tolerance value, if not, update prices and value functions and go to step 1 (set $i = i+1$). When updating the price function use a dampening parameter θ, such that

$$q(\mathbf{S})_{(i)} = \theta\tilde{q}(\mathbf{S}) + (1-\theta)q(\mathbf{S})_{(i-1)}$$

$$q^D(\mathbf{S})_{(i)} = \theta\tilde{q}^D(\mathbf{S}) + (1-\theta)q^D(\mathbf{S})_{(i-1)}$$

Details. I stop the algorithm when $r^q < 10^{-3}$ and $r^R, r^D < 10^{-5}$. The dampening parameter is set to $\theta = 0.1$. I use Gauss Hermite to compute integrals, with five nodes for each exogenous state.

D Proof of proposition 1

TO BE COMPLETED

References

Alesina, A. and S. Ardagna (2010): *Large Changes in Fiscal Policy: Taxes versus Spending*, The University of Chicago Press, 35–68.

Alesina, A., O. Barbiero, C. Favero, F. Giavazzi, and M. Paradisi (2015): "Austerity in 2009-2013," Working Paper 20827, National Bureau of Economic Research.

Alesina, A., C. Favero, and F. Giavazzi (2012): "The Output Effect of Fiscal Consolidations," Working Paper 18336, National Bureau of Economic Research.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98, 690–712.

Arellano, C. and Y. Bai (2016): "Fiscal Austerity during Debt Crises," Staff Report 525, Federal Reserve Bank of Minneapolis.

Auerbach, A. J. and Y. Gorodnichenko (2012): "Measuring the Output Responses to Fiscal Policy," *American Economic Journal: Economic Policy*, 4, 1–27.

Benjamin, D. and M. L. J. Wright (2013): "Recovery Before Redemption: A Theory of Delays in Sovereign Debt Renegotiations," Working paper, State University of New York, Buffalo and Federal Reserve Bank of Chicago.

Bi, H., E. M. Leeper, and C. Leith (2013): "Uncertain Fiscal Consolidations," *The Economic Journal*, 123, F31–F63.

Bianchi, J., P. Ottonello, and I. Presno (2019): "Fiscal Stimulus under Sovereign Risk," Working Paper 26307, National Bureau of Economic Research.

Blanchard, O. and R. Perotti (2002): "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output," *The Quarterly Journal of Economics*, 117, 1329–1368.

Bocola, L. and A. Dovis (2016): "Self-Fulfilling Debt Crises: A Quantitative Analysis," Working Paper 22694, National Bureau of Economic Research.

Born, B., G. J. Müller, and J. Pfeifer (2018): "Does austerity pay off?" Working paper.

Cafiso, G. and R. Cellini (2012): "Fiscal consolidations for debt-to-GDP ratio containment? Maybe ... but with much care," March, 20, VoxEU.org.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–99.

Corsetti, G. (2010): "Fiscal consolidation as a policy strategy to exit the global crisis," July, 7, VoxEU.org.

——— (2012): "Has austerity gone too far?" April, 2, VoxEU.org.

Cottarelli, C. (2012): "Fiscal adjustment: Too much of a good thing?" February, 8, VoxEU.org.

Cruces, J. J. and C. Trebesch (2013): "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117.

David, A., J. Guajardo, and J. Yepez (2019): "The Rewards of Fiscal Consolidation: Sovereign Spreads and Confidence Effects," Working Paper 19/141, International Monetary Fund.

de Córdoba, G. F., P. S. Pujolas, and J. L. Torres (2017): "Fiscal discipline and defaults," *Review of Economic Dynamics*, 24, 1 – 13.

Easterly, W., T. Irwin, and L. Servén (2008): "Walking up the Down Escalator," *World Bank Research Observer*, 23, 37–56.

Easterly, W. and L. Servén (2003): *The Limits of Stabilization : Infrastructure, Public Deficits, and Growth in Latin America*, Palo Alto, CA: Stanford University Press.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 48, 289–309.

Edwards, S. (2015): "Sovereign Default, Debt Restructuring, and Recovery Rates: Was the Argentinean "Haircut" Excessive?" *Open Economies Review*, 26, 839–867.

Fernandez-Villaverde, J. and J. F. Rubio-Ramirez (2007): "Estimating Macroeconomic Models: A Likelihood Approach," *Review of Economic Studies*, 74, 1059 – 87.

Giavazzi, F. and M. Pagano (1990): *Can Severe Fiscal Contractions Be Expansionary? Tales of Two Small European Countries*, MIT Press, 75–122.

Gros, D. (2011): "Can austerity be self-defeating?" November, 29, VoxEU.org.

Guajardo, J., D. Leigh, and A. Pescatori (2014): "EXPANSIONARY AUSTERITY? INTERNATIONAL EVIDENCE," *Journal of the European Economic Association*, 12, 949–968.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117 – 125.

Hatchondo, J. C., L. Martinez, and F. Roch (2015): "Fiscal rules and the sovereign default premium," Working paper, mimeo.

House, C. L., C. Proebsting, and L. L. Tesar (2017): "Austerity in the Aftermath of the Great Recession," Working Paper 23147, National Bureau of Economic Research.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

Krugman, P. (2015): "The Austerity Delusion," April, 29, The Guardian.

Lane, P. R. (2012): "The European Sovereign Debt Crisis," *Journal of Economic Perspectives*, 26, 49–68.

Martin, P. and T. Philippon (2014): "Inspecting the Mechanism: Leverage and the Great Recession in the Eurozone," Working Paper 20572, National Bureau of Economic Research.

Mendoza, E. G. (1995): "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations," *International Economic Review*, 36, 101–137.

Mendoza, E. G., L. L. Tesar, and J. Zhang (2014): "Saving Europe?: The Unpleasant Arithmetic of Fiscal Austerity in Integrated Economies," Working Paper 20200, National Bureau of Economic Research.

Mendoza, E. G. and V. Z. Yue (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 127, 889–946.

Nakamura, E. and J. Steinsson (2014): "Fiscal Stimulus in a Monetary Union: Evidence from US Regions," *American Economic Review*, 104, 753–92.

Nicoletta Batini, G. C. and G. Melina (2012): "Successful Austerity in the United States, Europe and Japan," Working Paper 12/190, International Monetary Fund.

Owyang, M. T., V. A. Ramey, and S. Zubairy (2013): "Are Government Spending Multipliers Greater during Periods of Slack? Evidence from Twentieth-Century Historical Data," *American Economic Review*, 103, 129–34.

Romer, C. D. and D. H. Romer (2010): "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks," *American Economic Review*, 100, 763–801.

Schmitt-Grohé, S. and M. Uribe (2016a): "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, forthcoming.

——— (2016b): *Open Economy Macroeconomics*, forthcoming Princeton University Press.

Shambaugh, J. C. (2012): "The Euro's Three Crises," Working paper, Brookings Institution.

Wren-Lewis, S. (2016): "A general theory of austerity," Working Paper BSG-WP-2016/014, Blavatnik School of Government, University of Oxford.

Yue, V. Z. (2010): "Sovereign default and debt renegotiation," *Journal of International Economics*, 80, 176 – 187.