Workers' Home Bias and Spatial Wage Gaps*

Lessons from the Enduring Divide between East and West Germany

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Abstract

Workers' birth-place shapes their labor market prospects: only few find a job elsewhere, and of those, a large fraction moves back home. Such home bias can have important aggregate effects if regions differ in their productivity. Moreover, understanding the drivers of the bias is paramount to shape regional policy. In this paper, we use Germany as a laboratory to shed light on the drivers and consequences of home bias. We first document, using matched employer employee data, that Germans born in the East enjoy large wage gains when moving West, but nonetheless they do so rarely: they mostly climb a local job ladder, which keeps them trapped in lower wage firms. We then build a tractable framework with worker reallocation across firms and space to structurally interpret the evidence. We show that East-born workers value one dollar earned in the West only as much as 74 cents earned in the East. They are also four times more likely to receive a job offer from the East, and are on average less skilled, but their skills are equally valuable in East and West. Counterfactuals show that removing either of the sources of home bias have sharp, but distinct, effects on aggregate wages and workers' utility.

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1 Introduction

Spatial wage gaps are ubiquitous: as extensively documented, workers with similar observable characterstics are paid different wages depending on their geographic location. Moreover, even within seemingly integrated labor markets, wage differences across regions persist over time and are not arbitraged away by labor mobility.[1] While the presence of sizable and persistent wage gaps could suggest that labor is spatially misallocated, it could also reflect geographic differences in amenities, sorting of workers based on their comparative advantage, or idiosyncratic tastes for a specific location. Identifying the right explanation is important: if misallocation were sizeable, the right policies could lead to large welfare gains.

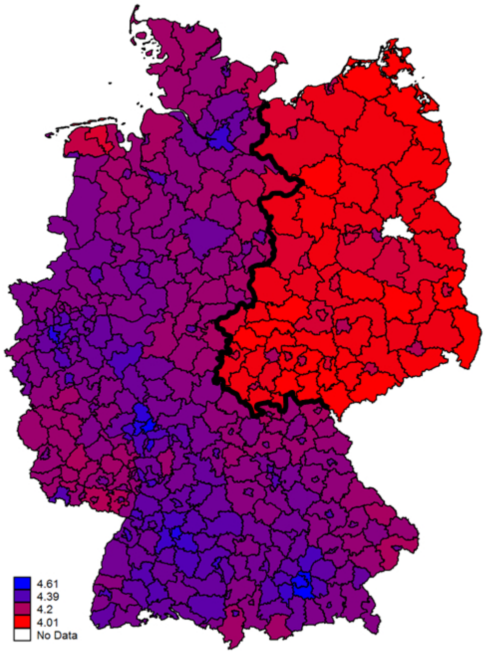
In this paper, we develop a methodology that can shed light on the origins of persistent spatial wage gaps. We adapt the tools and datasets from the frictional labor literature to the study of the spatial allocation of labor, and develop a novel framework that allows us to jointly study the reallocation of workers across firms and across space in one integrated setup with different spatial and reallocation frictions. Our framework enables us to make headway on two fronts. First, we can isolate spatial frictions that prevent the reallocation of workers across space – which have been the focus of the macro-development literature – from reallocation frictions that prevent the reallocation of workers to more productive firms – which have been the focus of the frictional labor literature. Second, we leverage matched employer-employee data to obtain direct estimates of mobility frictions instead of backing them out as a residual, which is necessary when one has to rely on cross-sectional data, as recently done by Hsieh, Hurst, Jones, and Klenow (2013) or Bryan and Morten (2017). For example, we use a revealed-preference approach to separately identify workers' *regional identity*, or taste for living in a specific region. Determining the relevance of this parameter is important, since it is unlikely to be affected by policies aimed at favoring internal migration.

We apply our methodology to the experience of Germany. Germany is an ideal laboratory to study spatial wage gaps. As Figure 1 highlights, a sizable wage gap between East and West Germany persists until today, despite the lack of any physical or legal barriers and a dynamic labor market in which one in five workers changes job each year. Furthermore, the average wage changes discretely at the former East-West border, suggesting that geographic amenities are unlikely to explain the wage gap. The border also allows us to have a predetermined, and non-arbitrary, definition of regions. Finally, Germany has available matched employer-employee data, which are necessary for our approach.[2]

[1] Moretti (2011), Gollin, Lagakos, and Waugh (2014), Kline and Moretti (2014), Redding and Rossi-Hansberg (2017).

[2] We use the administrative matched employer-employee records provided by the German Institute for Employment Research (IAB) via the longitudinal version of the Linked Employer-Employee Dataset (LIAB). The data cover the entire employment biography of about 1.9 million individuals during employment and unemploy-

Figure 1: Average Real Log Daily Wage by County, 2009-2014



Notes: Average daily log wages are obtained for a 50% random sample of establishments in Germany via the Establishment History Panel (BHP) of the Institute for Employment Research (IAB). Wages are adjusted by county-specific price levels from the Federal Institute for Building, Urban Affairs and Spatial Development (BBSR) in 2009, and deflated forward and backward in time using state-level price deflators from the Statistical Offices of the States. Former East-West border is drawn in black for clarification, there is no border today.

We proceed in three distinct steps. We start by documenting four features of the German labor market that motivate the focus of our model. We then develop a framework where heterogenous types of workers reallocate across firms and space. Finally, we bring the framework to matched employer-employee data to understand the drivers of the enduring divide between East and West Germany.

In Section 3 we use matched employer-employee data together with a half-census of all firms in Germany and document four features of the labor market that serve as motivation for our model and estimation. First, we show that labor market is characterized by high worker mobility, with workers changing firms on average every five years. Moreover, we show that gross flows between East and West Germany are sizable, while the net migration towards the West is minor. These facts motivate us to write a model with firm-firm reallocation both within and across regions. Second, we decompose workers' wages into an individual and an establishment component, using the methodology of Abowd, Kramarz, and Margolis (1999). The analysis

ment spells, matched to characteristics of the establishments at which these workers are employed, and allow us to track individuals as they move across establishments and space. We complement these data with establishmenty-level information for a 50% random sample of establishments in Germany drawn from the Establishment History Panel (BHP).

highlights that the wage gap in Germany is mostly driven by the establishment component, while sorting of high-skilled workers to the West only plays a minor role. This result contrasts for example with the case of the urban-rural wage gap, where spatial sorting is important (e.g., Young (2013); Hicks, Kleemans, Li, and Miguel (2017)) and suggests that some type of mobility friction or compensanting differential must play a role in explaining the East-West wage gap, generating an enduring wall between the two regions. Third, to investigate the drivers of this enduring gap we estimate a gravity equation that explains worker flows between counties as a function of distance as well as origin and destination county characteristics. We depart from the convential specification by allowing distance travelled to be more costly if a worker crosses the East-West border, and by interacting origin and destination fixed effects with individual birth location. We show that there is no additional cost from crossing the East-West border, but that East-born workers are much more "attracted" to East counties, and viceversa. In other words, individual identity, rather than current location is responsible for the enduring wall between East and West Germany. Fourth and last, we show that labor market frictions are present both across regions, as already discussed, but also within them. Using establishment-level data, we find that – in both East and West and even within narrowly defined industries – larger establishments pay a higher wage, which suggests the presence of frictions that prevent firm-firm reallocation. Moreover, establishments in East Germany are systematically larger, by about 1-2 workers, than West German establishments paying the same real wage, consistent with spatial frictions that allow East German establishments are able to hire and retain more workers for a given wage, and hence face a slacker local labor market.[3]

Section 4 develops a theoretical framework that allows us to unpack the wage gap into its different components. The nature of our data calls for a model with two types of labor reallocation: (i) spatial movements across East and West Germany; (ii) reallocation within each region across heterogeneous firms. Our main building block is a standard heterogenous firm job-posting model à la Burdett-Mortensen (e.g., Burdett and Mortensen (1998)), which we extend to a setting with an arbitrary number of regions and arbitrary many worker types. Each region is characterized by an exogenous productivity distribution of firms. Each worker type is characterized by a vector of region-specific skills, preferences, and wedges that define the relative chance of receiving a job offer from a given region. Firms choose their optimal wage and decide how many job vacancies to open, subject to a region-specific cost. Workers randomly receive offers and accept the offer that yields the higher utility, moving across firms both within and across regions. Despite the rich heterogeneity, we derive a tractable solution represented by a system of two sets of differential equations with several boundary conditions.

[3]This finding holds for all establishments except in the right tail, where East Germany has only relatively few establishments.

The unique feature of our model vis a vis the previous literature is its ability to distinguish between spatial frictions and reallocation frictions, and hence to compare how the mobility of workers across regions differs from the mobility of workers across firms within a given region. Reallocation frictions are the standard frictions present in frictional labor market models, and are generated in our setup by firms' cost of posting new vacancies and exogenous separation rates. The spatial frictions, which are our main focus, are generated by three type- and region-specific parameters: (i) workers' productivity (e.g., West German workers are more productive); (ii) workers' location preferences, and (iii) workers' probability of receiving an offer from each region. The allocation of labor across regions could be distorted by any of these previous margins. For example, within the spatial frictions, East German workers might be disproportionately attracted to the East if either they have a comparative advantage in East Germany, a stronger preference for the East, or are more likely to receive offers from East German firms.

We quantitatively estimate the model in Section 5. While all parameters are jointly estimated, we begin by performing reduced-form regressions to show that the different types of frictions can be separately identified in our data. First, workers' relative productivity can be identified by comparing the wages of observationally equivalent East and West German individuals employed at the same establishment. We find that West German workers' residual wage, after controlling for individual characteristics, is about 3% higher at both East and West German establishments. This result suggests a slightly higher absolute productivity of West German workers, but no comparative advantage of either type of worker in either region. Second, we estimate workers' relative location preferences by comparing the wage gains of East- and West-born workers when they switch jobs both within and across regions. We find that the average wage gain of an East-born job switcher from East to West Germany is about 45 percentage points higher than the same worker's wage gain when she switches jobs within East Germany, while a West-born worker only receives a 19 percentage point higher wage increase from moving across regions than from moving within the East. This difference suggests that East-born workers' relative preference for East Germany amounts to a 27 percentage point wage premium. Finally, we estimate workers' relative probability of receiving offers from each region using the mobility patterns of workers of each type. We find that flows of East German workers from the East to the West would need to increase by about a factor of ten to be consistent with an integrated market.

While Germany's example is particularly stark, persistent spatial wage gaps also exist within many countries, such as the Italian Mezzogiorno or the South-West of Spain. A large literature has sought to explain the existence of these gaps, arguing that they are only a manifestation of different local amenities (Rosen (1979), Roback (1982)), the result of sorting of workers with different ability (Young (2013)), or due to mobility frictions making it costly for workers to relocate (Bryan and Morten (2017)). Our work is, to our knowledge, the first to bring to bear detailed matched employer-employee data to disentangle these three types of frictions in a unified framework, and to relate frictions across space to frictions across establishments. Our finding that the persistent wage gap is mostly driven by a strong regional identity of workers may have far reaching implications. First of all, it casts doubts on the efficacy of policies aimed at an economic integration of the East and West German labor markets. Through the lens of our model, neither better infrastructure, nor labor market subsidies or even training programs, would have a discernible effect on the spatial wage gap or on the aggregate productivity. Instead, our results suggest that the two regional labor markets could be truly integrated only through the slow process of cultural and social assimilation necessary to reach a common German identity. Furthermore, we have shown that workers' preferences for East Germany shape the distribution of active firms in the East, and allow firms paying low wages to become relatively large. Our results thus provide a theory of spatial differences in firms' productivity distribution that takes as primitive worker preferences. By accepting to work for a lower wage in order to be able to stay in their home region, East born workers effectively keep alive firms that would be uncompetitive in a truly unified German labor market.

Literature. We are not the first to study spatial wage gaps. A large literature, at least since the work of Harris and Todaro (1970) on the rural-urban wage gap, has sought to explain the large observed differences in average wages across space. The literature can be broadly divided into two sets of papers. The first category assumes free labor mobility and homogenous workers and solves for spatial equilibria along the lines of the seminal work by Rosen (1979), Roback (1982), and more recently of Allen and Arkolakis (2014). The assumption of a spatial equilibrium implies that utility is equalized across space, and therefore the observed differences in wage gaps are simply a reflection of differences in local amenities. The second category, instead, has studied spatial wage gaps as a possible symptom of misallocation of labor across space. A core debate in this literature has been to distinguish between sorting of heterogenous workers based on their comparative advantages and frictions to labor mobility that generate wedges along the lines of the work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).[4] Our paper belongs to this second category of papers. As the more recent work in this literature, we allow for both sorting and frictions to explain the wage gap between East and West Germany. Our main contribution is to unpack the "spatial wedges" into several components, and hence to open up the black box of labor mobility frictions. In order to pursue this task we apply the toolset and the datasets of the frictional labor literature. In particular, our model adapts the work of Burdett and Mortensen (1998) to a setting with a non-trivial spatial dimension.

[4] See for example Bryan and Morten (2017); Young (2013); Hicks, Kleemans, Li, and Miguel (2017).

Moreover, we rely on matched employer-employee data , as now common in the labor literature,[5] and we show that they are crucial to distinguish spatial frictions from the general reallocation frictions across firms, which are the focus of the labor literature. From a purely methodological perspective, our paper bridges the gap between the macro-development misallocation literature and the frictional labor literature. We are – to the best of our knowledge – the first to use the tools of the latter within the context and research questions of the former.

Our work is informed by, and consistent with, the rich literature on migration. The idea that worker identity may be an important drivers of migration decisions is at least as old as the work of Sjaastad (1962), and more recently has been revived by the structural approach of Kennan and Walker (2011). This work has documented an important role for home preferences in explaining the dynamics of migration choices. Our contribution is to embed these core ideas into a labor model with reallocation across space and firms. We show that considering both dimensions of reallocation together is important to properly estimate the spatial frictions.

Last, our work is related to the (quite limited) literature that has examined East German convergence (or the lack thereof) after the reunification (e.g., Burda and Hunt (2001), Burda (2006)). This literature has in particular studied possible drivers behind the wage gap between East and West Germany and the nature of migration between the two regions (Krueger and Pischke (1995), Hunt (2001, 2002, 2006), Fuchs-Schündeln, Krueger, and Sommer (2010)). We use matched employer-employee data to examine the role of worker sorting, productivity differences, spatial, and reallocation frictions in a unified framework; a task that has not been attempted before.

2 Data

Our work relies on confidential micro data provided by the German Federal Employment Agency (BA) via the Institute for Employment Research (IAB). First, we use establishment-level data from the Establishment History Panel (BHP). This dataset contains a 50% random sample of all establishments in Germany with at least one employee liable to social security on the 30th June of a given year. The data are based on mandatory annual social security filings. Government employees and the self-employed are not covered. The BHP defines an establishment as a company's unit with at least one worker liable to social security operating in a distinct county and industry. Since several plants of the same company may operate in the same county and industry, the establishments in the BHP do not always correspond to economic units such as a plant (Hethey-Maier and Schmieder (2013)). Throughout the rest of this paper, we use the terms

[5] See for example Card, Heining, and Kline (2013).

establishment and firm interchangeably to refer to these entities. For each such establishment, the dataset contains information on the establishment's location, number of employees, employee structure by education, age, and occupation, and the wage structure. The underlying population of workers comprises about 80% of the German working population. The data are recorded as annual cross-sections since 1975 for West Germany and since 1991 for East Germany, which we combine to form a panel covering about 650,000 to 1.3 million establishments per year. Unless otherwise noted, we examine the period 2009 to 2014, the last available year in the data, to focus on persistent differences between East and West Germany. We focus on full-time employees only.

Our second, and most important, dataset is matched employer-employee data provided via the longitudinal version of the Linked Employer-Employee Dataset (LIAB). The dataset is a combination of establishment information from the BHP, the IAB Establishment Panel (a representative survey of German establishments), and individual-level information from the Integrated Employment Biographies (IEB). The IEB contains employment information and socio-economic characteristics of all individuals that were employed subject to social security or received social security benefits since 1993. The LIAB data are a representative sample of individuals from the IEB, linked to information about the establishments at which these individuals work. The sample is drawn by taking all establishments that are in the IAB Establishment Panel survey in any year between 2000 and 2008, and selecting all the individuals who were employed in one of these establishments at least one day during the time period from 1999 to 2013. The entire employment history of these individuals is then drawn for the period 1993-2014, including spells at other establishments, unemployment spells, etc., with exact beginning and end dates for each spell. The LIAB sample covers about 1.9 million individuals working for between 2,700 and 11,000 establishments per year. In addition to establishment information, the dataset records an individuals' education, location of residence and location of work, year of birth, occupation and daily wage. As with the BHP, we keep only full-time employment spells, and focus on 2009-2014.

We exploit the panel structure of the LIAB to impute each individual's birth location, which is not recorded in the data. We code an individual as born in East Germany (West Germany) if at the first time she appears in our full dataset since 1993 her residence location is in the East (West). Since, for employed workers, the residence location is not available before 1999, for these workers we use the location of the establishment before that. We compute whether an individual is currently located in the region in which she was born and label her a "native" in that case. We label her as "foreign" if she is currently in the other region.

Our third dataset contains information on cost of living differences across German counties from a study conducted by the Federal Institute for Building, Urban Affairs and Spatial Development for the year 2009 (BBSR (2009)), which we use to translate all wages into real wages. While regional differences in price levels within Germany are not measured by Germany's statistical offices, the BBSR's study assesses regional price variation across 393 German micro regions covering all of Germany that correspond to counties or slightly larger unions of counties. The data cover about two thirds of the consumption basket, including housing rents, food, durables, holidays, and utilities. Figure 11 in Appendix F shows the map of county-level price levels, and confirms that East Germany on average has lower prices, in particular in the cities. We adjust all wages in the BHP and in the LIAB in 2009 based on the BBSR's local price index, and then deflate the wages forward and backward in time using state-specific GDP deflators from the statistics offices of the German states. We complement our data with publicly available county-level unemployment rates from the Federal Employment Agency.

3 Does an Enduring Wall Still Separate East and West?

Germany was divided into two separate nations until 1990, governed by two distict economic systems. While West Germany was a market economy, the economy in East Germany (then called the German Democratic Republic, GDR) was planned and followed a communist regime. There was virtually no movement of workers between the two regions, and the border was tightly controlled. In 1990, with the fall of the "Iron Curtain", East Germany reunited with the West and adopted all its legal and economic institutions.[6] Workers were free to move between the two regions: there is no physical border, language, or legal barrier. Nonetheless, as Figure 1 shows, more than twenty years after the formal reunification a sizable and sharp wage gap remains, as if an enduring wall still separates East and West Germany.[7] In this section, we exploit our rich data to document four features of the German labor market that both provides suggestive evidence on the determinants of the persistent wage gaps and serve as motivation for the model we write in Section 4.

Labor Mobility. The simplest explanation for the persistency of the East-West wage gap is lack of labor mobility: in the presence of prohibitely large moving costs, a large wage gap between East and West could persist. The data, however, don't support this hypothesis, as shown in Table 1, which presents some mobility statistics for our core period of analysis, 2009-2014. In the first row of the table, we compute, separately for East- and West-born individuals, the total number of firm-firm moves in every year. For both groups, there is approximately one move in every five individuals per year. Moreover, a non-negligible share of these moves

[6]Section A in the Appendix provides more background on the reunification process

[7]Figure 12 in Appendix F shows a time series of real wages in East and West Germany and highlights that after an initial period of rapid convergence in the early 1990s, the gap has remained virtually constant.

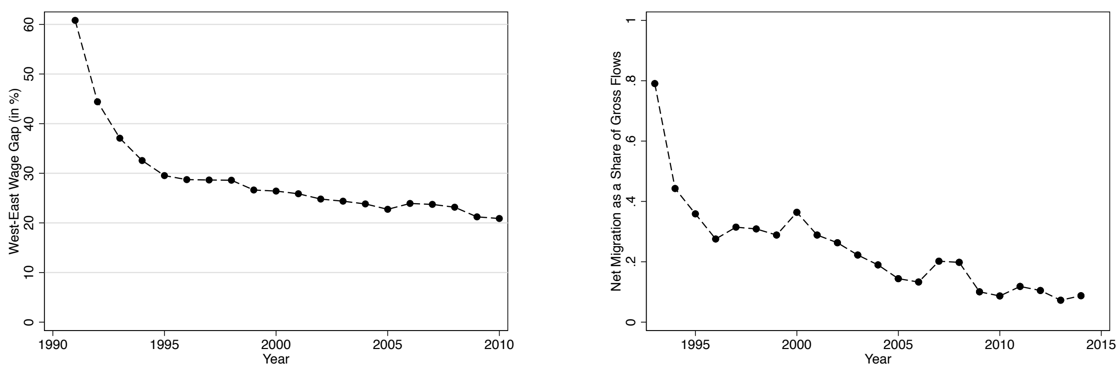
Table 1: Labor Mobility, 2009-2014

	West Born	East Born
$\frac{\text{Number of Firm to Firm Flows}}{\text{Number of Workers}}$	0.24	0.22
$\frac{\text{Gross Flows Across Regions}}{\text{Number of Firm to Firm Flows}}$	0.04	0.15
$\frac{\text{Net Flows Across Regions}}{\text{Number of Firm to Firm Flows}}$	0.004	0.009

Figure 2: Wage Gap and East-West Reallocation Over Time

(a) West Wage Premium

(b) Net Flows of East Born



– especially for East-born – are between two firms located on different sides of the East-West border (row 2). Thus, there is significant mobility between East and West Germany. At the same time, only a very small share of the overall moves are net flows across regions (row 3): that is, while several East born workers move to the West, almost as many move back to the East after a period working in the West.

Figure 2 presents the East-West wage gap (left panel) and the share of net flows of East workers to the West, that is, the share of East workers that moved West and did not return to the East during the sample period. We find that in the years immediately after the reunification, the wage gap quickly decreased, and most of cross-regional flows were net reallocation towards the West. In the more recent years, however, the economy seems to have converged to a steady state, with a quasi-constant wage gap and labor mobility both within and across regions.

Overall, the sizeable cross-region flows exclude the possibility that the wage gap is purely driven by an impermeable wall that prevents workers from moving West. Furthermore, given the substantial job-to-job flows not only across but also within regions, a setting with reallocation both across space and across firms is necessary to accommodate the data.

Sorting. The cross-sectional spatial wage gaps could be driven by sorting of more skilled workers to West Germany.[8] In order to investigate this hypothesis, we use the LIAB and follow four different approaches. Each approach follows standard methods in the literature. For this reason, we leave the details of the estimation to Appendix B.

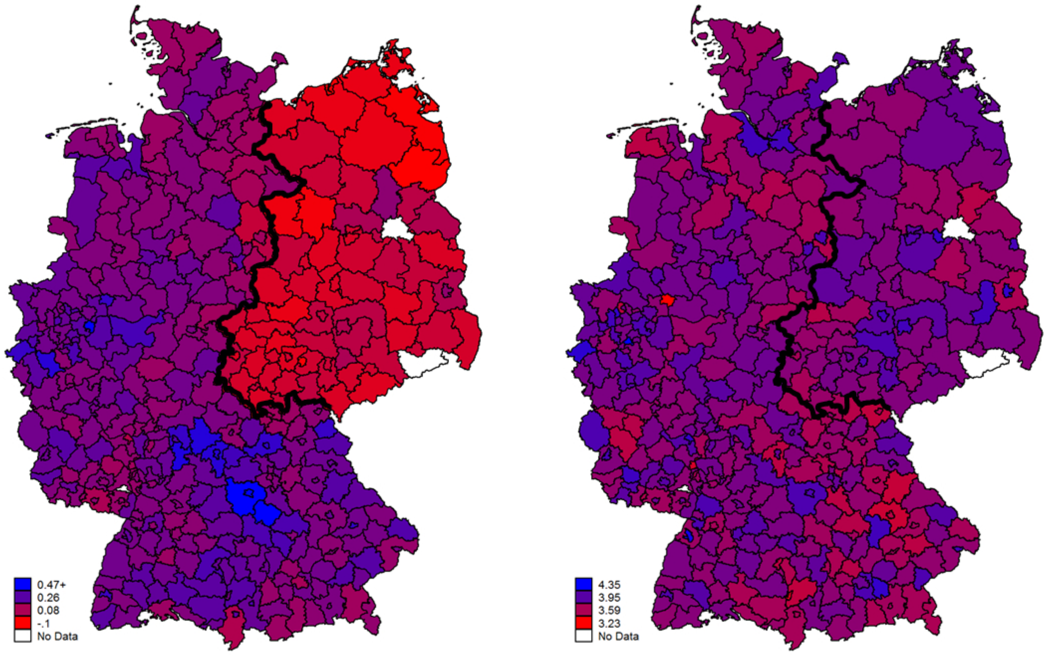
First, we run a simple Mincer wage regression where we additionally include a dummy for working in West Germany. We find that, controlling for experience and schooling, the wage gap is still very large: workers in the West earn a 27% higher real wage. Second, we follow the recent work of Roca and Puga (2017), which argues that a large share of spatial wage gaps are dynamic rather than static, and let returns to experience and schooling vary by region. Consistent with previous findings, the wage gap declines, but is still large, at 19.5%. Third, we control for unobservable characteristics, by including a worker fixed effect in the regression. The wage gap now shrinks quite considerably, to 13.5%, thus suggesting that sorting could play at least a partial role. However, we should be cautious in interpreting this result, since it may suffer from the migration selection bias (see McKenzie, Stillman, and Gibson (2010)). Our preferred estimates is the fourth and last approach. We leverage the match component between firms and workers, and fit in the data a linear model with additive worker and establishment fixed effects, as originally in Abowd, Kramarz, and Margolis (1999) and, more recently, in Card, Heining, and Kline (2013). We follow Card, Heining, and Kline (2013) closely, but use data from both East and West Germany. As is standard, we estimate the model on the largest connected set of workers in our data, since identification of workers and firm fixed effects requires firms to be connected through workers flows. We describe the estimation in more detail in Appendix B. The largest connected set includes approximately 97% of West and East workers in the LIAB. This approach identifies the characteristics of East and West workers by comparing their wages while working in the same firm. The results show that West-born workers are on average paid a slightly higher wage than East-born workers at the same firm, thus suggesting that they may be more skilled: the average worker fixed effect for West workers is 4.5% higher than for East workers. However, most of the spatial wage gaps is explained by firm fixed effects: the average firm fixed effect in West Germany is 19.7% higher than in East Germany. Figures 3a and 3b plot both county level averages of worker and firm fixed effects: firm effects clearly drive the average wage gaps. These results imply that, on average, workers moving from East to West face large real wage gains. However, as we just documented, the net migration rate is low, thus suggesting that either some sort of mobility friction or compensanting differential must play a role in explaining the East-West wage gap, generating a wall between the two regions that,

[8] Sorting has been shown to explain a consistent share of spatial wage gaps in several contexts, see for example Glaeser and Mare (2001); Combes, Duranton, and Gobillon (2008); Hicks, Kleemans, Li, and Miguel (2017); Young (2013)

Figure 3: Average Establishment and Worker Component, 2005-2014

(a) Average Establishment Component

(b) Average Worker Component



Source: LIAB, Authors' Calculations

Source: LIAB, Authors' Calculations

while permeable, is still enduring.[9]

Identity, not Geography. We next study worker flows across counties to investigate what forces keep the East and West labor market distinct. At an elementary level, the two labor markets could be separated by either a geographical barrier, which makes flows between East and West (and vice-versa) more costly, or by an identity barrier, which makes workers born in the East more attracted to East counties irrespective of their current location (and vice-versa). To distinguish between these two hypothesis, we run a gravity equation for workers flows between counties enriched with two features: distance travelled across the East-West border is allowed to be costlier, and destination and origin fixed effects are allowed to vary by region of birth. As we now describe in details, we find that geographic barriers don't play a role, while regional

[9]We use a slightly longer time period from 2005-2014 to increase the sample size for disclosure purposes. In Appendix F, we include further results: we plot the distribution of establishment and worker fixed effects in Figures 13a and Figures 13b, and we plot the maps for the other two covariance components of the decomposition in Figures 14 and 15. These results corroborate the main narrative that a shift in the average establishment fixed effects explains the majority of the average wage gap between East and West. In Appendix D, we perform a growth accounting exercise using aggregate data on GDP, capital, and labor from the statistics offices of the states. We show that West German real GDP per capita is about 40% higher than in the East, with the difference almost completely due to higher TFP in the West.

identity does.

The LIAB data allow us to track workers as they change jobs over time. We compute, for the period 2009-2014, the share of newly hired workers in county d from each origin county o, distinguishing workers by birth region b. Let $h_{o,d,b}$ be the total number of workers born in region b that were employed in the previous year in county o and are starting this year a job in county d. We compute the share of workers from county o moving to county d,

$$s_{o,d,b} = \frac{h_{o,d,b}}{\sum_{d \in \mathbb{D}} h_{o,d,b}}$$

where $\mathbb{D}$ is the set of all the 402 counties in both East and West Germany. We use $s_{o,d,b}$ to study the flows of workers across counties. We observe at least one worker flow, and hence a positive $s_{o,d,b}$, for $94,546$ out of the 161,000 possible origin-destination pairs. We use these pairs to fit the gravity equation

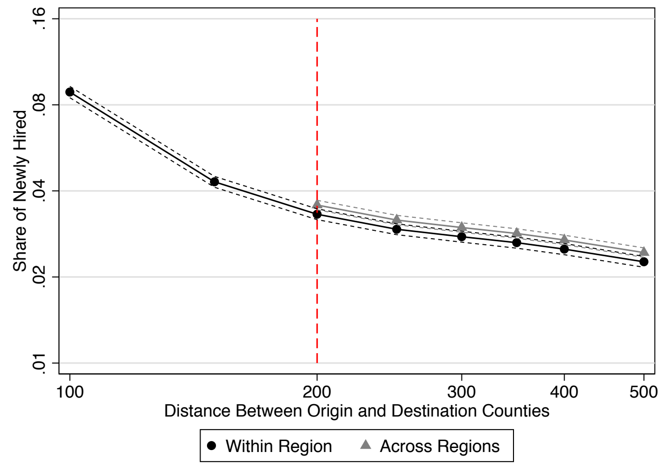
$$\log s_{o,d,b} = \delta_{o,b} + \gamma_{d,b} + g(dist_{o,d}) + \epsilon_{o,d,b}, \tag{1}$$

where $\delta_{o,b}$ and $\gamma_{d,b}$ are county of origin and destination fixed effects, which are allowed to vary by birth-region and capture the fact that some counties may be more attractive than others, either due to better market conditions or higher amenities; and $\epsilon_{o,d}$ is a mean zero i.i.d. error term. Furthermore, $g\left(dist_{o,d}\right)$ is a generic function of distance, $dist_{o,d}$, between the origin and destination counties in km. The sharp discontinuity near the former East-West border suggests that if mobility frictions are an important component of the explanation, they must change discountinuisly at the border. We allow for this possibility by specifying the effective distance between any two counties as a weighted average of the distance travelled within a region and distance travelled across regions. Define a set of buckets for the distance travelled, $\mathbb{X}$, which includes 50 km intervals from 50km-100km onward to 350km-400km, and an eighth group for counties that are further than 400 km apart. The omitted category is up to 50km, which includes the origin county. Similarly, we define another set of buckets for moves that are across regions, $\mathbb{Y}$, which includes four groups: 1-100, 100-150, 150-200, and 200+. We then define the function g as

$$g\left(dist_{o,d}\right) = \sum_{x \in \mathbb{X}} \phi_x D^x_{o,d} + \xi_{o,d} \sum_{y \in \mathbb{Y}} \psi_y D^y_{o,d},$$

where D^x_{od} is a dummy that takes value one if the distance between counties o and d is in the distance bucket x, $\xi_{o,d}$ is a dummy variable that takes value one if counties o and d are in different regions, and D^y_{od} is a dummy that takes value one for cross-region moves if the distance between county o and the closest county in the region of county d is in distance group y. If workers treat moves between East and West Germany in the same way as moves within their region, then the coefficients on these dummies, ψ_y, should be zero.

Figure 4: Geography



We visualize the results in Figure 4, which plots for an average county situated at 200 km from the regional border the share of workers that would be hired by a county in the same region at distances $x \in \mathbb{X}$ and by a county in the other region at distances $x \in \mathbb{X}$ for $x \geq 200$.[10] The detailed coefficients from the regression are presented in column (1) of Table 8 in Appendix G. Figure 4 shows, first, that workers are more likely to move to nearby counties, as evidenced by the declining share of workers hired from counties that are further away. This finding corresponds to the standard gravity result in the trade and labor literatures. More importantly, we find that after controlling for county fixed effects, there is no border effect: conditional on distance and fixed effects, workers are just as likely to move across regions than within region.[11]

Next, we turn to the analysis of the origin and destination fixed effects.[12] Workers born in the East and West may be attached to their respective home region, either due to preferences, comparative advantages, or possibly due to a social network that allows them to find job opportunities. To investigate this hypothesis, we study whether the origin and destination fixed effects vary systematically for East-born or West-born individuals. Consider first the destination fixed effects. For each county, we compute the difference between the destination fixed effect for East- and West-born. In the left panel of Figure 5 we plot the resulting fixed effects gaps as a function of the county distance to the East-West border, re-normalized in such a way

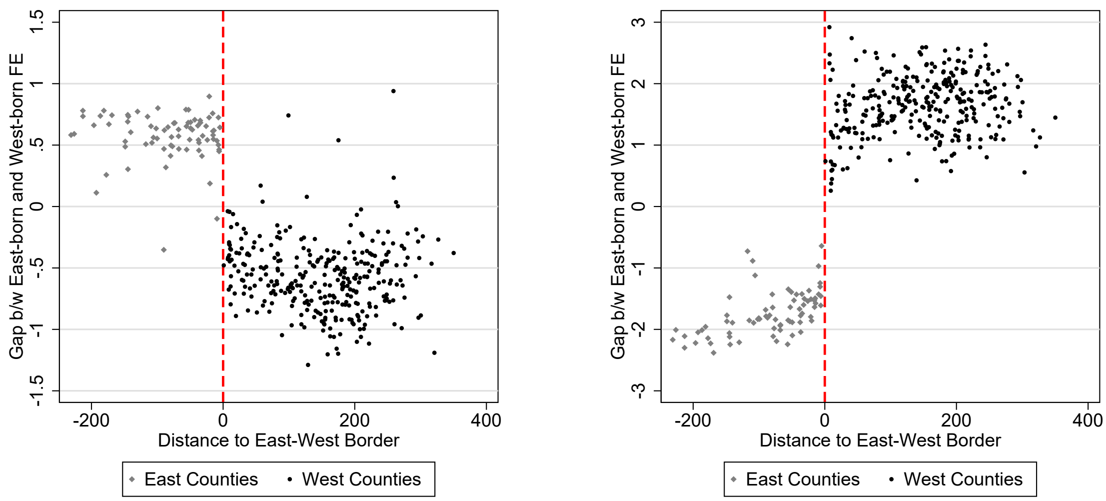
[10] We normalize the coefficients so that they sum to 1 including the excluded category, which is up to 50km.

[11] A border effect is instead present if we don't allow the county fixed effects to vary by birthplace. Figure (18) shows that such border effects can be large.

[12] As known in gravity equations, the level of the fixed effects is not identified. Therefore, for both origin and destination fixed effect, we normalize them, for both East-born and West-born workers, relatively to the average fixed effect, weighted by the number of within region counties, in such a way to assign equal weight to East and West Germany. This normalization is without loss of generality, since we are interested only in the relative fixed effects across counties, and not in their level.

Figure 5: Identity

(a) Destination FE (b) Origin FE



that East counties have negative distance. The figure shows a sharp border effect: East-born individuals are more attracted to counties in the East and vice-versa. The right panel of Figure 5 replicates the same analysis for the origin fixed effects: an identical pattern emerges, with inverted sign since a high origin fixed effect means that workers are not attracted to that county and are likely to move out of it. Overall, this analysis shows that both East and West workers have a strong "home bias" to the extent that are more attracted by counties in their own birth-region. Given the important role played by birth-place specific fixed effects, it is natural to wonder why East-born individuals are less attracted to West counties. The structure of the model will help us elucidate different drivers, and unpack labor market frictions from taste and comparative advantages. Even without relying on the model, however, we can make some progress exploiting a natural experiment. As documented in details in Burchardi and Hassan (2013), in the years 1946 to 1961, after World War II, a few millions individuals fled to West Germany after having spent several years in the East to pre-empt the construction of the wall. These "East-tied" individuals were more likely to settle in counties with available houses. As a result, we can use, replicating the same identification strategy of Burchardi and Hassan (2013), housing destruction due to WWII as an instrument for inflow of these individuals, but focus on the effect on migration flows. The results in Table 2 show that those counties in the West that exogenously received more East-tied individuals before 1961 are in our period of analysis, 2009-2014, relatively more attractive for East-born individuals. Specifically, columns (1) and (2) regress the gaps in destination and origin fixed effects, respectively, on the instrumented

Table 2: Inflows of East-tied Workers before 1961 and Nowdays Attraction of East-born Workers

	(1)	(2)	(3)	(4)
	$\gamma_{d,E} - \gamma_{d,W}$	$\delta_{o,E} - \delta_{o,W}$	$\gamma_{d,E}$	$\delta_{o,E}$
Share Expellees (Sov. Sec.) '61	0.29	-0.59*	0.45	-0.56**
	(0.40)	(0.35)	(0.30)	(0.28)
Income 1989 (p.c., log)	-0.17	0.06	0.05	-0.00
	(0.10)	(0.09)	(0.08)	(0.07)
Distance to East	0.02	-0.06	0.10	0.01
	(0.12)	(0.11)	(0.09)	(0.08)
Dest. FE for West-Born ($\gamma_{d,W}$)			0.55***	
			(0.05)	
Orig. FE for West-Born ($\delta_{o,W}$)				0.62***
				(0.04)
State Fixed Effects	Y	Y	Y	Y
N	291	291	291	291

Note: $*$ is significant at 10pp, $**$ at 5pp., *** at 1pp

inflows of East-tied individuals.[13] Coefficients are normalized in terms of standard deviations. The results have the expected sign – larger inflows lead to stronger attractiveness of a county – and are large in magnitude but either non significant (column 1) or marginally significant (column 2). Since the gap between East-born and West-born fixed effects is likely to be measured with significant noise, in column (3) and (4), we replicate the same analysis using simply the fixed effect for East-born and controlling for the ones of West-born. The point estimates are comparable and now have stronger statistical significance. We also note that the coefficient on West-born fixed effects is large and positive: within West Germany, East- and West-born individuals agree on which counties are more attractive, but East-born ones are simply less attracted to all counties in the West.[14]

Reallocation and Spatial Frictions. Our last step is to document, as two distinct concepts, the presence of frictions affecting firm-to-firm labor mobility within regions – or what we will refer to as *reallocation frictions* –, and the presence of frictions in labor mobility across regions – or *spatial frictions*. While there could be several alternative empirical approaches, our empirical investigation is guided by the framework of Burdett and Mortensen (1998), which we built upon in the model of the next section.

[13] The exact variable is the share of expellees through the Soviet Sector. See Burchardi and Hassan (2013) for details.

[14] This same fact can also be seen in Figure 19, in Appendix F, where we plot fixed effect of East-born as a function of fixed effects of West-born.

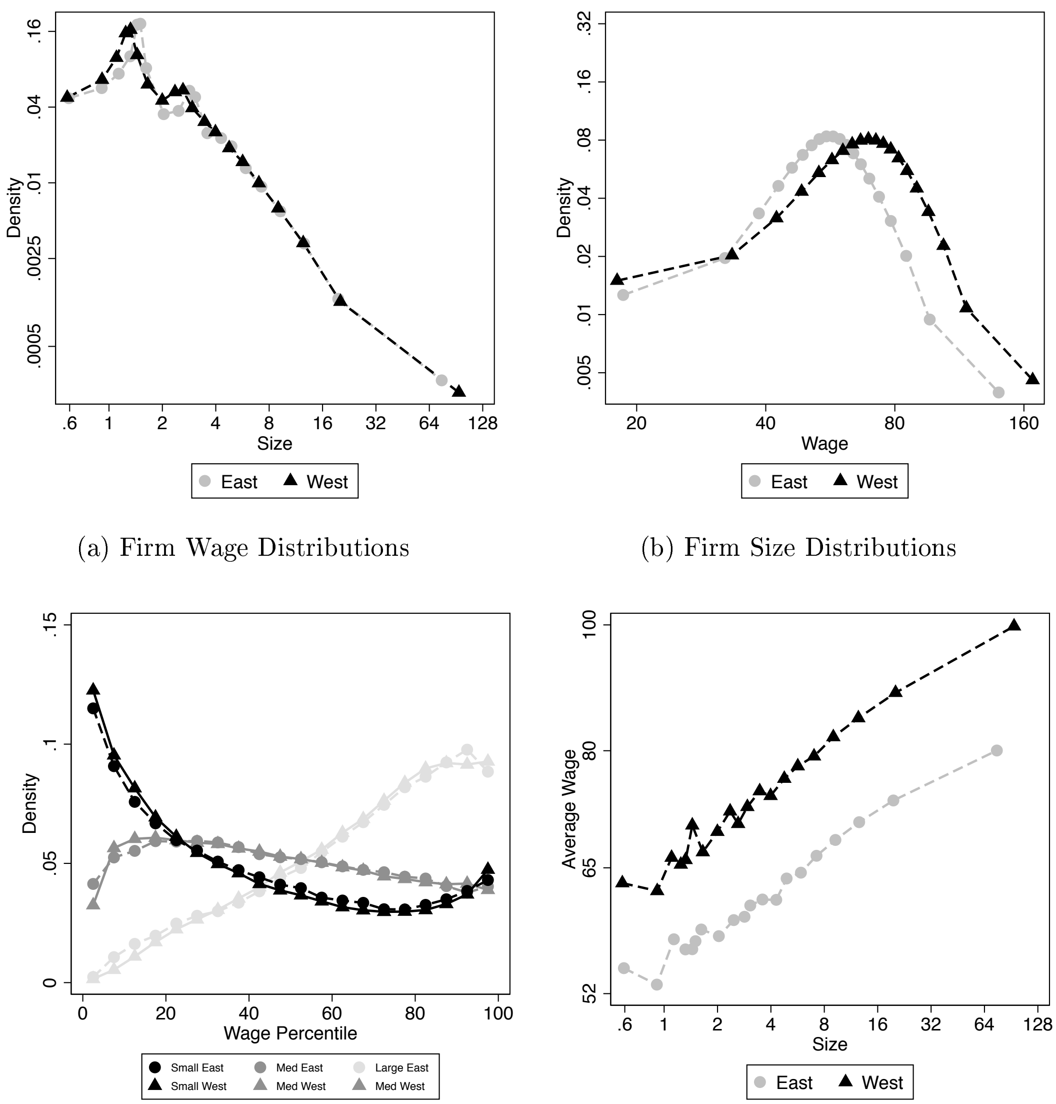
In frictional labor markets high and low productivity firms can coexist and offer different wages, as in Burdett and Mortensen (1998). Labor market frictions prevent immediate reallocation of workers from the low to the high paying firms, thus allowing the lower productivity firms to survive, but with a smaller size. Therefore, a natural way to document the presence of reallocation frictions is to show the existence of a job-ladder, where larger firms pay a higher wage. Along this same line of reasoning, the spatial wage gaps between East and West could simply be the byproduct of East firms being less productive while the German labor market is fully integrated. Under this scenario, we should observe that firms in the East are smaller – since their lower wage leads more workers to separate towards West firms – but conditional on size there should not be a wage gap since firm location does not matter in a fully integrated market. Therefore, a natural way to document the presence of spatial frictions is to show a positive size-specific wage gap.

We investigate these possibilities through a study of the joint distribution of firm wages and size in East and West using the BHP data. We first run, separately for East and West, firm level regressions of log size and log real wage on year and 3-digit industry fixed effects, and compute from this procedure residuals which capture the within-industry relative wage and size of each firm. We add to these residuals the region-level averages of log size and log real wage to maintain the overall gap between the two regions. Details are in Appendix C. This simple procedure recovers the real wage and size distributions, in East and West, controlling for industry composition. In Figure 6 we plot the results. In the top panels, we plot the wage and size distributions: firms in West Germany pay a higher real wage on average, but they are of the same size as those in the East. In the bottom-left panel, we plot the wage distribution for firms in the 1st, 10th, and 20th twentile of each region's size distribution. Large firms, as expected, pay on average higher wages, as evidenced by the rotation of the distribution to the right. Moreover, these joint distributions of wage and size are almost identical in East and West. As a result, given the shift of the wage distribution in West Germany to the right, firms in the West pay on average a higher real wage for each firm size, as shown in the bottom-right panel.

The analysis highlights that there is strong evidence in favor of a job ladder – possibly generated by reallocation frictions – both in the East and in the West, with larger firms paying higher wages. However, the ladder is at different levels in East and West, and thus the data are not consistent with reallocation frictions alone explaining the wage gaps, but suggest the presence of spatial frictions as well.[15]

[15] In order for reallocation frictions to explain the wage gap, we should have observed in the bottom-right panel that East firms, conditional on size, pay the same wage as the West, and that East firms pay a lower wage because they are smaller on average. Both these conditions are not satisfied in the data.

Figure 6: Distributions of Firms' Wages and Sizes.



(a) Firm Wage Distributions

(b) Firm Size Distributions

(c) Joint Wage-Size Distributions

(d) Wage-Size Relationships

Summary of Findings. We summarize the results of this section in the following four feature features of the German labor market.

Fact 1: *There is a persistent wage gap between East and West, despite sizable labor mobility.*

Fact 2: *The wage gap is mostly due to firms' and not workers' characteristics.*

Fact 3: *The barrier that separates the East and West labor market is mostly due to individual identity or "home bias" and not to geographical barriers to mobility.*

Fact 4: *There are two distinct, but overlapping, job ladders within the two regions.*

4 A Multi-Region Model of a Frictional Labor Market

We next approach the data through the lens of a model that is motivated by and that can be, if properly parametrized, consisent with the five facts on the German labor market. The model's aims are threefold: (i) it gives theoretical guidance on the types of spatial frictions that can generate the persistent wage gap; (ii) it provides a framework that we can use as a measuring device to identify the relative contribution of the different types of frictions; (iii) it provides a laboratory to perform counterfactual analysis in general equilibrium, taking into account the endogenous response of firms to changes in the labor supply.

Our model follows closely the work of Burdett and Mortensen (1998) and of more recent empirical applications, such as Moser and Engbom (2017). We depart from this previous work along two dimensions: we consider J distinct regional markets, each inhabited by a continuum of heterogeneous firms; and we consider I different types of workers, which are allowed to have a regional identity or to be biased towards one or more region. The regions are separated by permeable borders, and workers and firms all interact in one labor market that is subject to different types of reallocation and spatial frictions. To our knowledge, we build the first model that encompasses spatial and reallocation frictions within a unified framework. Our model differs from the literature in international trade (e.g.,Caliendo, Dvorkin, and Parro (2017)) because we introduce a frictional labor market in which heterogeneous firms optimally post vacancies and wages into a model of spatial mobility. In contrast to the trade literature, we abstract from trade in goods, and simplify on the worker side so that workers' migration decisions only depend on their flow utility.

4.1 Model Setup

We first provide a broad overview of the environment, then we study the problem of workers and firms, and last we discuss how the labor market clears.

Environment. Let time be continuous. There are J regions in the econom. The economy is inhabited by a continuum of mass 1 of workers of types $i \in \mathbb{I}$, where $\mathbb{I} = \{1, .., I\}$. We denote the mass of workers of type i by $\bar{D}^i$, where $\sum_{i \in \mathbb{I}} \bar{D}^i = 1$.

The workers differ in both their ability and in their taste for being in a given region. Specifically, a worker of type i produces θ^i_j units of output per time unit in region j, where we use superscripts for worker types and subscripts for regions. If this worker is employed at wage rate w per efficiency unit, he earns an income of $w\theta^i_j$. Furthermore, worker i has a preference parameter of τ^i_j for being in region j. Assuming linear utility as is standard in models following Burdett and Mortensen (1998), worker i's utility from receiving wage rate w in region j is $u^i_j = w\theta^i_j\tau^i_j$.

Workers operate in a frictional labor market and can either be employed or unemployed. A worker of type i faces an arrival rate of vacancy offers from region j of $\varphi^i_j\lambda_j$, where λ_j is the endogenous arrival rate of offers from region j, determined below, and φ^i_j is an exogenous wedge, which we normalize to satisfy $\sum_{i \in \mathbb{I}} \varphi^i_j \bar{D}^i = 1$. We assume that the arrival rate of offers does not depend on the region in which the worker is currently located, but only on the worker type. This is a strong assumption, which is needed to provide analytical tractability. Importantly, in our context, this assumption seems to be supported by the evidence: it is consistent with one sharp empirical fact shown in the previous section, namely that once we allow county fixed effects to depend on worker type (i.e. on where they are born), the current region does not affect the mobility patterns across counties. Workers draw offers from the endogenous distributions of wages F_j in all regions j, and must decide whether to accept an offer as soon as it is received. They separate into unemployment at rate δ^i irrespective of where they are working, and receive a utility flow equal to b^i when unemployed.[16] All agents discount future income at rate r.

On the firm side, there is a continuum of firms of mass one exogenously assigned to regions $j \in \mathbb{J}$. Γ_j is the mass of firms in region j, and by assumption $\sum_{j \in \mathbb{J}} \Gamma_j = 1$. Within each region, firms are distributed over productivity p according to density function $\frac{\gamma_j(p)}{\Gamma_j}$ with support on the positive real line. In each region j, the support of firms with positive mass is a region-specific closed set $[\underline{p}_j, \bar{p}_j] \subseteq \mathbb{R}^+$. Firms produce output from each vacancy with the production function $Y_j = p \sum_{i \in \mathbb{I}} \theta^i_j l^i_j$, where $\sum_{i \in \mathbb{I}} \theta^i_j l^i_j$ is the number of efficiency units of labor used by one vacancy

[16] The assumptions that δ^i and b^i depend only on worker's type mimic the assumption on the arrival rate. These three assumptions together guarantee that workers choose across wage offers only based on present utility flows and not on different continuation values.

of the firm.

Each firm p in region j decides how many vacancies $v_j(p)$ to post, subject to a vacancy cost $c_j(v)$, and what wage rate $w_j(p)$ to offer. Firms compete for all worker types in one unified labor market. To our knowledge, this is a novel feature of our wage-posting environment. Recent previous work with heterogenous types, see for example Moser and Engbom (2017), assumes that the labor market is segmented by type. In our framework, each firm posts a single wage rate $w_j(p)$, which will determine, endogenously, the composition of worker types it can attract. For example, firms posting a low wage rate in the East may only be able to attract workers born in the East but not West-born ones due to their preference for being in West Germany. Our setup will allow us to analyze how changing the wage impacts the share of East and West German workers hired by a firm. Each vacancy meets workers at a rate that we normalize, without loss of generality, to one.

We next turn to the problems of workers and firms.

Workers Under our assumptions, workers' decisions only depend on the flow utility received by an offer and not on the region they are currently in. Workers move from low utility to high utility jobs when the opportunity arises. However, they may move to a firm that pays a lower wage per efficiency unit if they are moving to a region where they have a stronger comparative advantage, θ_j^i: these type of moves would result in an increase in the worker's overall income but in a decline of the firm's wage rate. Workers may even accept a decrease in their own wage if they are moving to a region that provides them higher utility for a given labor income.
The expected discounted lifetime utility of an unemployed worker is the solution to

$$rU^i = b^i + \sum_{x\in\mathbb{J}} \varphi_x^i \lambda_x \left[\int \max\left\{U^i, W^i(\tilde{u})\right\} dF_x\left(\tilde{u}/(\theta_x^i \tau_x^i)\right) - U^i \right], \tag{2}$$

where we denote by $W^i(u)$ the value of an employed worker earning utility flow u. Thus, the value of an unemployed worker of type i consists of the worker's flow benefit plus the expected value from finding a job, which is only accepted if this value exceeds the value from continuing search.

The value of an employed worker receiving flow utility u solves

$$rW^i(u) = u + \sum_{x\in\mathbb{J}} \varphi_x^i \lambda_x \left[\int \max\left\{W^i(u), W^i(\tilde{u})\right\} dF_x\left(\tilde{u}/(\theta_x^i \tau_x^i)\right) - W^i(u) \right] + \delta^i \left[U^i - W^i(u)\right]. \tag{3}$$

Since $W^i(u)$ is increasing in u, there exists a reservation utility R^i for each type of worker such that $W^i(R^i) = U^i$. From equations (2) and (3), we obtain $R^i = b^i$. The reservation utility corresponds to a reservation firm wage which is region specific: in region j it is given by $\hat{w}_j = \frac{b^i}{\tau_x^i \theta_x^i}$.

Firms. Since the production function is linear, the firm-level problem of posting vacancies and choosing wages can be solved separately. Following the literature (e.g., Burdett and Mortensen (1998)), we focus on steady state: employers choose the wage rate that maximizes their steady state profits for each vacancy, which are

$$\pi_j(p) = \max_w (p - w) \sum_{i \in \mathbb{I}} \theta_j^i l_j^i(w). \tag{4}$$

Just as in the standard Burdett-Mortensen setup, the wage choice is determined by a trade-off between profit margins and firm size. On the one hand, a higher wage rate allows firms to hire and retain more workers, and hence the steady state firm size increases in w. On the other hand, by offering a higher wage, firms cut down their profit margin, $p-w$. The complementarity between firm size and productivity implies that more productive firms offer a higher wage, just as in the literature. However, unique to our framework, firms need to take into account that their wage posting decision also impacts the types of workers they attract, which introduces a non-convexity, since the labor function $l_j^i(w)$ may be discontinous, as we will show.

Once wages have been determined, firms choose the number of vacancies to post by solving

$$\varrho_j(p) = \max_v \pi_j(p) v - c_j(v),$$

where $\pi_j(p)$ are the maximized profits per vacancy from (4). The size of a firm p in region j is therefore given by $l_j(w_j(p)) v_j(p)$. Moreover, the vacancy posting policy from the firm problem gives us the endogenous arrival rate of offers from each region

$$\lambda_j = \int_{\underline{p}_j}^{\bar{p}_j} v_j(p) \gamma_j(p) \, dp, \tag{5}$$

and the wage policy gives us the endogenous distributon of offers

$$F_j(w_j(p)) = \frac{1}{\lambda_j} \int_{\underline{p}_j}^{p} v_j(p) \gamma_j(p) \, dp. \tag{6}$$

Finally, notice that allowing for the firm size to be affected by both wage and vacancy costs introduces an additional free parameter, which will allow us to match the data by decoupling the relationship between wage and size.

Market Clearing. To close the model, we need to describe how the distribution of workers to firms is determined. Let $\Upsilon^i(u) \equiv D^i(u)/\bar{D}^i$ be the share of workers that need to receive at least utility u in order to accept a new offer over their current one. Then $\Upsilon^i(u)$ is a cumulative distribution function (CDF) over workers' reservation utility. Since no worker accepts a wage providing lower utility than unemployment, the support of D^i is bounded below by b^i. The law of motion of $D^i(u)$, for $u \geq b^i$, is

$$\dot{D}^i(u) = \delta^i\left(\bar{D}^i - D^i(u)\right) - \sum_{x\in\mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{u}{\theta_x^i \tau_x^i}\right)\right) D^i(u),$$

where dots represent time derivatives, and F_j comes from the firm problem as just described. The first term captures worker flows from positions offering a higher utility than u into unemployment. The second term represents outflows of workers to positions offering a higher utility than u.

In steady state, $\dot{D}^i(u) = 0$, and we have

$$D^i(u) = \frac{\delta^i \bar{D}^i}{\delta^i + \sum_{x\in\mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{u}{\theta_x^i \tau_x^i}\right)\right)}, \tag{7}$$

if $u \geq b$, and $D^i(u) = 0$ for $u < b$. The unemployment rate of workers of type i is $D^i(b)/\bar{D}^i$.

Denote by $l_j^i(w)$ the measure of workers of type i employed at one vacancy posted in region j offering wage w. For $w\theta_j^i\tau_j^i \geq b^i$ the law of motion of $l_j^i(w)$ is given by

$$\dot{l}_j^i(w) = \varphi_j^i D^i\left(\theta_j^i \tau_j^i w\right) - q^i\left(\theta_j^i \tau_j^i w\right) l_j^i(w).$$

The first term of this expression represents worker inflows. These are given by the probability $\varphi_j^i D^i\left(\theta_j^i \tau_j^i w\right)$ that an offer contacts a worker of type i who is willing to accept, times the arrival rate of workers per vacancy, which is one. Outflows are equal to the mass of workers $l_j^i(w)$ multiplied by the rate at which workers separate either into unemployment or accept other offers,

$$q^i(u) = \delta^i + \sum_{x\in\mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{u}{\theta_x^i \tau_x^i}\right)\right). \tag{8}$$

Firms that offer a higher utility flow u are characterized by lower outflows, since workers are less likely to be poached by other firms. In steady state, using expression (7) for $D^i(u)$, we obtain a measure of workers per vacancy of

$$l_j^i(w) \quad = \quad \frac{\varphi_j^i \delta^i \bar{D}^i}{\left[q^i\left(\theta_j^i \tau_j^i w\right)\right]^2} \tag{9}$$

if $\theta_j^i \tau_j^i w \geq b^i$, and zero otherwise.

To conclude the model setup and summarize the discussion, we define the competitive equilibrium.

Definition 1: Stationary Equilibrium. *A stationary equilibrium consists of a set of wage and vacancy posting policies* $\{w_j(p), v_j(p)\}_{j \in J}$*, profits per vacancy* $\{\pi_j(p)\}_{j \in J}$*, firm profits* $\{\varrho(p)\}_{j \in J}$*, arrival rates of offers* $\{\lambda_j\}_{j \in J}$*, wage offer distributions* $\{F_j(w)\}_{j \in J}$*, firm sizes for each worker type* $\left\{l_j^i(w)\right\}_{j \in J, i \in I}$*, separation rates* $\left\{q_j^i(p)\right\}_{j \in J, i \in I}$*, and worker utility distributions* $\{D^i(u)\}_{i \in I}$ *such that*

1. *workers accept offers that provide higher utility, taking as given the wage offer distributions,* $\{F_j(w)\}_{j \in J}$*;*

2. *firms set wages to maximize per vacancy profits, and vacancies to maximize overall firm profits, taking as given the function mapping wage to firm size,* $\left\{l_j^i(w)\right\}_{j \in J, i \in I}$*;*

3. *the arrival rates of offers and wage offer distributions are consistent with vacancy posting and wage policies, according to equations (5) and (6);*

4. *firm sizes and worker distributions satisfy the stationary equations (7) and (9), where* $q_j^i(p) = q^i\left(\theta_j^i \tau_j^i w_j(p)\right)$.

4.2 Characterization of the Equilibrium

We next proceed to characterize the equilibrium. As mentioned, our model extends the class of job posting models à la Burdett and Mortensen to a setting with J regions and I types of workers that interact in one labor market subject to region-worker specific frictions, preferences, and comparative advantages. In order to understand the structure of our model, it is therefore useful to compare it with the benchmark Burdett-Mortensen model, which would be a special case of our model when all worker heterogeneity is shut down and there is only one region. In this case – as is well known – the equilibrium wage policy is as follows: the lowest productivity firm sets the minimum wage that allows it to hire workers from unemployment – i.e. $w\left(\underline{p}\right) = b$ –, and the wage policy is an increasing and continous function of productivity. We emphasize two key aspects of this solution. First, the equilibrium wage dispersion is given by the fact that firms that pay a higher wage are able to attract and retain more workers, and thus firm size is an increasing function of wage paid. Second, the wage policy must be continous. A discontinuity cannot be optimal, since a discrete jump in wage cannot lead to a discret jump in firm size, the reason being that firm size is purely determined by the ranking of wage offers and not by their level. In our setting these insights generalize, but need to be refined. Due to the presence of several types of workers and regions, discountinuities in the wage policy may arise. In fact, in our setting the size of the firm can increase discountinously if the wage increase is sufficient to attract workers of an additional type. More broadly, in our setting, within a given (i, j) pair only the ranking of wage matters for firm size, as in the benchmark model, but across regions and worker types also the level of the wage is relevant. As a result, we can show that the equilibrium is given by a set of piece-wise differential equations and boundary conditions for optimality.

Proposition 1. *The solution of the stationary equilibrium solves a set of J plus $J \times I$ differential equations*

$$\begin{aligned}
\frac{\partial w_j(p)}{\partial p} &= H_j\left(\left\{q_j^i(p)\right\}_{j \in J, i \in I}, \left\{w_j(p)\right\}_{j \in J}\right) \\
\frac{\partial q_j^i(p)}{\partial p} &= K_j^i\left(\left\{q_j^i(p)\right\}_{j \in J, i \in I}, \left\{w_j(p)\right\}_{j \in J}\right)
\end{aligned}$$

together with $J \times I$ boundary conditions for $q_j^i(\bar{p}_j)$, $J \times I$ cutoffs for firm types that have the lowest productivity to hire workers of type i in region j, defined by $\hat{p}_j^i = \min p_j$ such that $w_j(p_j) \geq R^i$, and $J \times I$ boundary conditions for $w_j\left(\hat{p}_j^i\right)$.

Proof. The proof formalizes the previous discussion on the economic forces present in our model, and provides the analytical expressions for the differential equations, which can be efficiently solved for any generic J and I. The formal proof for the general case is left to the Appendix E.1. Here, we highlight the main steps for a simple symmetric case, with two identical regions and two worker types, each biased towards one region.

Consider the parametrization $I = J = 2$, $b^1 = b^2 = b$, $\delta^1 = \delta^2 = \delta$, $\bar{D}^1 = \bar{D}^2 = \frac{1}{2}$, $\Gamma_1(p) = \Gamma_2(p) = \Gamma(p)$ with support on $p \in \left[\underline{p}, \bar{p}\right]$, where $\underline{p} \in \left[b, \frac{b}{\tau\theta}\right)$, $c_1(v) = c_2(v) = c(v)$, and

$$\theta_j^i = \begin{cases} 1 & \text{if } i = j \\ \theta < 1 & \text{if } i \neq j \end{cases} \qquad \varphi_j^i = \begin{cases} 1 & \text{if } i = j \\ \varphi < 1 & \text{if } i \neq j \end{cases} \qquad \tau_j^i = \begin{cases} 1 & \text{if } i = j \\ \tau < 1 & \text{if } i \neq j \end{cases}.$$

We focus on region 1. Due to symmetry, the problem for region 2 is identical.

We need to find the differential equations for wages and the separation rates together with their boundary conditions. We begin with wages. Taking the first order conditions from the wage posting problem (4) for a firm p in region 1 yields

$$\frac{\left(p-w_1\left(p\right)\right)\left(\frac{\partial l_1^1\left(w_1\left(p\right)\right)}{\partial w}+\theta\frac{\partial l_1^2\left(w_1\left(p\right)\right)}{\partial w}\right)}{l_1^1\left(w_1\left(p\right)\right)+\theta l_1^2\left(w_1\left(p\right)\right)} = 1, \tag{10}$$

where $l_1^1\left(w_1\left(p\right)\right)=\frac{\frac{1}{2}\delta}{\left[q^1\left(w_1\left(p\right)\right)\right]^2}$ if $w_1\left(p\right)>b$ and 0 otherwise, and $l_1^2\left(w_1\left(p\right)\right)=\frac{\frac{1}{2}\varphi\delta}{\left[q^2\left(\tau\theta w_1\left(p\right)\right)\right]^2}$ if $w_1\left(p\right)>\frac{b}{\tau\theta}$ and 0 otherwise. Since $\underline{p}<\frac{b}{\tau\theta}$ by assumption, the wage paid by the lowest productivity firm satisfies $w_1\left(\underline{p}\right)<\frac{b}{\tau\theta}$. Moreover, by the usual argument of Burdett and Mortensen model, we have that the first boundary condition is $w_1\left(\underline{p}\right)=b$. Therefore, the lowest productivity firm in region 1 only hires workers of type 1.

We next solve for the lowest productivity firm that finds it optimal to post a wage high enough to attract workers of type 2. We denote this firm by $\hat{p}_1^2$. Consider the per vacancy profits of a firm p when it hires only workers of type 1 – $\pi_1^1\left(p\right)$ – and when it hires both types of workers – $\pi_1^2\left(p\right)$. These profits are given by

$$\begin{aligned}\pi_1^1\left(p\right) &= \max_{w\geq b}\left(p-w\right)l_1^1\left(w\right)\\ \pi_1^2\left(p\right) &= \max_{w\geq\frac{b}{\tau\theta}}\left(p-w\right)\left(l_1^1\left(w\right)+l_1^2\left(w\right)\right).\end{aligned}$$

Since both π_1^1 and π_1^2 are continous, the firm $\hat{p}_1^2$ must be indifferent between hiring only workers of type 1 and hiring both types of workers, i.e.,

$$\pi_1^1\left(\hat{p}_1^2\right)=\pi_1^2\left(\hat{p}_1^2\right).$$

Figure (8a) plots the wage function $w_1(p)$ over the domain of productivity $[\underline{p},\bar{p}]$. In proximity of $\hat{p}_1^2$ the wage equation $w_1\left(p\right)$ is discontinous. Intuitively, it cannot be optimal for any firm to post a wage of just below $\frac{b}{\tau\theta}$, since in that case an infinitesimal wage increase to $\frac{b}{\tau\theta}$ would lead to a discrete jump in firm size by attracting the entire mass of unemployed workers of type 2, thus increasing profits. Firm $\hat{p}_1^2$ is indifferent between posting a lower wage and hiring only workers of type 1, or paying a premium to discretely increase the size of the firm by hiring also workers of type 2. The second boundary condition is thus given by

$$w_1\left(\hat{p}_1^2\right)=\arg\max_{w\geq\frac{b}{\tau\theta}}\left(\hat{p}_1^2-w\right)\left(l_1^1\left(w\right)+l_1^2\left(w\right)\right).$$

We next differentiate equation (10) with respect to p, and use equation (9) to get the step-wise differential equation for $w_1(p)$[17]

$$\frac{\partial w_1(p)}{\partial p} = -\left(p - w_1(p)\right)\left(\frac{\frac{\delta\left(\frac{\partial q_1^1(p)}{\partial p}\right)}{\left[q_1^1(p)\right]^3} + \mathbb{I}\left\{p \geq \hat{p}_1^2\right\}\frac{\varphi\theta\delta\left(\frac{\partial q_1^2(p)}{\partial p}\right)}{\left[q_1^2(p)\right]^3}}{\frac{\delta}{\left[q_1^1(p)\right]^2} + \mathbb{I}\left\{p \geq \hat{p}_1^2\right\}\frac{\varphi\theta\delta}{\left[q_1^2(p)\right]^2}}\right) \tag{11}$$

where $\mathbb{I}(p \geq \hat{p}_1^2)$ is an indicator function equal to one for $p \geq \hat{p}_1^2$. This completes the description of the differential equation for wages.

We next show how to derive the differential equations for the separation rates. Recall from equation (8) that in order to compute $q_1^2(p)$, and thus its derivative, we need to compute the share of offers from regions 1 and 2 that the different types of workers are willing to accept. Consider first offers from region 1. Irrespective of her type, within the same region a worker employed at wage $w_1(p)$ is willing to accept all offers that pay $w > w_1(p)$. For moves to region 2, workers of type 1 are willing to accept only offers that pay $w > \frac{w_1(p)}{\tau\theta}$, while workers of type 2 are willing to accept all offers that pay $w > \tau\theta w_1(p)$. We define the marginal productivity types $\psi_{12}^1(w_1(p))$ and $\psi_{12}^2(w_1(p))$ in region 2 which make workers exactly different between moving and not moving from firm p in region 1 as $w_2\left(\psi_{12}^1(w_1(p))\right) = \frac{w_1(p)}{\tau\theta}$ and $w_2\left(\psi_{12}^2(w_1(p))\right) = \tau\theta w_1(p)$. Notice that $\psi_{12}^1(w_1(p))$ and $\psi_{12}^2(w_1(p))$ may not exist for some p. Specifically, since we have defined $\hat{p}_1^2$ to be the lowest productivity firm that attracts workers from region 2, for each $p < \hat{p}_1^2$ we will get that $w_2\left(\underline{p}\right) > \tau\theta w_1(p)$, and thus $\psi_{12}^2(w_1(p))$ does not exist. Similarly, we define $\breve{p}_{12}^1$ such that $\frac{w_1(\breve{p}_{12}^1)}{\tau\theta} = w_2(\bar{p})$, that is, no firm in region 2 posts a wage that is sufficiently high to poach workers of type 1 that are employed in firms with productivity above $\breve{p}_{12}^1$. Then $\psi_{12}^1(w_1(p))$ does not exist for $p > \breve{p}_{12}^1$.

Differentiating equation (8) with respect to p, and using the offer distribution (6), we get the desired differential equations for the separation rates

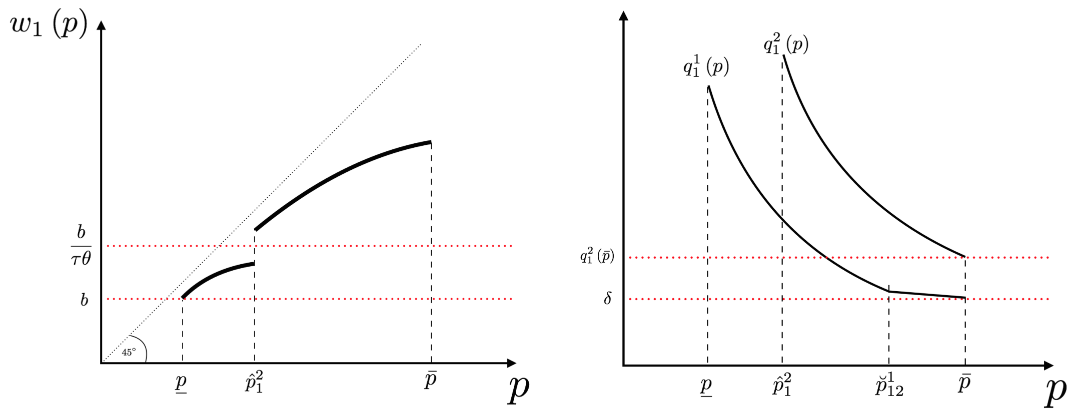
$$\begin{aligned}
\frac{\partial q_1^1(p)}{\partial p} &= -\upsilon_1\left(w_1(p)\right)\gamma_1\left(w_1(p)\right) - \mathbb{I}\left(p \leq \breve{p}_{12}^1\right)\left(\frac{\partial\psi_{12}^1(w_1(p))}{\partial p}\right)\upsilon_2\left(\psi_{12}^1(w_1(p))\right)\gamma_2\left(\psi_{12}^1(w_1(p))\right)) \\
\frac{\partial q_1^2(p)}{\partial p} &= -\upsilon_1\left(w_1(p)\right)\gamma_1\left(w_1(p)\right) - \mathbb{I}\left(p \geq \hat{p}_1^2\right)\left(\frac{\partial\psi_{12}^2(w_1(p))}{\partial p}\right)\upsilon_2\left(\psi_{12}^2(w_1(p))\right)\gamma_2\left(\psi_{12}^2(w_1(p))\right)),
\end{aligned}$$

where $\mathbb{I}(p < \breve{p}_{12}^1)$ and $\mathbb{I}(p \geq \hat{p}_1^2)$ are indicator functions.

Figure (8b) visualizes these separation rate functions. Their boundary conditions are determined as follows. The highest productivity firm in region 1 will only lose workers of type 1 to

[17]Readers familiar with the Burdett and Mortensen model should recognize that in the presence of only one worker type, equation (11) simplifies to the standard differential equation for wage.

Figure 7: Equilibrium in Region 1



(a) Wage Function: $w_1(p)$ (b) Separation Rates: $q_1^1(p)$ and $q_1^2(p)$

unemployment, and therefore the first boundary condition is $q_1^1(\underline{p}) = \delta$. The function $q_1^1(p)$ has a kink at $p = \check{p}_{12}^1$, since for $p > \check{p}_{12}^1$ workers of type 1 only quit for higher productivity firms within the same region. The boundary condition for $q_1^2(p)$ is

$$q_1^2(\bar{p}) = \delta + \lambda_2 \left(1 - F_2(\tau\theta w_1(\bar{p}))\right) = \delta + \lambda_2 \int_{\check{p}_{12}^2}^{\bar{p}} \upsilon_2(z)\, \gamma_2(z)\, dz.$$

Firms in region 2 can poach workers of type 2 even from the highest productivity firm in region 1, given these workers' preferences. □

We have completely characterized the system of differential equations and boundary conditions that pin down the equilibrium wage function and the separation rates in region 1. The solution for region 2 is identical. The proof for the general case follows a similar path, but needs to consider all the possible shapes of the wage function; that is, the fact that the wage function may have any number of discountinuities between 0 and $I - 1$. Moreover, it needs to take into consideration the fact that in some regions some workers type may not be hired even from the highest productivity firms.

4.3 Spatial and Reallocation Frictions

The main advantage of our model vis a vis the previous literature is that it allows us to distinguish explicitly between two types of frictions: general *reallocation frictions*, which, as in the class of models along the lines of Burdett and Mortensen (1998), prevent reallocation of workers to more productive firms; and *spatial frictions*, that distort the allocation between regions.

We will quantify the size of each type of friction in our quantitative exercise below. We next formally define each type of friction.

Definition 2: Reallocation Frictions. *The reallocation friction for a firm p in region j is*

$$\phi_j(p) \equiv \frac{\delta_j(p)}{\lambda_j(p)}$$

where $\lambda_j(p) = \upsilon_j(p)$ and $\delta_j(p) \equiv \frac{\sum_{i\in\mathbb{I}} \delta^i l_j^i(p)}{\sum_{i\in\mathbb{I}} l_j^i(p)}$. The average reallocation friction in region j is then

$$\phi_j \equiv \frac{\delta_j}{\lambda_j}$$

where $\lambda_j = \int \lambda_j(p)\gamma_j(p)\,dp$ and $\delta_j = \int \delta_j(p)\gamma_j(p)\,dp$.

Our definition of reallocation frictions follows closely the previous literature. As noticed by Mortensen (2005), the ratio of the job destruction (or separation) parameter, δ, to the contact (or offer) rate per worker, λ, has become known in the literature as the "market friction parameter". Definition 2 generalizes this definition to our setting, where both the offer and separation rates are endogenous and firm/region specific.[18]

For the spatial frictions, the previous literature does not provide us a lead, and hence we devise our own definition. We first describe the *spatial bias*, which captures how attracted a given worker type is to a specific region only as a consequence of the worker-specific characteristics – i.e. offer wedges, preferences, and skills. Next, we show how the spatial bias for each worker type can be summarized across regions, and finally how these type-specific measures can be aggregated to yield an overall measure of spatial frictions.

Definition 3: Spatial Biases and Frictions. *The spatial bias for workers of type i in favor of region j with respect to j' is*[19]

$$\Lambda^i(j,j') \equiv \underbrace{\log\varphi_j^i - \log\varphi_{j'}^i}_{\textit{Offer Bias}} + \underbrace{\log\tau_j^i - \log\tau_{j'}^i}_{\textit{Taste Bias}} + \underbrace{\log\theta_j^i - \log\theta_{j'}^i}_{\textit{Skill Bias}}.$$

[18] The offer rate is endogenous since it depends on the number of posted vacancies. The separation is endogenous since it depends on the relative share of workers of each type.

[19] Notice that in the knife-edge case when $\varphi_j^i = \varphi_{j'}^i = 0$ – or similarly for τ and θ – we will have that the bias is equal to $-\infty + \infty$, hence undefined. In that specific case, we pin down the "undefined" answer by assigning the relevant spatial bias to 0.

The spatial friction for workers i is then

$$\Upsilon^i \equiv Var\left[\Lambda^i\left(j, j'\right)\right]$$

The spatial friction for the overall economy is

$$\Upsilon \equiv \sum D^i \Upsilon^i.$$

To understand the spatial bias, consider two hypothetical regions j' and j, in which firms post identical vacancy and wage distributions. The value of $\Lambda^i\left(j, j'\right)$ captures the percentage difference in the expected value of offers from j' relative to j. By construction, the average spatial bias across all possible region pairs – $\frac{1}{J^2}\sum_{j\in\mathbb{J}}\sum_{j'\in\mathbb{J}}\Lambda^i\left(j, j'\right)$ – is equal to zero. Therefore, the larger are the spatial biases in absolute value, the more workers are attracted to some regions relative to others. It therefore becomes natural to define the spatial frictions as a norm of the spatial bias vector. Lacking specific guidance, we choose the variance for simplicity. Notice that if, for a given worker type, all regions are identical, then the spatial friction would be zero for this worker type. The spatial friction for the whole economy is simply the weighted average across all worker types.

If spatial frictions are present, then local labor markets are partially shielded from competition from other regions. Firms that face less competition are able to grow more without the need to offer high wages, and hence are larger on average for a given wage. We define a measure of relative labor market slack between different regions to capture this effect of spatial frictions in one summary statistic, which we then use to study the effect of these frictions.

Definition 4: Relative Labor Market Slack. *The relative labor market slack between regions j' and j for at wage level w is*

$$\kappa_{j'j}\left(w\right) \equiv \arg\min_{\kappa\in\mathbb{R}^+}\left|l_{j'}\left(w\right) - l_j\left(\kappa w\right)\right|.$$

The average relative labor market slack between regions j' and j is

$$\kappa_{j'j} = \int \kappa_{j'j}\left(w_{j'}\left(p\right)\right)\gamma_{j'}\left(p\right)dp.$$

Note that $\kappa_{j'j}$ is larger than one if the labor market in region j' is on average slacker than the one in region j, i.e., if firms in region j need to pay on average a higher wage to be as large as those in region j'. Furthermore, $\kappa_{j'j}$ solves $l_{j'}(w) - l_j(\kappa_{j'j}(w)\,w) = 0$ with equality for wages w where $l_{j'}(w)$ is within the range of firm sizes in region j'.

4.4 Three Special Cases

We next solve the equilibrium for three special cases, and show how spatial frictions affspect the equilibrium. Our aim is to build further intuition for the model mechanics and to show that the model can, at least qualitatively, match the empirical results shown in Section 3.

We consider the following cases. First, we consider distinct regional labor markets each with an associated worker type that has infinite spatial bias in favor of that region. Second, we discuss one perfectly unified labor market with no spatial frictions. Finally, we solve a semi-integrated labor market with two regions and two types of workers. The first two cases admit an analytical solution, while the third one needs to be computed numerically. To guide the discussion, in Figure 8 we plot, for each case, a computation of the solution of a two-regions, two-types economy. We interpret the two regions as East and West, and the two types as East- and West-born workers. In the figure, each column corresponds to one of three cases we study. For the rows, the first row shows wage functions, the second one employment of East born workers in each region, the third one employment of West born workers, and the fourth one the relationship between wage and size, which allows to visualize the labor market slack. In fact, the labor market slack is simply the horizontal gap between the two job ladders.

Distinct Labor Markets. Consider a distinct labor market where each worker type is willing to work in only one region.

Lemma 1. *Let $I = J$, and if $i = j$ then $\tau_j^i = \tau$, $\theta_j^i = \theta$, and $\varphi_j^i = \varphi$. Let the spatial bias satisfy $\Lambda^i(i,j) = \infty$ if $j \neq i$.*[20] *Assume that there is a Z_j such that $\Gamma_j(Z_j p) = \Gamma(p)$, $b^j = Z_j b$, and $c_j(v) = Z_j c(v)$. Finally, assume that $D^i = D$ and $\delta^i = \delta$. Then, the equilibrium is given by two functions $w(p)$ and $l(p)$ such that for all j and i*

$$w_j(p) = Z_j w(Z_j p)$$

and

$$l_j^i(Z_j p) = \begin{cases} l(p) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}.$$

Moreover, the relative labor market slack between any two regions is identical for each w and

[20] Notice that the following implies that if $i \neq j$, either $\tau_j^i = 0$ or $\theta_j^i = 0$, or $\varphi_j^i = 0$.

given by

$$\kappa_{j'j}(w) = \kappa_{j'j} = \frac{Z_j}{Z_{j'}}$$

and reallocation frictions satisfy

$$\begin{aligned}\phi_j(Z_j p) &= \phi(p)\\ \phi_j &= \phi.\end{aligned}$$

Proof. See Appendix E.2. □

Lemma 1 illustrates that, in this special case, each region are rescaled version of one another, shifted by the region specific parameter Z_j, which can be interpreted as an aggregate productivity parameter. The spatial bias between regions is infinite and therefore there is no mobility across regions. Moreover, for a given level of wage, firms in the low aggregate productivity regions face a slacker labor market, since they face effectively less competition from other firms. This first case matches some key salient feature shown in Section 3, in particular the wage gap between East and West and the shifted job ladders. However, it clearly misses the extensive reallocation of workers across regions.

Unified Labor Market without Spatial Frictions. Next, we consider a case where all regions are part of one unified labor market, with heterogenous worker types, but no spatial frictions.

Lemma 2. *Assume that for all j, $c_j(v) = c(v)$, and that there are no spatial frictions, that is*

$$\Upsilon = 0.$$

Then, conditional on productivity, wages and reallocation frictions are identical in all regions and the relative labor market slack is equal to one: for all j', j , p and w

$$\begin{aligned}w_{j'}(p) &= w_j(p)\\ \phi_{j'}(p) &= \phi_j(p).\\ \kappa_{j'j}(w) &= 1.\end{aligned}$$

Nonetheless, the average wage and aggregate reallocation frictions vary as a function of productivity distribution: the average wage is an increasing function of average firm productivity, while the reallocation friction is a decreasing one.

Proof. See Appendix E.3. □

Lemma 2 shows that a version of the model without spatial frictions would generate counterfactual implications. In fact, this version of the model predicts no regional identity, and hence workers' birth location does not affect their likelihood to work in one region or the other. Moreover, the model predicts that, conditional on wage, the establishment size in East and West is identical, which contradicts the evidence shown in Figure 6. At the same time, Lemma 2 also shows that data on spatial wage gaps alone are not sufficient to identify spatial frictions. In fact, Lemma 2 shows that even in the absence of any spatial frictions we can have lower wages in East Germany, simply through general reallocation frictions, which keep some workers at the lower productivity firms in the East.

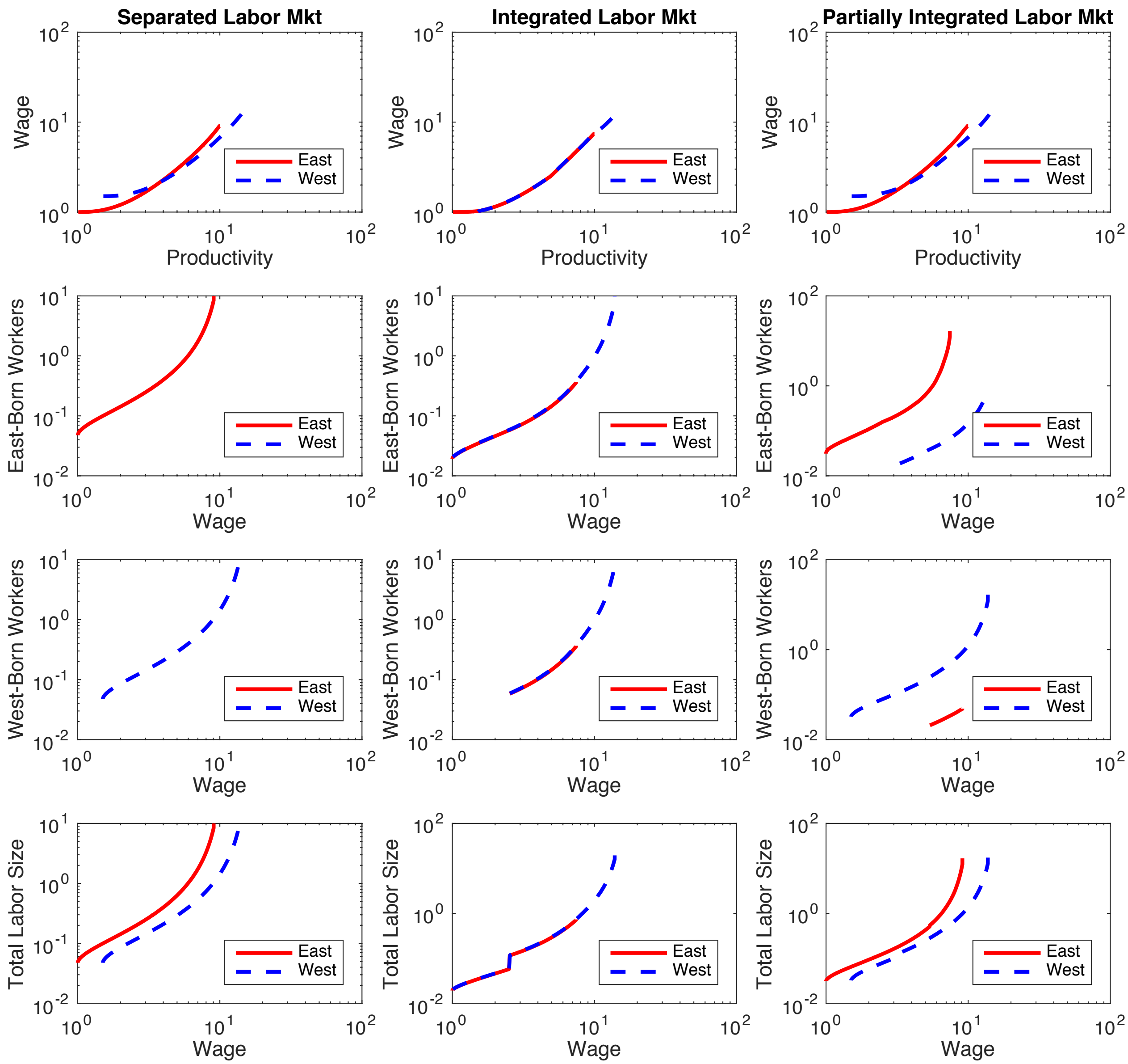
Finally, Figure 8 highlights one property of the solution that is unique to our framework. We have assumed that one type of workers (call them West workers) have a higher value of unemployment: $b^W > b^E$. As a result, the lowest productivity firms, in either region, only hire East born workers. Moreover, there exists a marginal firm that is the first one to post a wage sufficiently high to hire also West workers. At this marginal firm, the wage jumps discountinously.

Two-Regions Unified Labor Market with Spatial Frictions. Last, we consider a case with two partially integrated labor markets and two worker types, each biased towards one region. This is the case that most closely resembles Germany. Since we cannot provide an analytical solution, here we rely on computation of the model. We parametrize the model to have West Germany being more productive on average than the East.

The last column of Figure 8 shows the resulting solution. We emphasize several points. First, consider the first three rows of the figure. In both regions, the least productive firms only hire workers from their own region, since their wages are too low to attract foreign-born workers. In West Germany, these firms pay a higher wage than in the East, since unemployment benefits for West German workers are higher. In contrast to the standard framework, there is a discrete jump in the wage function at the point at which firms start hiring workers from the other region. The jump is less visible in this case than in the case of a fully integrated labor market, but it is present. In particular, the East German firm with idiosyncratic productivity level of around 2 is the marginal firm which is indifferent between hiring only East German workers at a relatively low wage, or paying a discretely higher wage and attracting also West German workers. Any firm with a larger idiosyncratic productivity hires both types of workers and pays a higher wage. Also, notice that only the very best East German firms are able to attract West German workers, while East German workers are willing to move even to relatively unproductive firms in the West. We empirically confirm this finding in the next section.

The last row presents the job ladders. Due to the spatial frictions, East firms face a relatively slacker labor market – i.e. firms paying the same wage are larger in the East – even though they do lose more workers to West firms than viceversa. To gain intuition for this result, consider the lowest productivity firm in the West. This firm is only able to hire West German born workers from unemployment. By contrast, an East German firm paying the same wage is significantly further up in the productivity distribution of East German firms. As a result, it is not only able to hire East German born workers from unemployment but also from all other firms below it with lower productivity, which increases its size. The same intuition holds for all other wage levels. Since aggregate productivity is lower in the East, a firm posting a given wage level must be at a higher position in the East German productivity distribution than in the West, which allows it to hire more workers.

Figure 8: Model Computation for three Special Cases



Notes: each column contains the solution of the equilibrium for one of the three special cases. The first row includes the wage function, for East and West firms, as a function of their productivity. The second and third rows plot respectively the number of East-born and West-born workers employed at firms either in the East or West as a function of the firms' wages. The fourth row includes the total labor size – i.e. the sum of East- and West-born workers – as a function of firms' wage.

5 Quantitative Analysis

We now quantify the size of the various frictions in the model for the case of East and

West Germany and perform counterfactuals. While all moments are jointly identified in general equilibrium, we first show that the structure and assumptions of the model provide a clear intuition for which moments identify the θ_j^i, τ_j^i, and φ_j^i. We then turn to a full structural estimation of the model. Finally, we use the estimated model to run counterfactual simulations.

5.1 Identification

Productivity Gap (θ_j^i)

We identify the relative productivities θ_j^i from the average wage gap within a given firm between two workers from different birth regions. From the model, the wage paid by firm p in region j to a worker from region i is given by $w_j^i(p) = w_j(p)\theta_j^i$. Therefore, the relative productivities are identified via $w_j^i(p) - w_j^{i'}(p) = \theta_j^i - \theta_j^{i'}$.

We estimate the wage gap empirically by first normalizing each worker's log real wage with the average log real wage of the worker's establishment in each year for 2009-2014. We then regress this difference on dummies for the worker's birth region and controls, according to[21]

$$\Delta \log w_{kp,t} = \beta_1 \gamma_{\text{East,k,t}} + \beta_2 \delta_{\text{born East,k}} + \beta_3 \gamma_{\text{East,k,t}} \delta_{\text{born East,k}} + \alpha X_{\text{k,t}} + \xi_t + \nu_{Ind(p)} + \epsilon_{k,t}, \quad (12)$$

where $\Delta \log w_{kp,t} = \log w_{k,t} - \log w_{p,t}$ is worker k's log wage relative to the wage of her establishment p in year t, $\gamma_{\text{East,k,t}}$ is a dummy that is equal to one if the worker's establishment is in the East, $\delta_{\text{born East,k}}$ is a dummy that is equal to one if worker k was born in the East, $X_{\text{k,t}}$ are characteristics of the worker which include age and age squared, a gender dummy, a dummy for whether a worker has an upper secondary school certificate or not, and dummies for the level of training. Furthermore, ξ_t are time fixed effects and $\nu_{Ind(p)}$ are fixed effects for establishment p's industry at the 3-digit level of the German National Industry Classification Scheme.[22] We cluster standard errors at the county-level. From this regression, the value for example for θ_E^E is obtained as $\beta_1 + \beta_2 + \beta_3$.

The implied productivity parameters θ_j^i, are presented in Table 3. We normalize the productivity of workers that are employed in their birth-region to one. The raw regression coefficients are presented in Table 9 in Appendix G. Table 3 shows that a West German worker on average receives a 3% higher real wage than a comparable East German worker employed in the same

[21] An analogous approach would be to include each worker's wage in absolute terms and to include firm fixed effects. The results are quantitatively very similar.

[22] These codes are based on the WS93 classification from the Federal Employment Agency, which we concord based on Eberle, Jacobebbinghaus, Ludsteck, and Witter (2011). Since the LIAB is only a subsample of the entire dataset, the average wage gaps of the observed workers are not necessarily zero within a given firm. Therefore, the industry fixed effects are not redundant.

Table 3: Idiosyncratic Productivity Differences

	(1) θ_W^W	(2) θ_W^E	(3) θ_E^W	(4) θ_E^E
Overall	1	0.97	1.03	1

firm in both regions. Thus, there appear to be meaningful differences in wages based on workers' birth location, which our model attributes to productivity differences. While this productivity difference is able to generate a (small) wage gap between regions, it cannot generate the "identity results" shown in the previous section because West workers have an absolute advantage in both regions. There is no comparative advantage for example of East German workers for the East, which would be necessary to explain why East-born workers are more attracted to the East (and vice-versa).

Preference Frictions (τ_j^i)

The preference frictions can be identified from the relative wage gains of job-to-job movers within and across regions for workers from different birth regions. Conditional on a worker's productivity, which can be identified as shown in the previous section, a worker of type i moves job-to-job from a firm in region j paying wage w to a firm in region j' offering wage w' through reallocation if and only if $\tau_{j'}^i w' > \tau_j^i w$. Define $G_j^i(w)$ to be the distribution function of workers of type i working in region j, and let

$$\eta_{j \text{ to } j'}^i \equiv \int \int_{(\tau_j^i w)/\tau_{j'}^i} dF_{j'}(w') dG_j^i(w) + \chi^i \int \int_{b^i/\tau_{j'}^i} dF_{j'}(w') dG_j^i(w)$$

be a weighted average of the probabilities that a randomly selected worker of type i in region j who receives an offer from region j' moves there. Accounting for churning, the average wage increase for a randomly drawn worker moving from region j to region j' is then

$$\begin{aligned} \Delta\overline{\log w_{j \text{ to } j'}^i} = & \frac{1}{\eta_{j \text{ to } j'}^i} \int \left(\int_{(\tau_j^i w)/\tau_{j'}^i} (\log w' - \log w)\, dF_{j'}(w') \right) dG_j^i(w) \\ & + \frac{\chi^i}{\eta_{j \text{ to } j'}^i} \int \left(\int_{(b^i/\tau_{j'}^i)} (\log w' - \log w)\, dF_{j'}(w') \right) dG_j^i(w). \end{aligned} \tag{13}$$

This expression consists of two terms: the expected wage gain from reallocation and the expected wage gain from churning. Both terms depend on the cdf of offers $F_{j'}$ and on G_j^i. If the offer distribution $F_{j'}$ puts a large amount of mass on relatively high wage offers, then the average wage gain from moving between j and j' is large since workers get very good offers from there.

A similar intuition holds if the wage distribution G^i_j in the origin region puts a lot of mass on low wages. Given $F_{j'}$ and G^i_j, which will be estimated through the model to match the variance of the wage distribution and the correlation between wages and firm size, and given χ^i, we can identify the preference friction $\tau^i_j/\tau^i_{j'}$ from the difference between the wage gain of workers moving from their native region to the other region compared to the wage gain of workers who return to their native region. A larger wage gap between these two moves implies a greater home preference.

Empirically, we construct workers' average wage gains by computing the change in each worker k's log daily real wage change $\Delta \log w_{k,t}$ from one wage record to the next. Since wage records are required to be filed annually, we observe each worker at least once every year, and more often if the worker changes jobs. For 2009-2014, we then run

$$\begin{aligned}\Delta \log w_{k,t} = &\beta_1 d_{\mathrm{k,t}} + \beta_2 d_{\mathrm{k,t}}\gamma_{\mathrm{East,k,t}} + \beta_3 d_{\mathrm{k,t}}\delta_{\mathrm{born\ East,k}} + \beta_4 d_{\mathrm{k,t}}\gamma_{\mathrm{East,k,t}}\delta_{\mathrm{born\ East,k}} \\ &+ \sum_{m\in\{\mathrm{migr,comm}\}}\sum_{x\in\{\mathrm{EW,WE}\}}\left[\alpha d_{\mathrm{k,t}}\rho_{\mathrm{m,x,k,t}} + \phi d_{\mathrm{i,t}}\rho_{\mathrm{m,x,k,t}}\delta_{\mathrm{born\ East,k}}\right] + \xi_t + \nu_k + \epsilon_{k,t},\end{aligned}$$

where $d_{\mathrm{k,t}}$ is a dummy that is equal to one if the individual changes jobs at time t, $\gamma_{\mathrm{East},k,t}$ is a dummy that is equal to one if the individual changes jobs within East Germany at time t, and $\delta_{\mathrm{born\ East,k}}$ is a dummy that is equal to one if the individual is born in East Germany. We additional control for the type of workers' mobility in the data. Define m as an index that records whether an individual migrates or commutes, where we record a worker as migrating if she transfers not only her place of work but also her place of residence from East Germany to West Germany or vice versa. Commuting only requires a change in the place of work. We index by $x = EW$ when a worker moves from East to West and by $x = WE$ if the worker moves from West to East. Then, $\rho_{\mathrm{m,x,k,t}}$ is a dummy that is equal to one if the migration decision is equal to m and the type of move is equal to x for worker k at time t. The regression includes time fixed effects ξ_t and worker fixed effects ν_k. The omitted category is the wage change for workers that remain at the same establishment. Standard errors are clustered at the county-level.

Table 4 shows the wage gains for different worker types and different types of moves. The detailed regression results are presented in Table 10 in Appendix G. Table 4 shows that a worker born in East Germany moving from East to West experiences on average a 53.5% real wage gain relative to when the same worker does not move firms (column 1). It compares to an 8.4% relative real wage gain for workers that move firm within East Germany (column 2). On the other hand, West German workers obtain only a 27% real wage increase when moving East to West, consistent with a relative preference for being in the West (column 3). The within-East wage gain is similar to the one for East German workers (column 4). We will estimate the τ^i_j from the relative wage gain of workers moving out of their native region relative to those coming

Table 4: Contribution of Taste Friction to Worker-Level Wage Gain

	(1) $\Delta w^i_{i \text{ to } j}$	(2) $\Delta w^i_{i \text{ to } i}$	(3) $\Delta w^j_{i \text{ to } j}$	(4) $\Delta w^j_{i \text{ to } i}$
i = East	53.5%	8.4%	27%	8.3%
i = West	37%	12%	11.5%	8.5%

Table 5: Size of the Contact Wedge

	(1) $\mu^i_{i \text{ to } j}$	(2) $\mu^i_{i \text{ to } i}$	(3) $\mu^j_{i \text{ to } j}$	(4) $\mu^j_{i \text{ to } i}$	DiD
i = East	1.3%	12.1%	12.4%	11.1%	~ 0.10
i = West	0.2%	13.9%	5.9%	13.0%	~ 0.03

back, $\Delta w^i_{i \text{ to } j} - \Delta w^i_{j \; to \; i}$.

Contact Wedge (φ^i_j)

We can gain intuition for the identification of the φ^i_j by noting that the rate at which workers of type i move from a job in region j to a job in region j' is

$$\mu^i_{j \text{ to } j'} = \lambda_j \varphi^i_j \times \int \left(\int_{(\tau^i_j w)/\tau^i_{j'}} dF_{j'}(w') \right) dG^i_j(w) + \chi^i \lambda_j \varphi^i_j \times \int \left(\int_{b/\tau^i_{j'}} dF_{j'}(w') \right) dG^i_j(w). \quad (14)$$

The first term represents the flows due to reallocation, while the second term represents moves due to churning. Taking the ratio of this expression for East and West born workers eliminates the λ_j. Conditional on the preference friction τ^i_j and the churning parameters χ^i, which were identified as described before, and given the $F_{j'}$ and G^i_j obtained through the structure of the model, we can then pin down the contact wedge. Specifically, the lower the relative flows of a given worker type $\mu^i_{j \text{ to } j'}/\mu^{i'}_{j \text{ to } j'}$, the smaller must be the ratio of contact rates, $\varphi^i_j/\varphi^{i'}_j$.

Table 5 shows the estimates for the average annual share of workers making a given type of transition, for the years 2009-2014 . The first column in the first row shows that 1.3% of East German born workers located in East Germany move to the West, compared to within-East transitions in 12.1% of cases (column 2). West German-born workers located in East Germany move from East to the West in 12.4% of cases (column 3) and within the East in 11.1% of cases. The second row shows the figures for moves from West to East and within the West. We will target these moments in the estimation.

5.2 Estimation [work in progress]

Let a time period be one quarter. To estimate the model, we parametrize the vacancy cost function as $c_j(v) = c_{0,j} v^{c_{1,j}}$, and let firms' productivity distributions be drawn from a Pareto distribution with region-specific mean Z_j and tail parameter ξ, which we assume is common across both regions. We normalize the within-region taste and offer wedge parameters to $\tau_i^i = \varphi_i^i = 1$.

Calibration of Exogenous Parameters

We set several of the model parameters exogenously. First, we normalize the total mass of workers $D^E + D^W = 1$, and set the mass of each type of workers based on the share of workers born in a given region in the LIAB.[23] We find $D^E = 0.32$ and $D^W = 0.68$. These figures are higher than the population share of workers in East Germany, since a sizeable fraction of East German born workers is living in the West. We set the mass of firms $M^E = 0.18$ and $M^W = 0.82$, using the number of establishments located in East and West Germany, respectively, in 2010 from the BHP. We estimate worker-type-speficic separation rates using the quarterly probability of East and West German workers of transitioning into unemployment during 2009-2014. These separation rates are $\delta^E = .0278$ and $\delta^W = .0191$, respectively. Thus the average East German worker in a job has an almost 50% higher quarterly probability of becoming unemployed than a West German one. We set the unemployment benefits to $0.9Z_j$ in both regions. Finally, we set the comparative advantages θ_j^i to the values in Table 3.

Identification of Other Parameters

In addition to τ_j^i and φ_j^i, we need to estimate values for the following nine parameters: Z_E, Z_W, $c_{0,E}$, $c_{0,W}$, $c_{1,E}$, $c_{1,W}$, ξ, χ^E, and χ^W. These parameters are identified as follows. First, we compute the 20 quantiles of the distribution of firms' average wages in each region, and compute the mean and variance across these quantiles. The mean of these average wages identifies the mean productivity parameters Z_E and Z_W. A larger value of a region's productivity parameter raises its average wage. The variance of wages helps us to pin down the distribution parameter ξ and the level of vacancy costs, $c_{0,E}$ and $c_{0,W}$. A smaller value of ξ increases the fatness of the productivity distribution's tail, increasing the variance of wages. Similarly, a higher level of vacancy costs increases the importance of a higher productivity, and raises the variance of wages as well. Second, we compute the average number of full time workers at each of the quantiles, and run a linear regression to estimate the relationship between firm size and wage

[23] Since the worker type in the model is based on the worker's birth region, we cannot use the share of workers currently living in a given region.

Table 6: Data Moments Versus Model Moments

	(1)	(2)	(3)	(4)
	Data		Model	
Birth region	East	West	East	West
Wage gain of E-W movers (%)	0.535	0.270	0.597	0.299
Wage gain of W-E movers (%)	0.115	0.370	0.137	0.418
Share of cross-movers	0.101	0.015	0.105	0.018
Average wage (EUR '000 per quarter)	5.637	6.735	5.892	7.119
Variance wage	0.179	0.228	0.148	0.215
Wage-size correlation	0.740	0.698	0.782	0.632
J (objective)	0.313			

in each region. These moments pin down $c_{1,E}$ and $c_{0,E}$, since a higher value of these parameters makes the vacancy cost function more convex, weakening the relationship between wage and size. Finally, we use the level of wage gains $\Delta w^i_{i \text{ to } j}$ from Table 4 to pin down the values of the χ^i. A larger value of the churning parameter reduces the size of the wage gain for any move.

Estimation

We estimate the 13 parameters jointly by targeting 12 data moments via an MCMC procedure based on Chernozhukov and Hong (2003), where we seek to minimize the squared percentage deviation between the model-implied moments and the moments in the data. Table 6 presents the moments in the data and compares them to the model-implied moments under our preferred calibration. Table 7 presents the estimated parameter values. The first row shows that the home region preference is roughly similar for both types of workers, and is in the order of $10-15\%$. The second row presents the importance of churning. East-born workers exhibit almost twice as many moves through churning as West-born ones, which contributes to flattening the observed job ladder of these workers. The third row shows the estimated matching frictions. With these values, East-born workers receive approximately 11% of offers from West German firms, while West-born workers receive approximately 5% of offers from the East. In the fourth row, we document that the observed average wage gap translate into an East German average productivity that is about 5% below that of the West. The last three rows present the tail parameter and the vacancy costs.

6 Conclusion

Our paper has documented that 25 years after reunification, there exists an "enduring wall" between East and West Germany. East Germany's real wage level is about 20% below that

Table 7: Estimated Parameters

	(1)	(2)
	$i=$East	$i=$West
Home preference ($\tau_j^{i\neq j}$)	0.86	0.88
Churning parameter (χ^i)	0.07	0.04
Matching friction ($\varphi_j^{i\neq j}$)	0.04	0.10
Productivity (Z_i)	4.25	4.49
Distribution tail (ξ)	1.24	
Vacancy cost level ($c_{0,i}$)	0.05	0.03
Vacancy curvature ($c_{1,i}$)	0.03	0.04

of the West, and unemployment is significantly higher. We find that this enduring wall is not the result of spatial sorting of more highly skilled East German workers to the West, but arises from a combination of more productive establishments in West Germany and a strong reluctance of workers to leave the region in which they are born. These preferences for workers' birth region affect worker mobility much more strongly than distance frictions. For example, an East German born worker that is currently in the West is as willing to move 100km within the West as she is to move 250km to get back to East Germany. To understand the relative contribution of spatial frictions versus frictions hindering the reallocation of workers to more productive firms, we develop an extension of the model by Burdett and Mortensen (1998) with multiple regions, multiple worker types, and heterogeneous firms. Our quantitative simulations show that preference frictions play an important role in generating the aggregate wage gap.

We believe that these results are of interest beyond the specific case of the German reunification. Large differences remain between regions in many countries, for example in the Italian Mezzogiorno or in Spanish Andalusia. Understanding the forces by which these regions continue to lag economically will enable policymakers to devise better policies to reduce regional gaps.

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Appendix

A Historical Overview

East and West Germany were separate countries before 1990. There was virtually no movement of workers between the two regions, and the border was tightly controlled. This separation gave rise to two distinct economic systems. While West Germany was a market economy, the economy in East Germany (then called the German Democratic Republic, GDR) was planned.

The German reunification completely removed the East German institutions of the planned economy and replaced them with West German ones. Starting on July 1, 1990, the two Germanys started a full monetary, economic, and social union, and introduced the regulations and institutions of a market economy to the GDR. These included for example the West German commercial code and federal taxation rules, as well as a reform of the labor market which imposed Western-style institutions (Leiby (1999)). At the same time, the West German Deutschmark (DM) became the legal currency of both halves of Germany. Wages and salaries were converted from Ostmark into DM at a rate of one-to-one, as were savings up to 400DM. While the currency reform implied an East German wage level of about 1/3 the West German level, in line with productivity, the switch meant that East German firms lost export markets in Eastern Europe, since customers there could not pay in Western currency. Additionally, East German customers switched to Western products, which were of much higher quality than East German ones (Smolny (2009)). West German unions negotiated sharp wage increases in many East German industries, which were not in line with productivity gains but driven by a desire to harmonize living conditions across the country (Burda and Hunt (2001), Smolny (2009)). As a consequence, East German unit labor costs rose sharply, and output and employment collapsed (Burda and Hunt (2001)). This trend was further excacerbated by the break-up and transfer of unproductive East German conglomerates to private owners, who usually downsized or closed plants.[24]

B Sorting and AKM Decomposition

We fit in the individual-level LIAB data a linear model with additive worker and establishment fixed effects, as originally in Abowd, Kramarz, and Margolis (1999) and, more recently, in Card, Heining, and Kline (2013). The model allows us to quantify the contribution of worker-specific and firm-specific

[24] This transfer was done via the *Treuhandanstalt*, a public trust, which was set up by the West German government to manage and ultimately sell the GDR's public companies. West German were initially slow to invest into East German firms. Eventually, most firms were sold at very steep discounts to the highest bidder, usually West German firms, which were often motivated by subsidies and had little interest in keeping their acquisitions alive (Leiby (1999)).

components to the real wage gap. Index full-time workers by k and time by t, and define by $\boldsymbol{Q}(k,t)$ the establishment that employs worker k at time t.[25]

We decompose the log daily real wage $\log w_{kt}$ according to

$$\log w_{kt} = \log \alpha_k + \log \psi_{\boldsymbol{Q}(k,t)} + x'_{kt}\beta + r_{kt}, \tag{15}$$

where α_k is a worker component, $\psi_{\boldsymbol{Q}(k,t)}$ is an establishment component, and x_{kt} is a set of year and age dummies, interacted with education. We specify r_{kt} as in Card, Heining, and Kline (2013) as three separate random effects: a match component $\eta_{k\boldsymbol{Q}(k,t)}$, a unit root component ζ_{kt}, and a transitory error ϵ_{kt},

$$r_{kt} = \eta_{k\boldsymbol{Q}(k,t)} + \zeta_{kt} + \epsilon_{kt}.$$

In this specification, the mean-zero match effect $\eta_{k\boldsymbol{Q}(k,t)}$ represents an idiosyncratic wage premium or discount that is specific to the match, ζ_{kt} reflects the drift in the persistent component of the individual's earnings power, which has mean zero for each individual, and ϵ_{kt} is a mean-zero noise term capturing transitory factors. Similar to Card, Heining, and Kline (2013), we estimate the model on the largest connected set of workers in our data. The largest connected set includes approximately 97% of West and East workers in the LIAB.[26]

Denote by $\omega_{kt} = \alpha_k \psi_{\boldsymbol{Q}(k,t)}$ individual k's predicted wage in year t net of the time varying age and education effects. We show that the average predicted wage in a region or county j, in year t, which we call $\bar{\omega}_{t,j}$, may be decomposed into

$$\bar{\omega}_{t,j} = \bar{\alpha}_{t,j}\bar{\psi}_{t,j}\eta_{t,j}\rho_{t,j}, \tag{16}$$

[25] Time is a continous variable, since, if a worker changes multiple firm within the same year, we would have more than one wage observation within the same year.

[26] While most workers are included in the sample, we miss approximately 10% of the establishments included in the LIAB dataset with at least one worker during 2009-2014 in East and 11% in the West. We find that we are more likely to miss establishments that pay lower wages. In fact, of the establishments in the bottom decile of the average wage distribution we miss 19% in the East and 21% in the West, while of the establishments in the top decile we miss 7% in the East and 5% in the West. We miss more establishments than workers since – due to the nature of the exercise – large establishments are more likley to be included in the connected set.

where

$$
\begin{aligned}
\bar{\theta}_t &\equiv \frac{1}{J_t}\sum_{j\in\mathbb{J}_t}\theta_t^j \\
\hat{\theta}_t &\equiv \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\theta_{t,i} \\
\eta_t &\equiv 1+\mathrm{Cov}\left(\frac{n_t^j}{\bar{n}_t},\frac{\theta_t^j}{\bar{\theta}_t}\right)=\frac{\hat{\theta}_t}{\bar{\theta}_t} \\
\rho_t &\equiv 1+\mathrm{Cov}\left(\frac{\theta_{t,i}}{\hat{\theta}_t},\frac{\psi_i}{\bar{\psi}}\right)=1+\frac{1}{N_t}\sum_i\left(\frac{\theta_{t,i}}{\hat{\theta}_t}-1\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)
\end{aligned}
$$

and the j superscript indicates the firm, and $\mathbb{J}_t$ is the set of firms.

Each object in the decomposition is easily interpretable. $\bar{\theta}_t$ captures the effect of firm characteristics. $\bar{\psi}$ captures the effect of workers characteristics, η_t captures the correlation between firm size and firm fized effect, and last, ρ_t is the pure effect of sorting, that is, it captures the correlation between firm and workers fixed effects.

We next show the details of the derivation

$$
\begin{aligned}
\bar{w}_t &\equiv \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\theta_{t,i}\psi_i \\
&= \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\theta_{t,i}-\hat{\theta}_t\right)\psi_i+\hat{\theta}_t\bar{\psi} \\
&= \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\theta_{t,i}-\hat{\theta}_t\right)\left(\psi_i-\bar{\psi}_i\right)+\hat{\theta}_t\bar{\psi} \\
&= \bar{\theta}_t\bar{\psi}\left[\frac{1}{\bar{\theta}_t}\frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\theta_{t,i}-\hat{\theta}_t\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)+\frac{\hat{\theta}_t}{\bar{\theta}_t}\right] \\
&= \bar{\theta}_t\bar{\psi}\left[\frac{\hat{\theta}_t}{\bar{\theta}_t}\frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\frac{\theta_{t,i}}{\hat{\theta}_t}-1\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)+\frac{\hat{\theta}_t}{\bar{\theta}_t}\right] \\
&= \bar{\theta}_t\bar{\psi}\frac{\hat{\theta}_t}{\bar{\theta}_t}\left[\frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\frac{\theta_{t,i}}{\hat{\theta}_t}-1\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)+1\right] \\
&= \bar{\theta}_t\bar{\psi}\frac{\hat{\theta}_t}{\bar{\theta}_t}\left[1+\mathrm{Cov}\left(\frac{\theta_{t,i}}{\hat{\theta}_t},\frac{\psi_i}{\bar{\psi}}\right)\right]
\end{aligned}
$$

finaly notice that

$$
\frac{\hat{\theta}_t}{\bar{\theta}_t}=\frac{\frac{1}{N_t}\sum_{j\in\mathbb{J}_t}n_t^j\theta_t^j}{\frac{1}{J_t}\sum_{j\in\mathbb{J}_t}\theta_{t,i}}=\frac{\frac{1}{N_t}\sum_{j\in\mathbb{J}_t}\left(n_t^j-\bar{n}_t\right)\theta_t^j+\bar{\theta}_t}{\bar{\theta}_t}=\frac{\frac{1}{J_t}\sum_{j\in\mathbb{J}_t}\left(n_t^j-\bar{n}_t\right)\left(\theta_t^j-\bar{\theta}_t\right)+\bar{\theta}_t}{\bar{\theta}_t}=1+\mathrm{Cov}\left(\frac{n_t^j}{\bar{n}_t},\frac{\theta_t^j}{\bar{\theta}_t}\right)\blacksquare
$$

where I've defined $\bar{n}_t$ as average firm size, and I used the fact that $N_t=\bar{n}_tJ_t$.

We perform this decomposition separetely for East and West Germany in each year between 2009 and 2014 and then take the expectation of the logs across these years. Thus, for example, $E[\log \bar{\omega}_W] \equiv \frac{1}{6}\sum_{t=2009}^{2014} \log \bar{\omega}_{W,t}$. We obtain:

$$\begin{aligned}
\underbrace{E[\log \bar{\omega}_W] - E[\log \bar{\omega}_E]}_{20.2\%} \quad = \quad & \underbrace{E[\log \bar{\alpha}_W] - E[\log \bar{\alpha}_E]}_{4.5\%} + \underbrace{E\left[\log \bar{\psi}_W\right] - E\left[\log \bar{\psi}_E\right]}_{19.7\%} + \\
& \underbrace{E[\log \bar{\eta}_W] - E[\log \bar{\eta}_E]}_{\text{-3.0\%}} + \underbrace{E[\log \bar{\rho}_W] - E[\log \bar{\rho}_E]}_{-1.0\%}.
\end{aligned}$$

The decomposition shows that the majority of the average wage gap is explained by differences in establishment fixed effects. Workers' fixed effects are also larger in West Germany, but this effect contributes only a small fraction to the overall average wage gap. The covariance terms are instead larger in East Germany. However, the size of the effect is relatively small. Moreoever, as discussed in Footnote 26, we should be careful in interpreting the covariance terms due to the fact that the LIAB sample is not a representative sample of establishments. Additionally, the procedure by Abowd, Kramarz, and Margolis (1999) restricts the sample to the set of connected firms, generating a bias towards larger firms.

C Computation of Joint Wage-Size Distribution

D Growth Accounting

West German GDP per capita in real terms is still about 40% larger than in the East (Figure 9). We perform a standard accounting exercise to decompose this GDP gap into its different components. We follow the literature and assume an aggregate Cobb-Douglas production function, with elasticities to labor and capital equal to $1-\alpha$ and α, respectively. We set, as usual, α equal to $\frac{2}{3}$. Aggregate GDP in East and West in a year t is therefore given by

$$\begin{aligned} Y_{E,t} &= A_{E,t} K_{E,t}^{\alpha} N_{E,t}^{1-\alpha} \\ Y_{W,t} &= A_{W,t} K_{W,t}^{\alpha} N_{W,t}^{1-\alpha}, \end{aligned}$$

where we observe in the data provided by the statistics offices of the states, for each year and separetely for East and West Germany, employment N, capital K and GDP Y. We can so use the formula above to compute the implied total factor productivity term, A. We rewrite the previous equation in per capita terms, that is

$$\begin{aligned} y_{E,t} &= A_{E,t} k_{E,t}^{\alpha} n_{E,t}^{1-\alpha} \\ y_{W,t} &= A_{W,t} k_{W,t}^{\alpha} n_{W,t}^{1-\alpha}, \end{aligned}$$

where $y \equiv \frac{Y}{L}$, $k \equiv \frac{K}{L}$, $n \equiv \frac{N}{L}$ and L is total population, also observed in the data. Last, we decompose the percentage difference in GDP per capita between West and East into its three components, that is

$$\underbrace{\log y_{W,t} - \log y_{E,t}}_{\text{GDP p.c. gap}} = \underbrace{\log A_{W,t} - \log A_{E,t}}_{\text{TFP gap}} + \underbrace{\alpha \left(\log k_{W,t} - \log k_{E,t}\right)}_{\text{Capital p.c. gap}} + \underbrace{(1-\alpha)\left(\log n_{W,t} - \log n_{E,t}\right)}_{\text{Workers p.c. gap}}.$$

In Figure 10 we plot each component of this decomposition. The initial convergence in GDP per capita is both due to a convergence in capital per capita and in TFP. However, virtually all of the current gap between East and West Germany is due to a lower level of TFP in East. This result aligns with the larger establishment component of West German establishments in the AKM decomposition.

Figure 9: Real GDP per capita

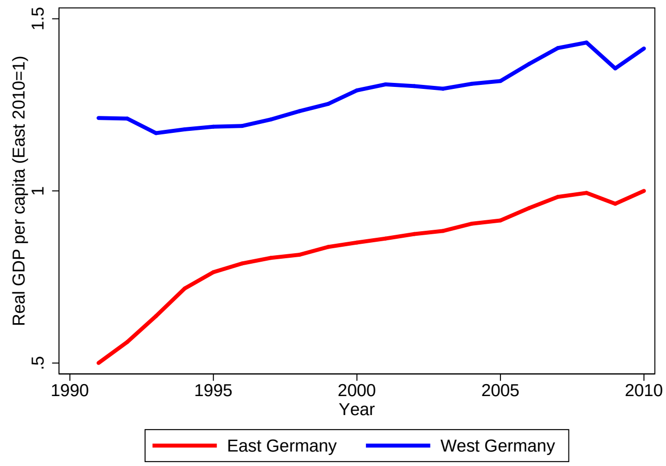
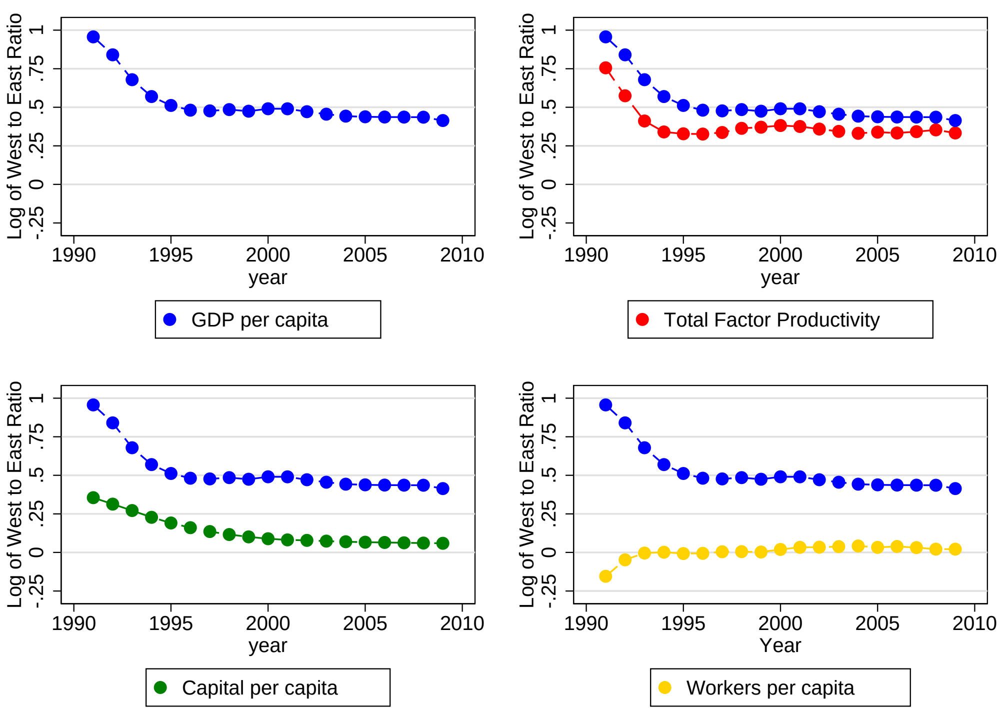


Figure 10: Decomposition of the difference in GDP per capita



E Proofs

E.1 Proof of Proposition 1 1

The proof is constructive, and solves the optimization problem to show that it leads to a system of differential equations, for which we provide analytical expressions.

Consider first the wage posting problem (4). Using equation (9), the first-order condition of this problem is

$$\frac{(pZ_j - w_j(p))\left(\sum_{i\in\mathbb{I}}\theta_j^i\frac{\partial l_j^i(w_j(p))}{\partial w}\right)}{\left(\sum_{i\in\mathbb{I}}\theta_j^i l_j^i(w_j(p))\right)} = 1, \tag{17}$$

where for any type i we have $l_j^i(w_j(p)) > 0$ if $\theta_j^i\tau_j^i w_j(p) \geq b^i$ and $l_j^i(w_j(p)) = 0$ otherwise. Define the ordered set for each region j

$$\mathbb{N}(j) \equiv \left\{\frac{b^{\iota(j,1)}}{\theta_j^{\iota(j,1)}\tau_j^{\iota(j,1)}}, ..., \frac{b^{\iota(j,I)}}{\theta_j^{\iota(j,I)}\tau_j^{\iota(j,1)}}\right\},$$

where the set $\mathbb{N}(j)$ ranks worker types according to the minimum wage rate they require to work in region j, starting with the worker type that requires the lowest wage. Specifically, $\iota(j,x)$ is an operator such that $\iota(j,1) = \arg\min_{i\in\mathbb{I}}\frac{b^i}{\theta_j^i\tau_j^i}$, $\iota(j,2) = \arg\min_{i\in\mathbb{I}\setminus\iota(j,1)}\frac{b^i}{\theta_j^i\tau_j^i}$, and so on, up to $\iota(j,I) = \arg\max_{i\in\mathbb{I}}\frac{b^i}{\theta_j^i\tau_j^i}$.

Next, differentiating equation (17) with respect to p and using equation (9), we obtain a set of differential equations for wages $\left\{\frac{\partial w_j^{\iota(j,1)}(p)}{\partial p}, ..., \frac{\partial w_j^{\iota(j,I)}(p)}{\partial p}\right\}$ of the form

$$\frac{\partial w_j^{\iota(j,n)}(p)}{\partial p} = -\frac{\left(pZ_j - w_j^{\iota(j,n)}(p)\right)\sum_{i=1}^n\theta_j^{\iota(j,i)}\left(\frac{2\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^i\left(\frac{\partial q_j^{\iota(j,i)}(p)}{\partial p}\right)}{\left[q_j^{\iota(j,i)}(p)\right]^3}\right)}{\left(\sum_{i=1}^n\theta_j^{\iota(j,i)}\frac{\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^{\iota(j,i)}}{\left[q_j^{\iota(j,i)}(p)\right]^2}\right)} \tag{18}$$

which define firms' optimal wage posting within each non-overlapping interval $w_j^{\iota(j,n)}(p) \in [\frac{b^{\iota(j,n)}}{\theta_j^{\iota(j,n)}\tau_j^{\iota(j,n)}}, \frac{b^{\iota(j,n+1)}}{\theta_j^{\iota(j,n+1)}\tau_j^{\iota(j,n+1)}})$ for $n < I$, and $w_j^{\iota(j,I)} \in [\frac{b^{\iota(j,I)}}{\theta_j^{\iota(j,I)}\tau_j^{\iota(j,I)}}, \infty)$. The overall differential equation for wage, $\frac{\partial w_j(p)}{\partial p} = H_j\left(\left\{q_j^i(p)\right\}_{j\in J, i\in I}, \left\{w_j(p)\right\}_{j\in J}\right)$, is going to be given by the union of the piece-wise ones shown in (18). To find this function, we need to know the relevant domains of each region's productivity distribution that map into each wage support.

Towards this aim, we now show how to determine the cutoffs $\hat{p}_j^i$ as defined in Proposition 1, which provide the lowest productivity firm in region j hiring workers of type i, and consequently the boundary conditions for the differential equation. Define $\pi_j^{\iota(j,n)}(p_j)$ to be the profit function for a firm p_j that posts a wage high enough to attract workers of type $\iota(j,n)$ and behaves optimally, hence

$$\pi_j^{\iota(j,n)}(p_j) = \max_{w \geq \frac{b^{\iota(j,n)}}{\theta_j^{\iota(j,n)} \tau_j^{\iota(j,n)}}} (p_j Z_j - w) \sum_{i=1}^{n} \theta_j^{\iota(j,i)} l_j^{\iota(j,i)}(w).$$

Next, define $\underline{\tilde{p}}_j = \min p_j$ s.t. $p_j \geq \frac{b^{\iota(j,1)}}{\theta_j^{\iota(j,1)} \tau_j^{\iota(j,1)}}$. The firm with productivity $\underline{\tilde{p}}_j$ is the lowest productivity firm active in region j, since firms with $p_j < \underline{\tilde{p}}_j$ would make losses at the wage necessary to attract even the lowest reservation wage workers. Similarly, define for each $n \geq 2$

$$\tilde{p}_j^{\iota(j,n)} = \min p_j \quad s.t. \quad \pi_j^{\iota(j,n)}(p_j) \geq \max_{x<n} \pi_j^{\iota(j,x)}(p_j), \tag{19}$$

where $\tilde{p}_j^n$ is thus the lowest productivity firm that has a weakly higher profit by hiring workers of types $\iota(j,1), ..., \iota(j,n)$ rather than any subset of workers of type lower, in the reservation wage sense, than n. In general, it may be the case that $\tilde{p}_j^{\iota(j,n+1)} < \tilde{p}_j^{\iota(j,n)}$, for example if posting a higher wage and attracting type $n+1$ as well allows a firm to significantly raise its profits relative to the case in which only workers up to type n are hired. Therefore, we define

$$\hat{p}_j^{\iota(j,n)} = \min_{x \geq n} \tilde{p}_j^{\iota(j,x)}. \tag{20}$$

Equation (20) defines the productivity of the marginal firm that hires workers of type i, $\hat{p}_j^i$ for $i \in \mathbb{I}$.

Since it is possible to have $\hat{p}_j^i = \hat{p}_j^{i'}$ for some or even all pairs (i, i') of worker types, we define $\mathbb{D}$ as the set of worker types that have distinct cutoffs. By continuity of the profit function, the $n \in \mathbb{D}$ types are the ones that satisfy

$$\hat{p}_j^{\iota(j,n)} = \tilde{p}_j^{\iota(j,n)}.$$

Due to the usual complementarity argument between productivity and size, it must be that $\hat{p}_j^{\iota(j,n+1)} \geq \hat{p}_j^{\iota(j,n)}$ for all n, that is, higher reservation wage types are hired on average by higher productivity firms within the same region j. Call d_x the x^{th} element of $\mathbb{D}$.[27] We need to find

[27] For example, assume that $I = 4$ and that $\hat{p}_j^{\iota(j,1)} < \hat{p}_j^{\iota(j,2)} = \hat{p}_j^{\iota(j,3)} < \hat{p}_j^{\iota(j,4)}$, therefore the set $\mathbb{D} = \{1, 3, 4\}$, adn $d_1 = 1$, $d_2 = 3$, and $d_3 = 4$.

$|\mathbb{D}|$ restrictions to solve for cutoffs values $\hat{p}_j^{\iota(j,d_x)}$ for $d_x \in \mathbb{D}$. Given these $|\mathbb{D}|$ cutoffs, we can find the cutoffs for the remaining types $n \notin \mathbb{D}$ using equation (20).

For the first cutoff, we have

$$\hat{p}_j^{\iota(j,1)} = \underline{\tilde{p}}_j.$$

Then, since the profit function $\pi_j^{\iota(j,n)}(p_j)$ is continous,[28] it must be that for all $d_x \in \mathbb{D}$ with $x \geq 2$, we have

$$\pi_j^{\iota(j,d_{x-1})}\left(\hat{p}_j^{\iota(j,d_{x-1})}\right) = \pi_j^{\iota(j,d_x)}\left(\hat{p}_j^{\iota(j,d_x)}\right),$$

which gives us the remaining $|\mathbb{D}| - 1$ restrictions. Thus, we have found the restrictions for the I cutoffs.

The boundaries of the differential equation are then given by, for each $n \in \mathbb{D}$

$$w_j\left(\hat{p}_j^{\iota(j,n)}\right) = \arg \max_{w \geq \frac{b^{\iota(j,n)}}{\theta_j^{\iota(j,n)}\tau_j^{\iota(j,n)}}} \left(p_j Z_j - w\right) \sum_{i=1}^{n} \theta_j^{\iota(j,i)} l_j^{\iota(j,i)}(w).$$

Therefore, we have shown that wage function satisfies the following step-wise differential equation

$$\frac{\partial w_j(p)}{\partial p} = -\frac{\left(pZ_j - w_j^{\iota(j,n)}(p)\right)\sum_{i=1}^{n} \theta_j^{\iota(j,i)} \left(\frac{2\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^i\left(\frac{\partial q_j^{\iota(j,i)}(p)}{\partial p}\right)}{\left[q_j^{\iota(j,i)}(p)\right]^3}\right)}{\left(\sum_{i=1}^{n} \theta_j^{\iota(j,i)} \frac{\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^{\iota(j,i)}}{\left[q_j^{\iota(j,i)}(p)\right]^2}\right)} \text{for } p \in \left[\hat{p}_j^{\iota(j,n)}, \hat{p}_j^{\iota(j,n+1)}\right),$$

where the cutoffs and the boundaries are as previously described, and we define $\hat{p}_j^{\iota(j,I+1)} = \infty$.

We next turn to the derivation of the derivative of the separation rate with respect to p, $\frac{\partial q_j^i(p)}{\partial p}$, which appears in the differential equation for the wage. From (8), this derivative depends on the distribution of wage offers F from each region x. Since the wage is increasing in firms' productivity, the probability that a worker of type i in region j receiving utility flow $u = \theta_j^i \tau_j^i w_j(p)$ rejects a random offer from region x is given by

$$F_x\left(\frac{\theta_j^i \tau_j^i w_j(p)}{\theta_x^i \tau_x^i}\right) = \frac{\int^{\psi_{jx}^i(w_j(p))} v_x(z)\gamma_x(z)\, dz}{\lambda_x}, \tag{21}$$

[28]This profit function is continous due to the fact that we are defining it keeping constant the set of hired types.

using equation (6). Equation (21) contains a productivity cut-off $\psi_{jx}^{i}(w)$, which is defined via

$$\begin{cases} \theta_x^i \tau_x^i w_x \left(\psi_{jx}^i(w_j(p))\right) = \theta_j^i \tau_j^i w_j\left(p\right) & \text{if } \psi_{jx}^i(w_j(p)) \in \left[\underline{p}_x, \bar{p}_x\right] \\ \psi_{jx}^i(w_j(p)) = \bar{p}_x & \text{if } \theta_x^i \tau_x^i w_x\left(\bar{p}_x\right) < \theta_j^i \tau_j^i w_j\left(p\right) \\ \psi_{jx}^i(w_j(p)) = \underline{p}_x & \text{if } \theta_x^i \tau_x^i w_x\left(\underline{p}_x\right) > \theta_j^i \tau_j^i w_j\left(p\right). \end{cases} \tag{22}$$

Intuitively, $\psi_{jx}^{i}(w_j(p))$ is the firm with the highest productivity in region x whose offer makes a worker of type i in region j earning $w_j(p)$ just indifferent between accepting and rejecting, provided that such a firm exists. For wage offers made by firms in the same region j as the worker, $\psi_{jx}^{i}(w_j(p)) = p$. If no firm exists in region x to provide the worker with a sufficiently high utility, the probability that the worker accepts is zero (second line). The acceptance probability is one if any offer would induce the worker to move (third line). For unemployed workers, the expressions are identical with $w_j(p) = \frac{b^i}{\theta_j^i \tau_j^i}$.

Next, rewrite equation (8) as a function of p, by substituting the utility $u = \theta_j^i \tau_j^i w_j\left(p\right)$

$$q^i\left(\theta_j^i \tau_j^i w_j\left(p\right)\right) = \delta^i + \sum_{x \in \mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{\theta_j^i \tau_j^i w_j\left(p\right)}{\theta_x^i \tau_x^i}\right)\right),$$

differentiate with respect to p, use (21), and the definition of $q_j^i\left(p\right)$, to yield a differential equation for the separation rate $q_j^i\left(p\right)$

$$\frac{\partial q_j^i\left(p\right)}{\partial p} = -\sum_{x \in \mathbb{J}} \varphi_x^i \left(\frac{\partial \psi_{jx}^i(w_j(p))}{\partial p}\right) v_x\left(\psi_{jx}^i(w_j(p))\right) \gamma_x\left(\psi_{jx}^i(w_j(p))\right). \tag{23}$$

Equation (23) defines the $J \times I$ differential equations for the separation rates introduced in the proposition: $\frac{\partial q_j^i(p)}{\partial p} = K_j^i\left(\left\{q_j^i\left(p\right)\right\}_{j \in J, i \in I}, \left\{w_j\left(p\right)\right\}_{j \in J}\right)$. The equation shows that the change in the separation rate is negatively related to the change in the marginal firm, which in turn depends on the slope of the wage functions. When the wage schedule in region x is relatively flat compared to the wage schedule in region j, a small change in p can significantly reduce the separation rate of workers from j to x.

We then need to find the $J \times I$ boundary conditions for the separation rate. For each type of worker i, these are given by the rate at which the worker leaves the most productive firm in each region j,

$$q_j^i\left(\bar{p}_j\right) = \delta^i + \sum_{x \in \mathbb{J}} \varphi_x^i \int_{\psi_{jx}^i(w_j(p))} v_x\left(z\right) \gamma_x\left(z\right) dz. \tag{24}$$

In the region in which the most productive firm provides the highest utility flow for this type of worker, indexed by $\bar{p}^i_j = \text{argmax}\left\{\theta^i_{j'}\tau^i_{j'}w_{j'}(\bar{p}_{j'})\right\}^{\mathbb{J}}_{j'=1}$, the worker only quits exogenously and therefore $q^i_j(\bar{p}^i_j) = \delta^i$.

Summing up, we have shown that the solution of the equilibrium satisfies a set of differential equations, with a rich set of boundary conditions. Therefore we have proved Proposition 1.

E.2 Proof of Lemma 11

The proof proceeds in two steps. First, we follow the standard steps in Burdett and Mortensen (1998) to show that the equilibrium in a closed economy can be described by two differential equations. We then show that if we have an equilibrium in the West, then the conjectured equilibrium relationships constitute an equilibrium in the East.

Step 1: Equilibrium in the closed economy

Consider the problem described in the main text for an economy in isolation, i.e., $\varphi = 0$. We normalize the mass of workers in the economy to $N = 1$. The separation rate from equation (**??**) is

$$q(w) = \delta + \theta + \lambda\left[1 - F(w)\right]$$

and the hiring rate $h(w)$ from equation (**??**) is

$$h(w) = u + (1-u)G(w).$$

We have the flow equations

$$\dot{u} = (\delta + \theta)(1-u) - \lambda u$$

and

$$\dot{D}(w) = \lambda F(w)\, u - (\delta + \theta) D(w) - \lambda\left(1 - F(w)\right) D(w).$$

In steady state, these equations lead to

$$\frac{u}{1-u} = \frac{\delta + \theta}{\lambda}$$

and

$$G(w) = \frac{\delta F(w)}{\delta + \lambda\left(1 - F(w)\right)}. \tag{25}$$

As in the main text, equation (**??**), firms maximize profits by choosing vacancies and wages:

$$\max_{(w,v)\geq(b,0)} \left[(p-w)\frac{h(w)}{q(w)}v - c(v)\right].$$

Define profits per vacancy as $\pi(p,w) = (p-w)h(w)/q(w)$. The optimal vacancy posting satisfies

$$\pi^*(p) = c'(v) \quad \Rightarrow \quad v = \xi(\pi^*(p)),$$

where ξ is the inverse marginal cost function and $\pi^*(p)$ are the profits under the profit-maximizing wage policy.

To obtain the wage function, we use the expressions for $h(w)$ and $q(w)$ and the steady state equations for unemployment and $G(w)$ to obtain

$$\pi(p,w) = \frac{(\delta+\theta)(p-w)}{[\delta+\theta+\lambda[1-F(w)]]^2}.$$

The first-order condition of this expression yields

$$\frac{2\lambda F'(w)(p-w)}{\delta+\theta+\lambda\left[1-F(w)\right]} = 1. \tag{26}$$

Using the expression for $F(w)$ from equation (**??**) and differentiating with respect to p gives

$$F'(w(p))w'(p) = \frac{v(p)\gamma(p)}{\int_b^{\bar{p}} v(z)\gamma(z)dz} = \frac{v(p)\gamma(p)}{\bar{\lambda}},$$

where $w(p)$ is the wage posting as a function of productivity and the second equality follows from (**??**). Combining this expression with equation (26) yields a differential equation for the wage function

$$w'(p) = \frac{2v(p)\gamma(p)(p-w(p))}{q(p)}. \tag{27}$$

We can re-write this expression by noting that the overall separation rate as a function of productivity is

$$q(p) = \delta+\theta+\lambda\left[1-F(w(p))\right] = \delta+\theta+\int_p^{\bar{p}} v(z)\gamma(z)dz,$$

which has the derivative

$$q'(p) = -v(p)\gamma(p).$$

Applying this expression to equation (27) gives

$$w'(p) = \frac{-2q'(p)(p-w(p))}{q(p)},$$

which has the boundary condition

$$w(\underline{p}) = b.$$

On the other hand, from optimal vacancy posting we can re-express the derivative of the separation function as

$$q'(p) = -\xi\left(\frac{(\delta+\theta)(p-w(p))}{[q(p)]^2}\right)\gamma(p), \tag{28}$$

which has boundary condition

$$q(\bar{p}) = \delta+\theta.$$

The two differential equations together with the boundary conditions provide the solution to the problem without cross-regional mobility.

Step 2: Equilibrium in the East

Assume that $\Gamma_E(\kappa p) = \Gamma_W(p)$ with $\kappa < 1$, and $b_E = \kappa b_W$, $\delta_E = \delta_W$, and $\bar{c}_E = \kappa \bar{c}_W$ as given in the proposition. We show that if under these assumptions

$$\left(\lambda_W, w_W(p), q_W(p), G_W(w(p)), F_W(w(p))\right)$$

are an equilibrium in the West, then

$$\left(\lambda_E, w_E(p), q_E(p), G_E(w(p)), F_E(w(p))\right)$$

with

$$\begin{aligned}
\lambda_E &= \lambda_W \\
w_E(\kappa p) &= \kappa w_W(p) \\
q_E(\kappa p) &= q_W(p) \\
G_E(w(\kappa p)) &= G_W(w(p)) \\
F_E(w(\kappa p)) &= F_W(w(p))
\end{aligned}$$

are an equilibrium in the East.

We begin by verifying that our conjectures $q_E(\kappa p) = q_W(p)$ and $w_E(\kappa p) = \kappa w_W(p)$ are correct. Rewriting equation (28) and using the expression for optimal vacancies (**??**), we have for West Germany that

$$q_W(p) = \left(-\frac{\delta(p - w_W(p))}{\bar{c}_W\left[q'_W(p)\right]^{\chi}}\right)^{\frac{1}{2}}\gamma_W(p)^{\chi/2}.$$

If our guess that $q_E(\kappa p) = q_W(p)$ is correct, then $q'_E(\kappa p) = \frac{1}{\kappa}q'_W(p)$. From the assumptions for Γ_E and Γ_w we have that $\gamma_E(\kappa p) = \frac{1}{\kappa}\gamma_W(p)$. Replacing these expressions in the equation for East Germany yields

$$q_E(\kappa p) = \left(-\frac{\delta(\kappa p - w_E(\kappa p))}{\bar{c}_E\left[\frac{1}{\kappa}q'_W(p)\right]^{\chi}}\right)^{\frac{1}{2}}\left(\frac{1}{\kappa}\right)^{\chi/2}\gamma_W(p)^{\chi/2}$$

Given our conjecture for the wage function,

$$q_E(\kappa p) = \left(-\frac{\kappa\delta\,(p - w_W(p))}{\bar{c}_E \left[q'_W(p)\right]^{\chi}} \right)^{\frac{1}{2}} \gamma_W(p)^{\chi/2}.$$

Using the fact that $\bar{c}_E = \kappa\bar{c}_W$ yields the desired result, and so indeed $q_E(\kappa p) = q_W(p)$.

We also need to verify the guess for the wage. Our conjecture implies that $w'_E(\kappa p) = w'_W(p)$. Using the differential equation for the wage and the relationship between the separation function in the East and in the West, we obtain

$$w'_E(\kappa p) = \frac{-2q'_E(\kappa p)(\kappa p - w_E(\kappa p))}{q_E(\kappa p)} = \frac{-2\frac{1}{\kappa}q'_W(p)\,(\kappa p - \kappa w_W(p))}{q_W(p)} = w'_W(p).$$

Thus this guess is also verified.

Note that the boundary conditions hold. For the wage function,

$$\kappa w_W(\underline{p}) = \kappa b_W \quad \Leftrightarrow \quad w_E(\kappa\underline{p}) = b_E.$$

For the separation function,

$$q_W(\bar{p}) = q_E(\kappa\bar{p}) = \delta + \theta.$$

We next verify that $\lambda_W = \lambda_E$. From optimal vacancy posting,

$$v_E(\kappa p) = \left[\frac{\delta(\kappa p - w_E(\kappa p))}{\bar{c}_E\left[q_E(\kappa p)\right]^2}\right]^{1/\chi} = \left[\frac{\delta(p - w_W(p))}{\bar{c}_W\left[q_W(p)\right]^2}\right]^{1/\chi} = v_W(p). \tag{29}$$

Given the definition of λ,

$$\lambda_E = \int_{b_E}^{\bar{p}_E} v_E(z)\,\gamma_E(z)\,dz,$$

we obtain

$$\lambda_E = \int_{b_E}^{\bar{p}_E} v_E(x)\,\gamma_E(x)\,dx = \int_{b_E/\kappa}^{\bar{p}_E/\kappa} v_E(\kappa y)\,\gamma_E(\kappa y)\,\kappa dy = \int_{b_W}^{\bar{p}_W} v_W(y)\,\gamma_W(y)\,dy = \lambda_W,$$

as claimed, where the second equality holds by a change in variable.

We finally need to focus on the accounting equations, and show that G and F hold as well. Note that $u_W = u_E$ holds trivially, since $\lambda_W = \lambda_E$, $\theta_E = \theta_W$, and $\delta_W = \delta_E$.To verify that the relationship for F holds, we use that

$$\begin{aligned} F_E(\kappa p) &= \frac{\int_{b_E}^{\kappa p} \upsilon_E(x)\gamma_E(x)dx}{\int_{b_E}^{\bar{p}_E} \upsilon_E(x)\gamma_E(x)dx} \\ &= \frac{\int_{b_E/\kappa}^{p} \upsilon_E(\kappa y)\gamma_E(\kappa y)\kappa dy}{\int_{b_E/\kappa}^{\bar{p}_E/\kappa} \upsilon_E(\kappa y)\gamma_E(\kappa y)\kappa dy} \\ &= \frac{\int_{b_W}^{p} \upsilon_W(y)\gamma_W(y)dy}{\int_{b_W}^{\bar{p}_W} \upsilon_W(y)\gamma_W(y)dy} \\ &= F_W(p). \end{aligned}$$

The condition for G can then be verified using the flow equation (25). This concludes the proof. Since we assumed that the W functions are an equilibrium for the West, than the properly defined E functions are an equilibrium for the East.

E.3 Proof of Lemma 2 2

If $b_E = b_W$, then from equation (**??**) we have that $U_E = U_W$. Therefore, unemployed workers are indifferent between unemployment in either region. It follows from equation (**??**) that the probability of accepting a job offer when unemployed is equal to $\hat{P}_i(w) = 1$ in both regions. Firms post wages $w \geq b_i$, and the East German firms with $p < b_i$ exit the market and do not post wage offers. From equation (**??**), the steady-state unemployment ration in a region i is then

$$\frac{u_i}{n_i} = \frac{\delta_i + \theta_i}{\bar{\lambda}_i + \bar{\lambda}_j}. \tag{30}$$

Since $\delta_E = \delta_W$ and $\theta_E = \theta_W$ by assumption, the unemployment to employment ratio is the same in both regions.

To see that allocational quality is not necessarily the same in both regions, note that since workers are indifferent between being in either region, they will move to any firm that offers them a higher job. Hence, for a firm posting wage w, the hiring probability $h_i(w)$ and the quit probability $q_i(w)$ will be the same in both regions. From equation (**??**), a firm with the same productivity p will therefore post the same wage w in both regions. However, if the cost of vacancy posting $\bar{c}_i$ differs across regions, then firms of the same productivity level will have a different size in both regions. Furthermore, the lower tail of the East German firms does not post wages since $p < b_E$. Dependent on the shape of the upper tail of the firm productivity distribution, the ratio of the weighted average wage to the unweighted average may either be higher or lower in the East than in the West. Hence, in general, $\rho_E \neq \rho_W$.

F Additional Figures

Figure 11: Price level in 2009

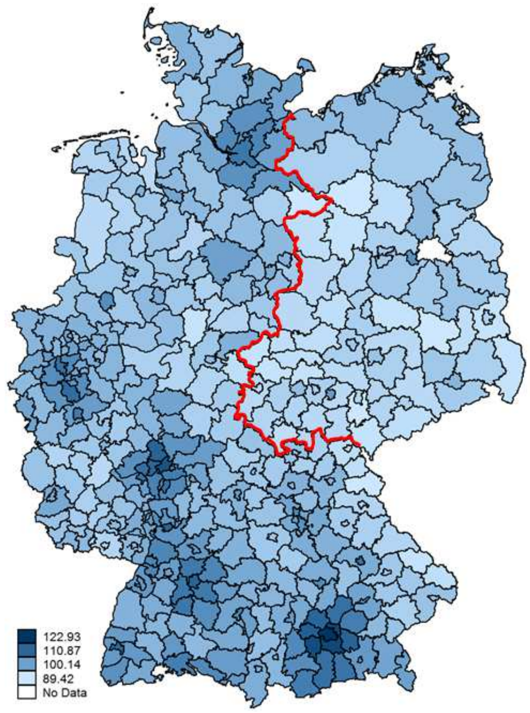
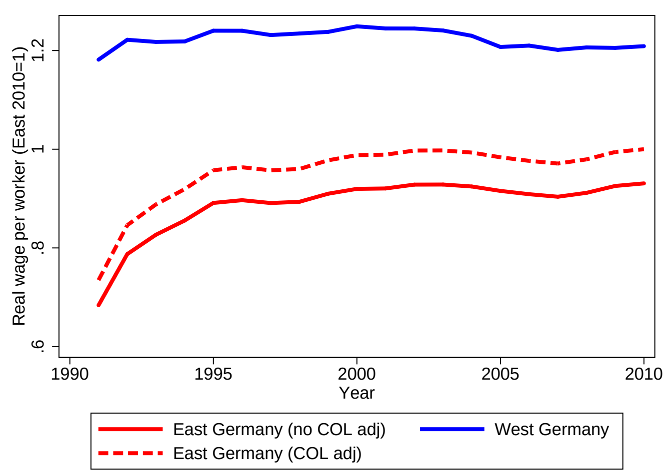


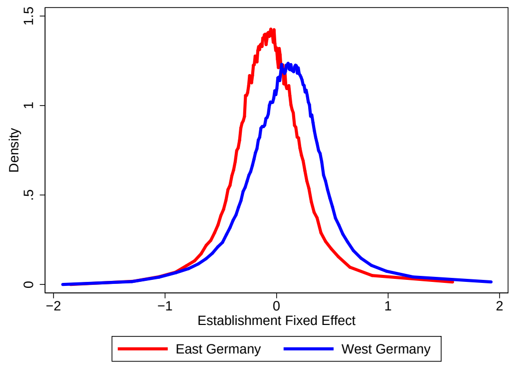
Figure 12: Average real wage with and without cost of living (COL) adjustment



Notes: Wages for East and West Germany (excluding Berlin) are obtained from the statistics offices of the German states and deflated by region-specific price deflators from the same source. We normalize East Germany in 2010 to one. The dashed line is constructed by adjusting the wage level in 2009 based on the population-weighted average price level in East Germany from the BBSR.

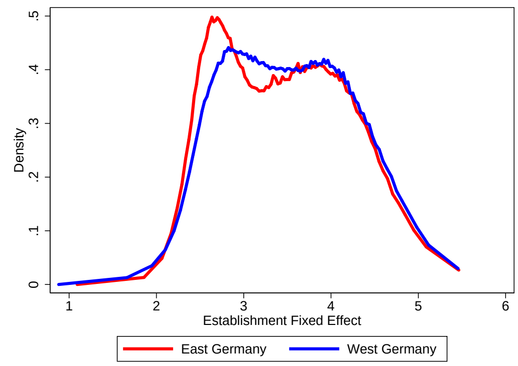
Figure 13: Distribution of Establishme and Workers Component, 2009-2014

(a) Establishment Fixed Effects



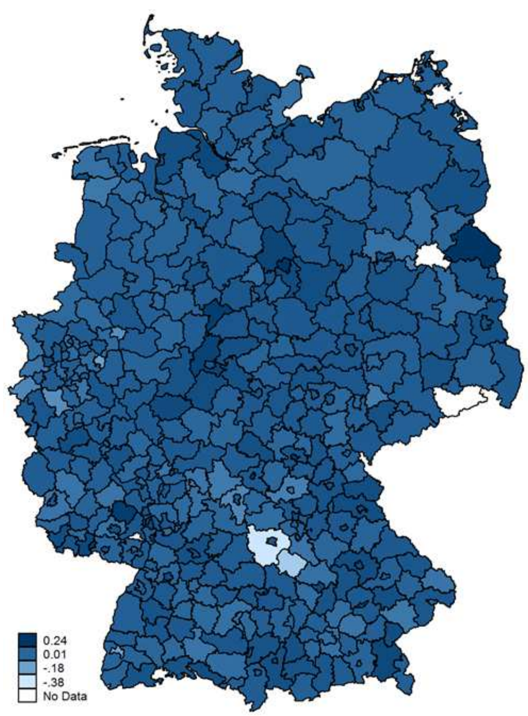
Source: LIAB, Authors' Calculations

(b) Worker Fixed Effects



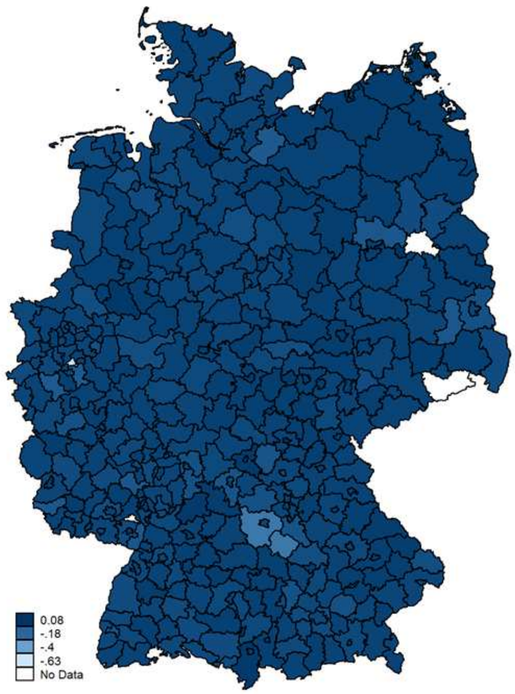
Source: LIAB, Authors' Calculations

Figure 14: Covariance between Establishment Size and Fixed Effect (η)



Source: LIAB, Authors' Calculations

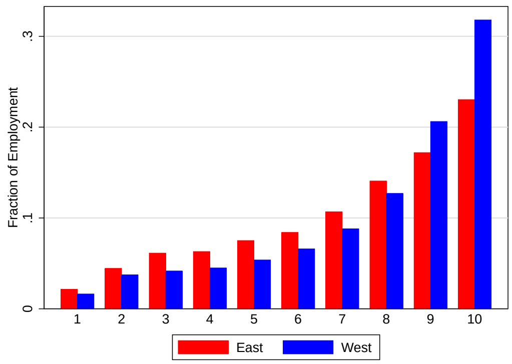
Figure 15: Covariance between Worker and Establishment Component (ρ)



Source: LIAB, Authors' Calculations

Figure 16: Employment by wage decile, 2010

(a) Males



(b) Females

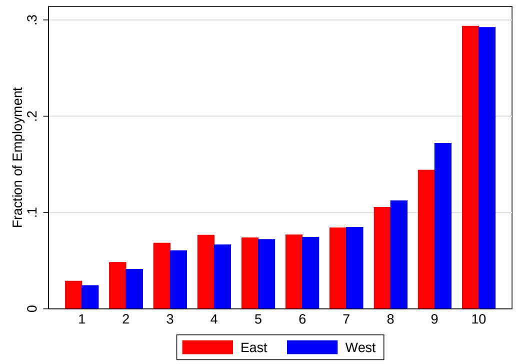
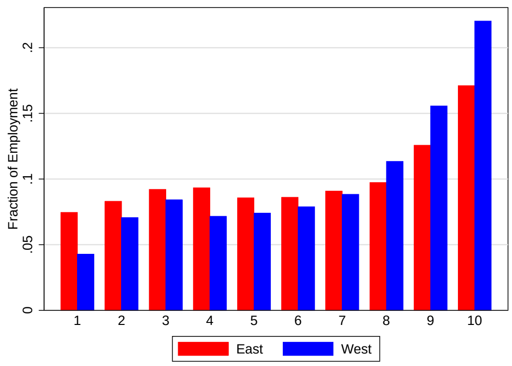
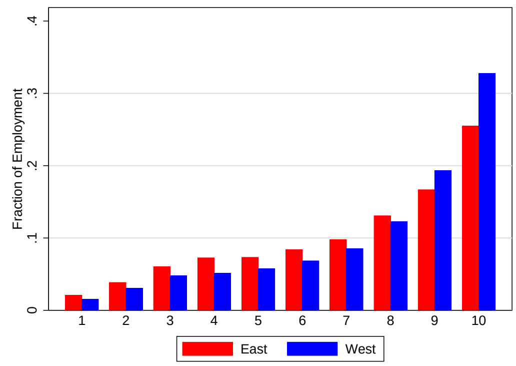


Figure 17: Employment by wage decile, 2010

(a) Low-skilled (no vocational training)



(b) Medium-skilled (high-school plus vocational training)



(c) High-skilled (university degree)

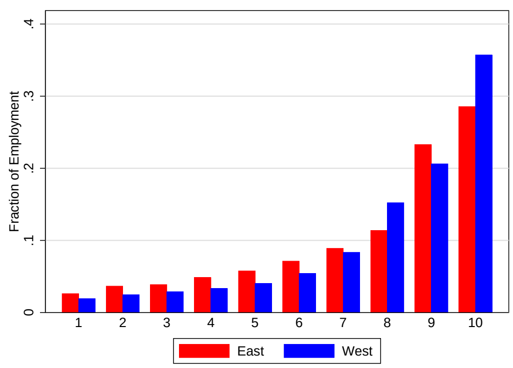


Figure 18: Flows Between Counties

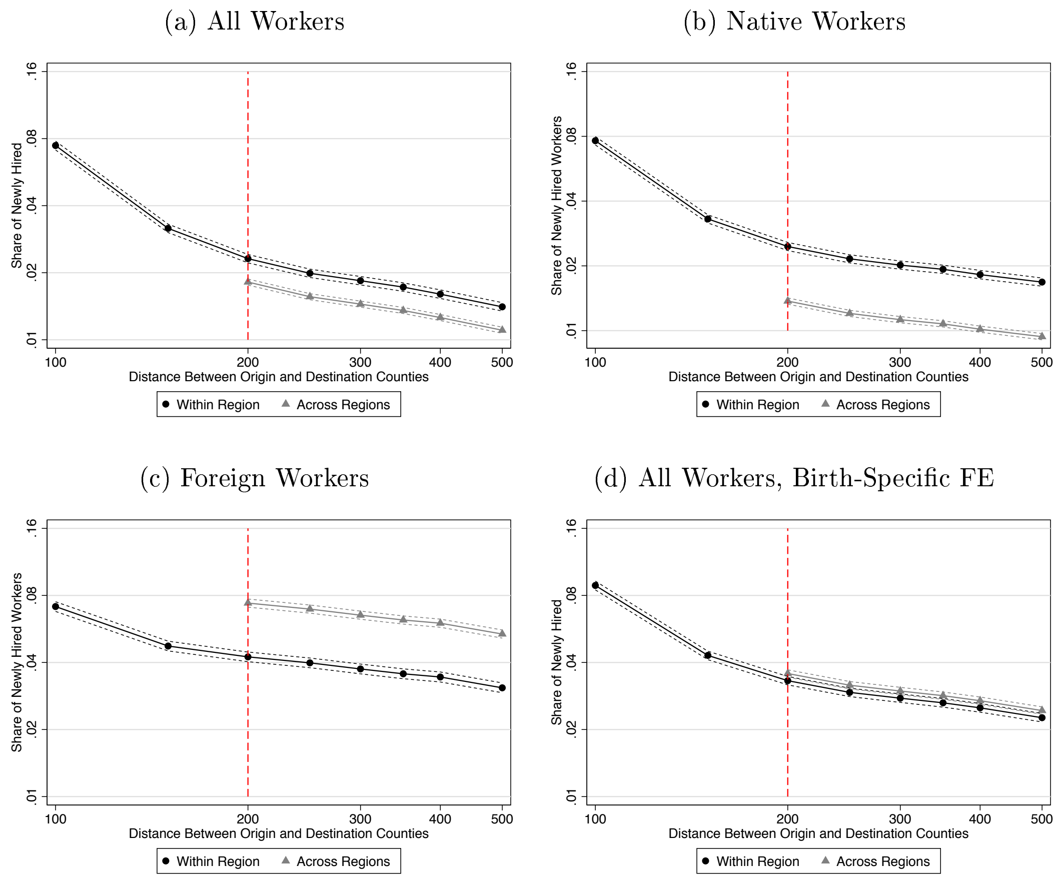


Figure 19: County Fixed Effects by Workers Birthplace

Notes: in both figures, a dot represents one county. We plot the point estimates for East-born workers as a function of the point estimates for West-born workers.

G Additional Tables

Table 8: Gravity Equation of Worker Flows

Distance in Km	(1) All Workers	(2) Natives	(3) Foreigners	(4) Birth-Specific FE
Point Estimates for Effect of Distance Within Region				
50-100	**−2.240*** **	**−2.22*** **	**−1.355*** **	**−1.783*** **
	(0.023)	(0.023)	(0.026)	(0.018)
101-150	**−3.096*** **	**−3.057*** **	**−1.763*** **	**−2.506*** **
	(0.022)	(0.022)	(0.026)	(0.018)
151-200	**−3.410*** **	**−3.352*** **	**−1.875*** **	**−2.767*** **
	(0.022)	(0.022)	(0.025)	(0.018)
201-250	**−3.561*** **	**−3.484*** **	**−1.937*** **	**−2.888*** **
	(0.022)	(0.022)	(0.026)	(0.018)
251-300	**−3.638*** **	**−3.551*** **	**−2.00*** **	**−2.948*** **
	(0.022)	(0.023)	(0.026)	(0.019)
301-350	**−3.704*** **	**−3.596*** **	**−2.05*** **	**−2.996*** **
	(0.022)	(0.023)	(0.026)	(0.019)
351-400	**−3.777*** **	**−3.653*** **	**−2.083*** **	**−3.048*** **
	(0.023)	(0.023)	(0.026)	(0.019)
>400	**−3.908*** **	**−3.733*** **	**−2.195*** **	**−3.150*** **
	(0.022)	(0.022)	(0.026)	(0.019)
Point Estimates for Additional Effect of Distance Across Regions				
1-100	**−0.257*** **	**−0.405*** **	**0.105*** **	−0.008
	(0.012)	(0.010)	(0.016)	(0.010)
101-150	**−0.250*** **	**−0.425*** **	**0.348*** **	**0.041*** **
	(0.016)	(0.018)	(0.022)	(0.014)
151-200	**−0.242*** **	**−0.584*** **	**0.556*** **	**0.074*** **
	(0.016)	(0.019)	(0.022)	(0.015)
>200	**−0.152*** **	**−0.471*** **	**0.502*** **	**0.140*** **
	(0.015)	(0.019)	(0.022)	(0.015)

Notes: robust standard errors are in parentheses. $***$ indicates significance at the 1% level. For foreigners, we report the standard error of the interaction terms – i.e. of ϕ^f and ψ^f.

Table 9: Productivity Regression

	All
$\gamma_{\text{East,k,t}}$	**.0284***
	(0.0117)
$\delta_{\text{born East,k}}$	**$-$.0283***
	(0.0053)
$\gamma_{\text{East,k,t}}\delta_{\text{born East,k}}$	.0006
	(0.0056)
Obs	5,172,074

$*** = p < 0.01$. Standard errors clustered at the county-level.

Table 10: Wage Gain Regression

	Coefficient	S.E.
$d_{\text{i,t}}$	**.1199***	(0.0057)
$d_{\text{i,t}}\gamma_{\text{East,i,t}}$	**$-$.0373***	(0.0105)
$d_{\text{i,t}}\delta_{\text{born East,i}}$	**$-$.0358***	(0.0060)
$d_{\text{i,t}}\gamma_{\text{East,i,t}}\delta_{\text{born East,i}}$	**.0364***	(0.0104)
$d_{\text{i,t}}\rho_{\text{migr,EW,i,t}}$	**.1539***	(0.0547)
$d_{\text{i,t}}\rho_{\text{migr,EW,i,t}}\delta_{\text{born East,i}}$	**.2968***	(0.0577)
$d_{\text{i,t}}\rho_{\text{migr,WE,i,t}}$	**.2485***	(0.0491)
$d_{\text{i,t}}\rho_{\text{migr,WE,i,t}}\delta_{\text{born East,i}}$	**$-$.2125***	(0.0661)
$d_{\text{i,t}}\rho_{\text{comm,EW,i,t}}$	$-$**.0856**	(0.0368)
$d_{\text{i,t}}\rho_{\text{comm,EW,i,t}}\delta_{\text{born East,i}}$	**.1241***	(0.0325)
$d_{\text{i,t}}\rho_{\text{comm,WE,i,t}}$	$-$.0094	(0.0169)
$d_{\text{i,t}}\rho_{\text{comm,WE,i,t}}\delta_{\text{born East,i}}$	$-$.0313	(0.0209)
Obs	6,182,842	

$*** = p < 0.01$. Standard errors clustered at the county-level.