

Law enforcement and the size of the informal sector*

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Abstract

I assemble new cross-country evidence showing that contrary to the standard view, the relationship between the size of the informal sector and tax rates is, at best, ambiguous. Law enforcement and informality also show no clear relation. Motivated by these findings I augment a standard two-sector -formal and informal- DSGE model with endogenous law enforcement that depends on the size of the informal sector (measured by its assets) and government expenditure. I use a micro-dataset from Colombia to show that both taxes and law enforcement are necessary to match the data. In the absence of law enforcement, tax evasion incentives imply a counterfactually large informal sector. However, law enforcement motivates households to reduce the informal sector and lower the probability of detection. Capital thus flows into the formal sector, which in turn, increases tax collection and law enforcement. The dependence of the latter on government expenditure creates a non-linearity between tax rates and the size of the informal sector through a Laffer curve. The model implies that lowering tax rates would not necessarily reduce the size of the informal sector since there is a tradeoff with the effectiveness of law enforcement.

Keywords: Informal sector, Law enforcement, Tax policy, Small open economy.

JEL Codes: E26, E62, F41, O17

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1 Introduction

The size of the informal sector in emerging markets is large, it accounts for more than one third of output (Vuletin, 2008), and more than thirty per cent of employment (Todaro and Smith, 2015). This goes in line with the fact that developing countries have a larger share of labor working in the informal sector and are less productive than developed countries (Pratap and Quintin, 2006a). This sector, among other issues, emphasizes the small-scale, self-financed, and unskilled labor-intensive economic activities (Pratap and Quintin, 2006b).

Considering these facts, some governments have explicitly pointed out the large share of the informal sector in the economy to be a problem for the long-run economic growth, as well as for the sustainability of public finances. To address this issue, some countries have implemented policies that encourage firms to hire workers by decreasing the rate of taxes that these firms have to pay per worker. One example is the case of Colombia, where such tax reform was implemented in the year 2012, and seems to have had an impact on reducing the share of workers in the informal sector by 2.3 percentage points (Osorio-Copete, 2016). However, those policies are based on the assumption that the optimal policy is such that taxes on formal wages are the main determinant of the size of the informal sector relative to the formal sector, without considering the effects of other possible distortions such as: Taxes on capital to formal firms, taxes on formal income to households, and the probability that the government fines informal firms by confiscating a fraction of their output (i.e. Law enforcement). The latter is specially important considering that, theoretically, other policies could prove to be more relevant in reducing the size of the informal sector than only lowering tax rates, as shown by Quintin (2008), indicating that policies taken by governments in emerging markets are still far from being as deep and effective as they could be¹. As a result, it is important to understand the determinants of the size of the informal sector

¹The mechanism in Quintin (2008) is through the enforcement of contracts, such that agents who choose to operate in the informal sector can evade taxes, but they have no access to official means of contract enforcement. Event though this mechanism is not explored in this paper, it is useful to illustrate how other policies different than tax policies applied to the formal sector might be more effective in reducing its size.

relative to the formal sector in an environment richer than one with merely distorting taxes.

Additionally, mainly due to different measurement challenges, previous literature has focused on explaining the size of the informal sector in terms of labor or output but, with a few exceptions, not in terms capital stock. I focus on studying the relative size of the informal sector in terms of capital stock, as well as output. An important contribution in this direction is the work done by [Leal Ordonez \(2014\)](#), who shows for the case of Mexico that the fact that the formal sector is usually more productive implies that the allocation of capital across formal and informal firms might be an important source of missallocation, and therefore lower output². This author includes endogenous Law enforcement depending on informal capital stock, such that informal firms have incentives to remain small. I complement that framework by making Law enforcement endogenous not only to informal capital stock, but also to government expenditure, which creates a trade-off between tax policies, Law enforcement and informality.

I use cross-country data to show that both the relations between tax rates and informality, and Law enforcement and informality are ambiguous. These results are at odds with the standard view that tax rates and Law enforcement are, respectively, positively and negatively correlated with the size of the informal sector³. In other words, there does not seem to be a clear role for tax policies and Law enforcement in determining the size of the informal sector⁴.

This paper attempts to untangle those relations by showing how Law enforcement, defined as the probability of an informal firm to be audited and fined, contributes to explain the relative size of the informal sector in terms of capital stock and output. For this purpose I

²[Leal Ordonez \(2014\)](#) focuses on the role of imperfect enforcement in generating missallocation of capital across sectors, which leads to output losses. This work is in the spirit of the frameworks developed by [Restuccia and Rogerson \(2013\)](#), [Bento and Restuccia \(2017\)](#), among others. For a thorough literature review on missallocation see [Hopenhayn \(2014\)](#).

³See [Pratap and Quintin \(2006a\)](#) for further detail on the standard view.

⁴Additionally, I build cross-country evidence that shows that countries with bigger size of the informal sector have lower GDP per capita (as seen in previous literature, e.g., [Maloney 2003](#), [Schneider 2004](#), [Schneider and Buehn 2017](#)), and that countries with better quality of institutions have a smaller size of the informal sector (as predicted by [Quintin 2008](#)).

build a standard two-sector (formal and informal) small open economy model where the formal sector has higher total factor productivity than the informal but it is regulated through taxes (as in [Fernández and Meza 2015](#)), and operating informally implies a probability of being audited and fined that is increasing in capital stock and government expenditure (i.e., Law enforcement). Using a micro-dataset from Colombia that allows me to gauge formal and informal capital stock, I show that the model can account better for the relative size of the informal sector than a framework without Law enforcement. The mechanism is that endogenous Law enforcement creates a wedge, increasing in informal capital stock and government expenditure, between the marginal productivity of informal capital and the rate of return; which further lowers the relative size of the informal sector compared to a framework without this mechanism.

I perform policy experiments suggesting that reducing tax rates is not always optimal to reduce the size of the informal sector due to the fact that the effectiveness of Law enforcement is related to tax collection. In terms of tax policies, considering taxes on firm's capital returns, labor, and income, marginal reductions in the income tax rates are effective up to a given level due to the aforementioned non-linearity, which resembles a standard Laffer curve. Reducing the tax rate on capital paid by firms always decreases the size of the informal sector, and changes in the tax rate on wages paid by firms have negligible effects. Exogenous improvements in the institutional efficiency of Law enforcement reduce the size of the informal sector but must be accompanied with reductions in other tax rates to avoid hindering aggregate output, otherwise excessive monitoring would disincentive the supply of capital and labor in the informal sector, which would not be offset by the supply of those two inputs to the formal sector.

This paper contributes to at least three strands of the literature: First, to the empirical literature dedicated to measure the size of the informal sector and its relation with the economy (e.g. [Schneider and Enste 2002](#), [Maloney 2003](#)). I provide a new measure of Law enforcement based on cross-country microdata, and show how this is different from the

quality of institutions and has no clear relation with the size of the informal sector, as well as taxes; which creates a puzzle. Second, to the theoretical strand of literature that models the informal sector, its determinants and implications for the macroeconomy ([Loayza 1999](#), [Pratap and Quintin 2006a](#), [Amaral and Quintin 2006](#), [Quintin 2008](#), [Hamann et al. 2011](#)), by developing a framework that augments an otherwise standard two-sector model with endogenous Law enforcement that matches better the relative size of the informal sector in terms of capital stock and output. Within this literature, I expand the previous frameworks that studied the role of Law enforcement in determining the size of the informal sector in terms of labor ([Ihrig and Moe 2004](#), [Posada and Mejía 2012](#), [Restrepo-Echavarria 2014](#), [Bardey and Mejía 2019](#)), by showing how Law enforcement endogenous to capital stock and government expenditure affects the allocation of capital stock across sectors. Third, to the small open economy (SMOE) literature by developing a business cycle model for this type of economies which includes the informal sector and allows for the study of business cycle features in the presence of informality ([Restrepo-Echavarria 2014](#), [Finkelstein Shapiro 2014](#), [Finkelstein Shapiro 2015](#), [Fernández and Meza 2015](#))⁵.

The rest of the paper is organized as follows: Section 2 shows cross-country empirical regularities regarding the size of the informal sector, output, taxes and Law enforcement. Section 3 shows evidence for the relative size of the informal sector in terms of capital stock (as well as output and labor) using microdata from Colombia. Section 4 develops the model. Section 5 shows the quantitative analysis with the model calibrated to Colombia. Section 6 performs policy experiments, and within each of them discusses their implications. Section 7 provides a discussion of the results in relation to normative and positive implications, as well as future venues of research. Section 8 concludes.

⁵The mainstream small open economy business cycle theory developed by [Mendoza \(1991\)](#) and then pursued by [Schmitt-Grohé and Uribe \(2003\)](#), [Aguiar and Gopinath \(2007\)](#) and others, is the base for the model here. However, I do not focus on business cycle analysis, even though the model performs well in replicating standard business cycle facts in emerging markets (consumption more volatile than output and countercyclical trade balance).

2 Stylized Facts

In this section I use cross-country data to summarize four relevant stylized facts regarding the relation between informal sector, output, tax rates and Law enforcement. With them, I point out the fact that the relation between informality and tax rates and informality and Law enforcement is ambiguous. Section A.1 in the appendix describes the sources of the cross-country data used in this section.

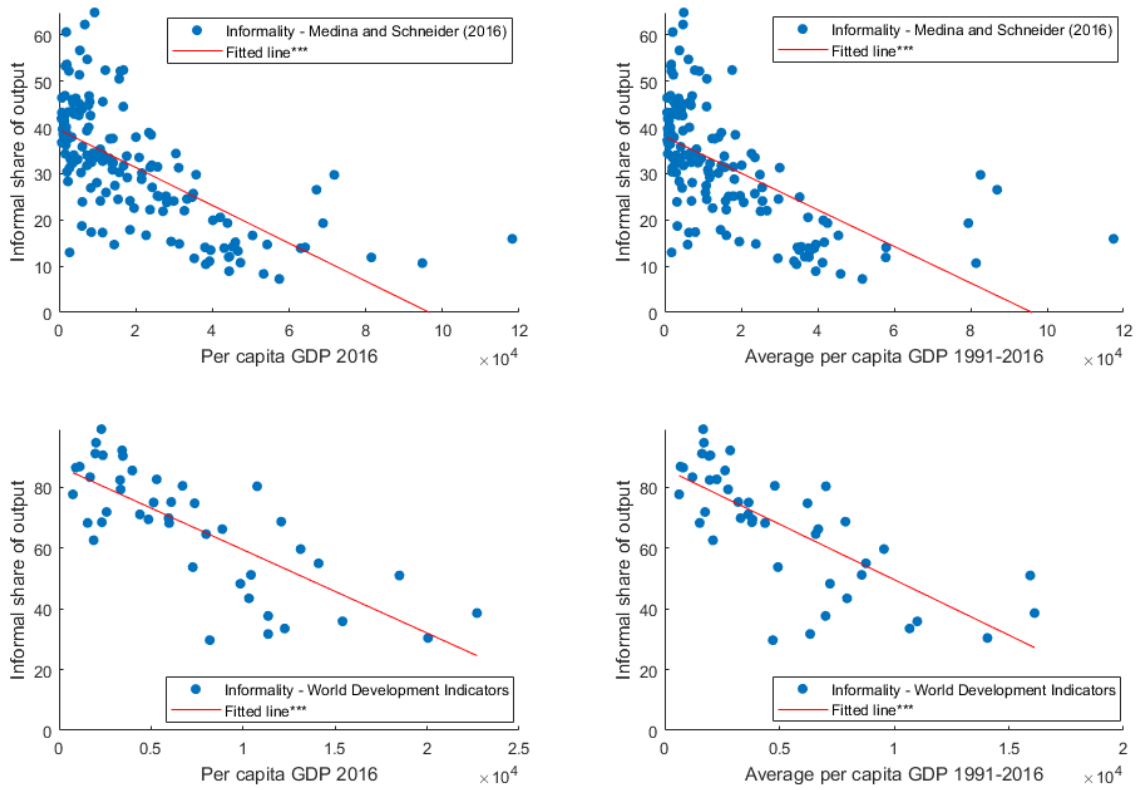
2.1 Fact 1: Countries with higher informality exhibit lower levels of per capita GDP

In line with previous literature, I find for that per capita GDP and the size of the informal sector are inversely correlated. Clearly there is nothing new about this fact, but it is relevant in light of any modeling strategy that aims to rationalize the relative size of the informal sector; such that this negative correlation should be an outcome of any model. The upper panel in [Figure 1](#) shows on the horizontal axis the per capita GDP, and on the vertical axis it shows the share of GDP produced by the informal sector. The figure on the left in the upper panel uses the per capita GDP of the year 2016, and the figure on the right uses the average per capita GDP between 1991 and 2016. The lower panel in [Figure 1](#) shows the same relationship but now using as a measure of the size of the informal sector the share of workers that are informal. In both cases The correlation is negative and the slope is significant at the 5% level.

2.2 Fact 2: The relationship between informality and tax rates is, at best, ambiguous

Contrary to the standard view that lower tax rates imply lower levels of informality (see [Pratap and Quintin 2006a](#)), I find evidence (with no causal claim) pointing out that such relationship is not clear. [Figure 2](#) shows that this relationship is, at best, ambiguous: the

Figure 1: Per capita GDP (PPP) vs. size of the informal sector



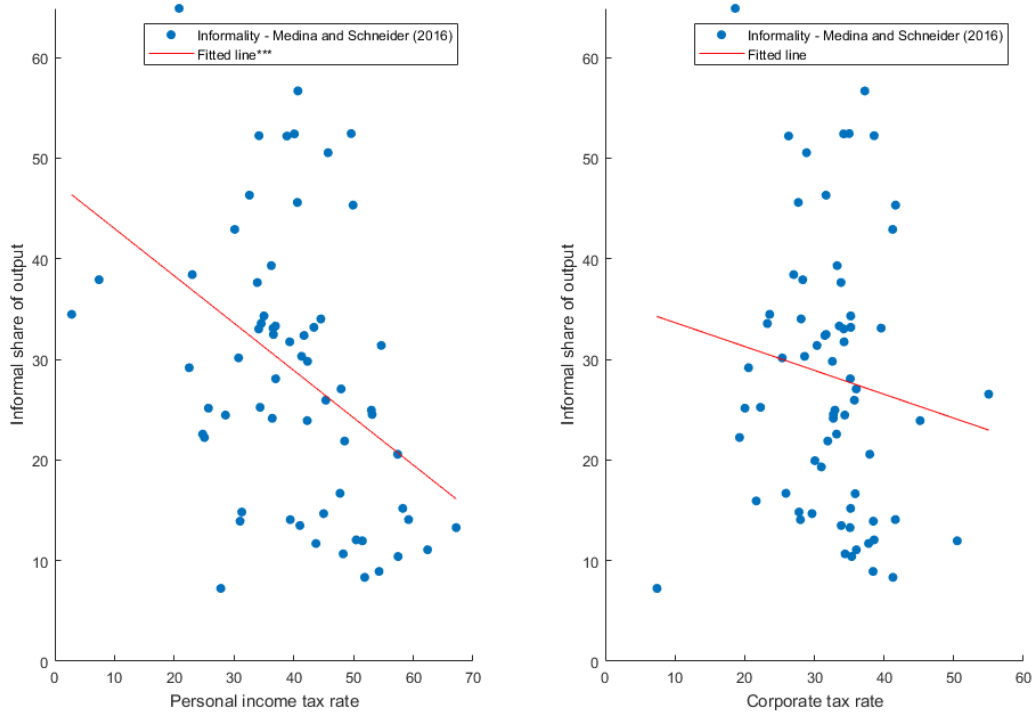
Notes: Upper panel: Left figure shows the size of the Per capita GDP (PPP) in 2016 vs. informal sector measured as a share of output (vertical axis). The right figure shows the same relationship but now Per capita GDP is, for each country, the average between 1991 and 2016.

Lower panel: Left and right figures are similar to those in the upper panel but now the size of the informal sector is measured as the share of labor working in the informal sector (vertical axis).

*** The linear regression represented by the fitted line in each panel is significant at a 99% level.

left panel shows that countries with lower personal income tax rates (horizontal axis) have higher informality (the fitted line is significant), and the right panel shows that there is no clear relationship between corporate income tax rates and informality (the fitted line is not significant). This suggests that, given the fact that higher tax rates provide incentives to operate informally, there must be an offsetting force that is not letting the relationship between taxes and informality to be positive.

Figure 2: Personal income tax rates and corporate tax rates vs. size of the informal sector



Notes: The figure on the left shows the relationship between the personal income tax rate and the size of the informal sector (vertical axis). The figure on the right shows the relationship between personal the corporate tax rate and the size of the informal sector (vertical axis).

*** The linear regression represented by the fitted line in the first figure is significant at a 99% level.

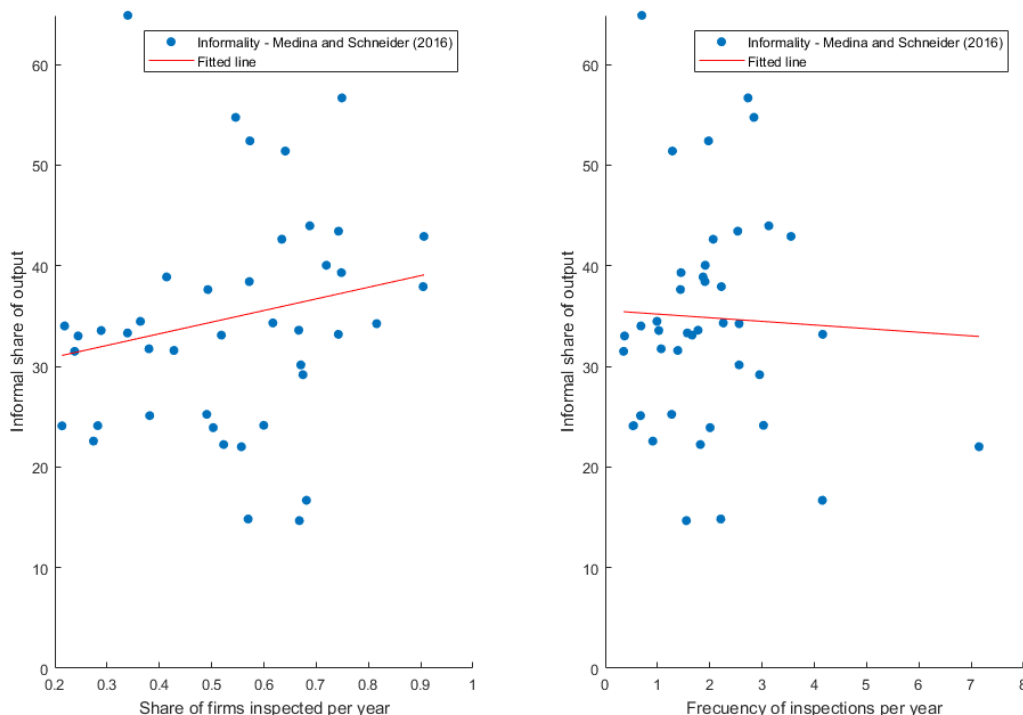
The linear regression represented by the fitted line in the second figure is not significant at any standard level.

2.3 Fact 3: The relationship between informality and Law enforcement is ambiguous

Considering that I define Law enforcement as the (unconditional) probability of being audited operating informally, [Figure 3](#) shows on the left panel the relation between Law enforcement (horizontal axis) and the size of the informal sector (vertical axis). I find that the correlation between those variables is positive, albeit not significant. This result means that there is an ambiguous relationship between Law enforcement and informality. The right-hand panel shows an alternative measure of Law enforcement, which is the average frequency

of inspections per firm. The slope of this graph is not significant at any standard level. Contrary to the standard belief that simply more Law enforcement implies lower levels of informality, cross-country data show that such correlation is not evident.

Figure 3: Law enforcement vs. size of the informal sector



Note: The figure on the left shows the relationship between Law enforcement (i.e. share of firms expected each year) and the size of the informal sector (vertical axis). The figure on the right shows the relationship between the average frequency of inspections of a firm per year (i.e. an alternative measure of Law enforcement) and the size of the informal sector (vertical axis).

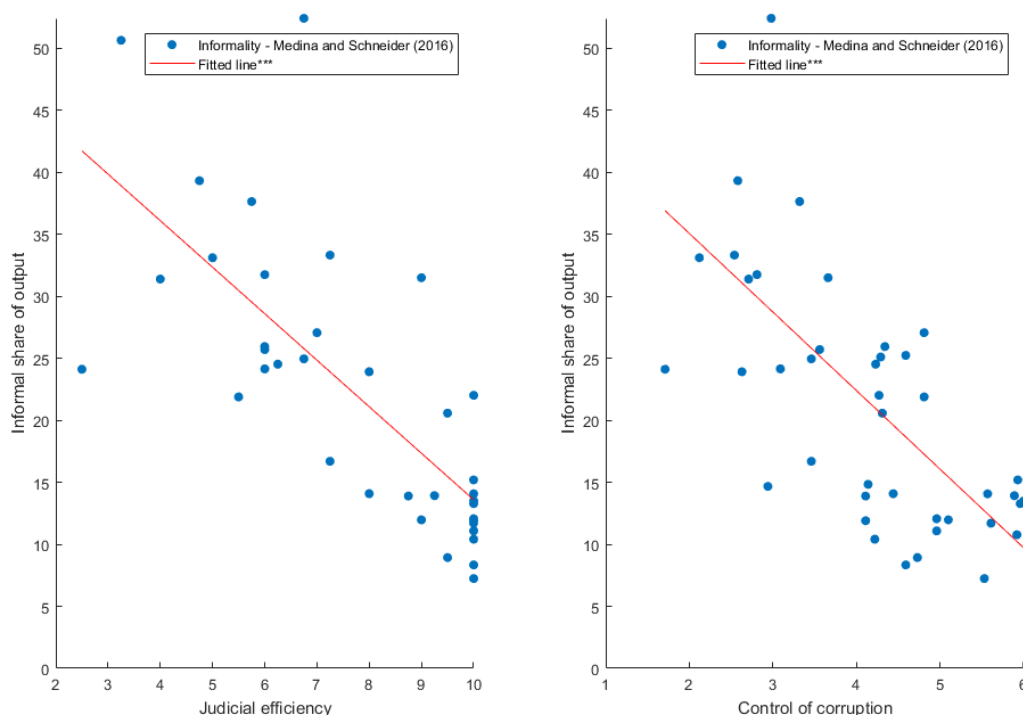
The linear regressions represented by the fitted lines in both figures are not significant at any standard level.

2.4 Fact 4: Countries with higher informality exhibit lower quality of institutions

Due to the government-operated nature of Law enforcement, this variable is closely related to institutional quality. One possible reason why the relationship between Law enforcement and informality documented in Fact 3 is ambiguous, is an ambiguous relation between insti-

tutional quality and informality. However, considering qualitative measures of institutional quality, such as judicial efficiency and control of corruption, I find that countries with higher quality of institutions exhibit lower informality. Figure 4 shows these relations. This finding is in line with the view that better institutions provide benefits to formal firms (e.g. Amaral and Quintin 2006). In other words, the puzzle found in Fact 3 is not driven by a puzzle related to the quality of institutions and informality.

Figure 4: Strength of institutions vs. size of the informal sector



Note: The figure on the left shows corruption efficiency as a proxy for strength of institutions vs the size of the informal sector (vertical axis). The right-hand side figure uses instead control of corruption vs the size of the informal sector (vertical axis).

*** The linear regression represented by the fitted line in each figure is significant at a 99% level.

2.5 Takeaways

From the stylized facts documented above, the main takeaway is that there is not a clear direct negative relationship between informality and Law enforcement, let alone a clear positive relationship between taxes and informality. However, the fact that measures of institutional quality are negatively correlated with informality, and considering that Law enforcement is held by the government (requiring public funds to be executed), suggest that there is a trade off between fiscal policy and Law enforcement for a given efficiency of the latter. This motivates the need to disentangle those two interacting forces to understand how they affect the size of the informal sector. Understanding the reasons behind the puzzles summarized by Facts 2 and 3, as well as gauging how Law enforcement related to fiscal policy contributes to explain the size of the informal sector, are the main purposes of this work. The next section provides specific measures of the size of the informal sector in terms of labor, output and, more importantly, capital stock, using microdata from a developing country.

3 Informal capital stock: The case of Colombia

One of the contributions in this paper is to study the role of Law enforcement in determining the size of the informal sector, specially in terms of capital stock. However, estimating informal capital stock has usually been a challenge because measuring informal assets, which by definition are not reported by individuals, implies that official institutions in charge of taking stock of the economy's assets are not able to observe a considerable bulk of those that are not reported. As a result, there is not, to my knowledge, a cross-country dataset that documents the informal capital stock. Notwithstanding these limitations, some exceptions exist, focusing on specific countries, such as [Leal Ordonez \(2014\)](#) does in the case of Mexico, or [Hamann et al. \(2011\)](#) do for the case of Colombia. In an attempt of measuring the size of both the formal and informal sector in terms of capital stock, as well as output and labor,

and due to data accessibility, I follow the latter authors and use confidential microdata from a survey held in Colombia that allows me to differentiate formal and informal firms⁶.

3.1 Microdata

I use a confidential database from the Colombian office of statistics, DANE, named “Microestablishments survey”. This survey follows initially randomly chosen business over a number of years with a confidentiality clause such that businesses cannot be fined or monitored directly as a result of any of the answers provided. The data I use is for the year 2016, since this is the only period where the survey asks to each firm about the value of total assets (i.e. the measure of capital stock). I drop the companies that did not report any asset or any worker (if only the owner works for the firm, the number of workers reported should be one). This yields a total 33,013 firms that are spread out about the main metropolitan areas, proportional to their population, and across the three main sectors of the economy (agriculture, manufacturing and services) proportional to their participation on output. Aside from allowing for the identification of firms’ assets, this dataset also allows for the classification of firms among formal or informal according to different definitions. In [subsection 3.2](#) below I explain how I do this classification.

This dataset has an important caveat: the firms surveyed have less than 10 workers; leaving out some medium or big size firms. In spite of this, there are reasons to believe that this dataset is representative of the informal sector: First, as in most developed countries, in Colombia official statistics show that more than 80% of firms are small and medium size. Second, [Maloney \(2003\)](#) shows that in the informal sector, at least 80% have less than 10 workers. Thus, most likely, disciplining a model with this data is leaving out big formal firms, which could lead to understate the size of the formal sector. However, since large firms in Colombia by law have more than 200 workers and exhibit higher rates of survival than less productive firms ([Eslava et al., 2013](#)), which usually are small and medium firms, I

⁶I thank the extremely helpful collaboration of the national institution of statistics in Colombia, DANE, and its employees in charge of the microdata for their help and support while I was processing the data.

do not focus on them and for the sake of the analysis focus only on firms of the type shown in the microdata⁷.

3.2 Identifying informal firms

As pointed out by [Busso et al. \(2012\)](#), informal firms are those that even though they engage in legal activities, do not pay social security to workers and do not comply with taxes or other regulation. The data allows us to identify informality with three questions. The survey asks firms if they paid social security to workers in 2016 (and how much), if they are registered in the official system by the Colombian internal revenue authority (named DIAN) and if they do and officially report bookkeeping records of their income statements and balance sheets. I define an informal firm as a firm that does not pay social security to its workers, is not registered and does not report to have any bookkeeping record⁸.

3.3 Empirical findings

For each formal and informal firm I compute their capital stock, measured by the value of assets, their value added, measured by total revenue minus the cost of intermediate inputs, and the total amount of workers. Since our objective is to obtain observed statistics of the relative size of the informal sector, for each sector I take the average of the capital stock and output, and compute the formal to informal ratios. For labor I simply calculate the share of labor working at each sector.

Assuming a Cobb-Douglas production function, I estimate the elasticity of output with respect to capital and labor, and then the total factor productivity (TFP). I then calculate the relative TFP of the formal sector. [Table 1](#) shows the moments for the key variables, namely the relations found for capital, output and labor between both sectors. The mean

⁷This means that I assume that big firms, and small and medium firms are segmented, such that all the action of capital accumulation will happen in the latter types of firms.

⁸Alternative definitions of informality could be defined by relaxing how many questions out of the three determine whether a firm is informal or not. See [Hamann et al. \(2011\)](#) for a detail break down of different degrees of informality with this data.

relative size of the formal sector with respect to the informal in terms of capital stock and output are respectively 2.94 and 2.1 (first and second rows of the table). The share of labor in the formal sector is 0.51 (third row). The mean elasticities of formal and informal output to their respective capital stock are 0.29 and 0.15 (fourth and fifth row), and the mean relative TFP of the formal sector is 1.19. Aside from the first moments of each statistic, [Table 1](#) shows also their median and standard deviation. As expected, the mean (and median) size of the formal sector is larger both in terms of capital and output than the informal sector, however roughly half of the labor force works at each sector (which is consistent with households' survey data for Colombia in the year 2016).

Towards the purpose of explaining the relative size of the informal sector, a successful model should perform well in matching the relative capital stock and relative output. Thus, the results in the first two columns in [Table 1](#) serve as a benchmark to evaluate the performance of the model in the next session⁹.

Table 1: Main moments for key variables in the Microdata

Key Variable	Meaning	Mean	Median	Std. Dev
k^F/k^I	Relative capital stock: formal to informal sectors	2.94	3	0.88
y^F/y^I	Relative output stock: formal to informal sectors	2.1	1.7	0.7
n^F/n	Share of formal labor	0.51	0.5	0.03
α_F	Formal output elasticity to capital stock	0.29	0.18	0.01
α_I	Informal output elasticity to capital stock	0.15	0.1	0.002
z^F/z^I	Relative TFP: formal to informal sectors	1.19	1.1	0.02

Notes: The table shows, for each formal to informal key ratio in first three rows, their median, mean and standard deviation. The last three rows show the same moments for key parameters of the production function. All moments and parameters are described in the second column.

⁹Alternatively, one could define a successful model one that matches relative output and the share of total labor working at each sector. Since my focus is on the size of the informal sector in terms of capital stock, I will use the share of labor working at each sector as a target moment to calibrate the model, and leave the relative capital stock and output free to be matched.

4 Baseline model

I build a two-sector, formal and informal, small open economy DSGE model. Households are identical and supply labor and capital to both sectors. There are two goods, one for each sector, which are imperfect substitutes and are produced respectively by a representative formal firm and by households using constant returns to scale (CRS) technologies. Since households are identical and the production at each sector follows CRS technologies, I refer to production of both types of goods as formal and informal sectors. The formal sector is regulated, thus the households' formal income is subject to an income tax, τ^Y . The representative formal firm pays taxes on the use of labor, τ^w , and capital, τ^r . In spite of these costs, operating in the formal sector has the benefit that it is exogenously more productive. Operating informally implies not paying taxes to the use of inputs and informal income, but is subject to imperfect enforcement: There is a probability of being audited and fined when operating in the informal sector, which is increasing in both informal capital stock and government expenditure. In absence of enforcement the model is akin to the framework developed by [Fernández and Meza \(2015\)](#)¹⁰.

4.1 Households

There is a continuum of identical households of mass 1 that derive utility from a consumption composite, c_t , and disutility from aggregate labor, n_t . Thus, the expected present value of utility at time zero is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \tag{1}$$

¹⁰Although in our model, contrary to [Fernández and Meza \(2015\)](#), shocks only come from the formal TFP process and there is no direct propagation mechanism of TFP shocks across sectors. The main purpose of these authors is to explain the fact that informal employment in Mexico is countercyclical, lags the cycle and is negatively correlated with output; as well as studying how the imperfect propagation of shocks across sectors contributes to explain those facts.

Where $\beta \in (0, 1)$ is the standard discount factor. As it is standard for SMOEs the utility function is assumed to be of the GHH form (see [Aguiar and Gopinath 2004](#))¹¹: $U(c_t, n_t) = \frac{(c_t - (n_t)^\omega)^{1-\sigma} - 1}{1-\sigma}$. The parameter ω is the labor supply elasticity to wages and σ is the degree of relative risk aversion. The consumption composite is described by the CES aggregator:

$$c_t = A(c_t^F, c_t^I) = [a(c_t^F)^\rho + (1 - a)(c_t^I)^\rho]^{1/\rho} \quad (2)$$

With c_t^F and c_t^I being the consumption of the formal and informal goods, respectively. The parameter a is the weight that households give to the formal good in aggregate consumption, and ρ is the parameter that governs the intratemporal elasticity of substitution between the two goods. Aggregate labor is the sum of labor supplied to both sectors:

$$n_t = n_t^F + n_t^I \quad (3)$$

4.1.1 Informal income and Law enforcement

Households have access to the following CRS technology for producing informal goods:

$$y_t^I = F^I(k_t^I, n_t^I) = z_t^I (k_t^I)^{\alpha_I} (n_t^I)^{1-\alpha_I} \quad (4)$$

Where y_t^I , k_t^I , and n_t^I are respectively the informal: output, capital stock, and labor. The parameter z_t^I in equation is the exogenous TFP of the informal sector, and α_I is the informal output elasticity to informal capital stock.

I introduce Law enforcement as defined in [section 2](#), the probability of being audited and fined when operating informally¹². This variable is a bivariate *c.d.f.* denoted by $H(k_t^I, g_t)$, such that $H_1(\cdot) > 0$, $H_2(\cdot) > 0$, $H(0, 0) = 0$ and $0 \leq H(k_t^I, g_t) \leq 1$. Thus, households'

¹¹As it is well known, these type of preferences shut down the wealth effect on labor supply; such that the latter only depends on the wage. For emerging SMOEs such as Colombia, [Chang and Fernández \(2013\)](#) and [Neumeyer and Perri \(2005\)](#) show that these preference explain better business cycle comovements observed in the data than preferences with active wealth effects on labor supply.

¹²I assume that all audits to the informal sector lead to a fine. Therefore, there is no room for bribery. However, in practice the bribe acts as a tax.

informal income at period t is random with the following expected value:

$$E(\text{income}_t^I) = y_t^I[1 - \tau_y\phi]H(k_t^I, g_t) + y_t^I[1 - H(k_t^I, g_t)] \quad (5)$$

Where $\phi \geq 1$ is the additional penalty for being informal (i.e. the fine). The first term in the right-hand side of Equation 5 is the households' income from producing informal output when it is audited and fined by the government, and the second term is the informal income when it is not audited.

4.1.2 Households' problem

Households can borrow from abroad by issuing one period non-state contingent bonds. International borrowing can only be in formal goods, meaning that only those goods can be traded in international markets. This debt is denoted by d_{t+1} with price q_t at period t . The households' budget constraint is.

$$c_t^F + p_t c_t^I + i_t^F + p_t i_t^I + d_t = (w_t n_t^F + r_t k_t^F)(1 - \tau_y) + p_t y_t^I[1 - \tau_y\phi H(k_t^I, g_t)] + q_t d_{t+1} + g_t \quad (6)$$

The right-hand side of Equation 6 represents the households' stream of income. The first term is the formal income, where w_t is the wage paid to formal workers, and r_t is the return to capital in the formal sector. This formal income is taxed at the rate τ_y . The second term represents the expected informal income in Equation 5. The third term is the period- t value of debt to be paid the next period, and the last term is the government's expenditure which is rebated to households. The left-hand side of Equation 6 represents the households' stream of expenditure and contains, respectively, consumption and expenditure in both types of goods, and d_t , the stock of debt acquired at period $t - 1$ that is due on period t , all in terms of formal goods such that p_t is the relative price of informal goods with respect to formal goods. The law of motions for capital stock in boths sectors are:

$$k_{t+1}^F = i_t^F + k_t^F(1 - \delta) \quad (7)$$

$$k_{t+1}^I = i_t^I + k_t^I(1 - \delta) \quad (8)$$

Where δ is the depreciation rate, which is assumed to be the same for both sectors¹³

The households' problem is then to maximize the expected net present value of utility described by 1, subject to Equations 2 to 4, 6, and the no-Ponzi constraint:

$$\lim_{t \rightarrow \infty} q_t^t D_t \leq \bar{D} \quad (9)$$

4.2 Formal firm

There is a representative formal firm that is regulated, it pays taxes on the use of labor, τ^w , and capital, τ^r . Every period the representative formal firm chooses k_t^F and n_t^F to maximize its profits, π_t :

$$\pi_t = y_t^F - (1 + \tau^w)w_t n_t^F - (1 + \tau^r)r_t k_t^F \quad (10)$$

Output is produced with the following CRS technology:

$$y_t^F = F^F(k_t^F, n_t^F) = z_t^F (k_t^F)^{\alpha_F} (n_t^F)^{1-\alpha_F} \quad (11)$$

The parameter α_F is the formal output elasticity to formal capital stock. The parameter z_t^F is the formal's sector TFP and it is the sole source of uncertainty in the economy. It is assumed to follow an AR(1) process of the form:

$$\ln(z_t^F) = \rho^F \ln(z_t^F) + \epsilon_t; \quad \epsilon_t \sim i.i.d(0, (\sigma^F)^2) \quad (12)$$

Parameters ρ^F and σ^F are respectively the first order autocorrelation of z_t^F and the

¹³There is no compelling empirical evidence to assume that δ is different in both sectors.

standard deviation of the unexpected shock ϵ_t .

4.3 Government and interest rates

Each period the government follows a balanced budget rule of the form:

$$\tau_y \phi H(k_t^I, g_t) p_t y_t^I + \tau_w^F w_t n_t^F + \tau_r^F r_t k_t^F + (w_t n_t^F + r_t k_t^F) \tau^y = g_t \quad (13)$$

The first term in the right-hand side of [Equation 13](#) is the fine paid by the informal sector in terms of formal goods, the second and third terms are the tax collection paid by the formal firm, and the last term is the tax collection on formal income paid by households.

As it is standard in small open economies, the interest rate is determined by the (gross) foreign interest rate, R^* , and a debt-elastic interest rate premium. Following [Schmitt-Grohé and Uribe \(2003\)](#), this is:

$$\frac{1}{q_t} = R^* + \psi [e^{(d_{t+1} - \bar{d})} - 1] \quad (14)$$

The left-hand side of [Equation 14](#) is the gross domestic interest rate. The second term in the right-hand side of the equation is the debt-elastic interest rate premium with parameters ψ , the sensitivity of the domestic interest rate to the debt premium, and $\bar{D}$, the exogenous steady state level of (net) foreign debt in the economy.

4.4 Market clearing

By defining g_t^F as the government's revenue from the formal sector and g_t^I as the one from the informal sector (in terms of formal goods), we have:

$$g_t^F = \tau_w^F w_t n_t^F + \tau_r^F r_t k_t^F + (w_t n_t^F + r_t k_t^F) \tau^y \quad (15)$$

$$g_t^I = \tau_y \phi H(k_t^I, g_t) p_t y_t^I \quad (16)$$

The market clearing conditions of the baseline model are:

$$y_t^F = c_t^F + i_t^F + g_t^F + d_t - q_t d_{t+1} \quad (17)$$

$$y_t^I = c_t^I + i_t^I + g_t^I \quad (18)$$

$$nx_t = \frac{y_t^F - c_t^F - i_t^F - g_t^F}{y_t^F} \quad (19)$$

$$y_t = y_t^F + p_t y_t^I \quad (20)$$

Equations 17 and 18 represent, respectively, the supply and demand equilibrium in the formal and informal sectors. Equation 19 is the trade balance as a share of formal output, and equation 20 is aggregate output in terms of formal goods¹⁴.

4.5 Optimality and equilibrium

The first order conditions (F.O.Cs) of the households' problem are:

$$[c_t^F] : \quad U_c(c_t^F, c_t^I, n_t^F, n_t^I) A_1(c_t^F, c_t^I) = \lambda_t \quad (21)$$

$$[c_t^I] : \quad U_c(c_t^F, c_t^I, n_t^F, n_t^I) A_2(c_t^F, c_t^I) = \lambda_t p_t \quad (22)$$

$$[n_t^F] : \quad U_n(c_t^F, c_t^I, n_t^F, n_t^I) \omega (n_t^F + n_t^I)^{\omega-1} = \lambda_t (1 - \tau^y) w_t \quad (23)$$

¹⁴I introduce the trade balance for completeness, because it is an important object of study in small open economies, albeit the study of this variable is not within the scope of this work.

$$[n_t^I] : \quad U_n(c_t^F, c_t^I, n_t^F, n_t^I) \omega(n_t^F + n_t^I)^{\omega-1} = \lambda_t p_t [1 - \tau^y \phi H(k_t^I, g_t)] F_2^I(k_t^I, n_t^I) \quad (24)$$

$$[D_{t+1}] : \quad \lambda_t q_t = \beta E_t(\lambda_{t+1}) \quad (25)$$

$$[k_{t+1}^F] : \quad \lambda_t = \beta E_t(\lambda_{t+1} [(1 - \tau^y) r_{t+1} + 1 - \delta]) \quad (26)$$

$$[k_{t+1}^I] : \quad \lambda_t p_t = \beta E_t \{ \lambda_{t+1} p_{t+1} [(1 - \tau^y \phi H(k_t^I, g_t)) F_1^I(k_{t+1}^I, h_{t+1}^I) - \tau^y \phi H_1(k_t^I, g_t) F^I(k_{t+1}^I, h_{t+1}^I) + 1 - \delta] \} \quad (27)$$

Combining [Equations 21](#) and [22](#) yields:

$$p_t = \frac{A_2(c_t^F, c_t^I)}{A_1(c_t^F, c_t^I)} \quad (28)$$

The first order conditions of the households' problem are standard, with λ_t being the Lagrange multiplier of the budget constraint. However, it is worth pointing out some important features: First, from [Equation 23](#), the disutility of labor supplied to the formal sector is equal to the wage net of income taxes in utility units; meaning that income taxes distort the labor supply to this sector. Second, from [Equation 24](#), the disutility of labor supplied to the informal sector is equal to the marginal productivity of labor (MPL) in the informal sector net of the fine paid by operating informally, measured in utility units. This MPL net of the fine is equivalent to the informal wage, but it is seen directly in [Equation 24](#) because the household has direct access to the informal production technology. Since the fine is proportional to the income tax, such tax also distorts the labor supply to the informal sector. Third, from [Equation 27](#), which is the Euler equation for informal capital stock,

the marginal disutility of accumulating informal capital stock is equal to: The expected marginal productivity of capital (MPK) in the informal sector net of the fine, the additional cost brought about by Law enforcement due to future capital accumulation in the informal sector, and depreciation, all in utility units. This implies that capital accumulation in the informal sector has dynamic effects over the allocation of capital across different periods.

The F.O.Cs of the formal firm's problem are:

$$[k_t^F] : \quad F_1^F(k_{t+1}^F, h_{t+1}^F) = (1 + \tau^r)r_t \quad (29)$$

$$[n_t^F] : \quad F_2^F(k_{t+1}^F, h_{t+1}^F) = (1 + \tau^r)w_t \quad (30)$$

Clearly, both taxes to capital and labor paid by the formal firm act as wedges between the marginal products of each input and their respective rental rate; distorting the optimal allocation of both inputs. The competitive equilibrium is:

Definition 1 *Competitive equilibrium: Given the initial conditions k_0^F , k_0^I and d_0 , and the stochastic process for ϵ_t , a competitive equilibrium is a set state-contingents allocations: $c_t, c_t^F, c_t^I, n_t, n_t^F, n_t^I, d_{t+1}, i_t^F, i_t^I, y_t, y_t^F, y_t^I, g_t, k_{t+1}^F, k_{t+1}^I, \lambda_t, nx_t$ and prices w_t, r_t, p_t, q_t such that:*

- I Given prices and law of motions for the stock of capital in both sectors given by [Equations 7 and 8](#), the allocations solve the households' problem: [Equations 2 to 4, 6, 9](#), and [22 to 28](#).*
- II Given prices, the allocations solve the formal firm's problem: [Equations 29 and 30](#).*
- III The government satisfies its budget constraint each period described by [Equation 13](#)*
- IV Domestic interest rates satisfy the debt-elastic interest rate condition in [Equation 14](#).*
- V Markets clear for capital, labor, and goods in all sectors of the economy: [Equations 17 to 20](#).*

5 Quantitative analysis

I evaluate the models' performance by its accuracy in matching the ratios $\frac{k_t^F}{p_t k_t^I}$ and $\frac{y_t^F}{p_t y_t^I}$ observed in the data. The first ratio is the relative formal to informal capital stock, and the second is relative formal to informal output. Hence, the inverse of both ratios are respectively the relative size of the informal sector in terms of capital and output. Thus, I focus on the model's steady state¹⁵. I solve for the non-stochastic steady state using standard non-linear numerical methods. I also solve the stochastic model by standard perturbation methods, and obtain the policy functions for each variable. The mean of the ergodic distribution for both ratios using Markov Chain Monte Carlo Simulations yield virtually the same results as the analysis of the non-stochastic steady state¹⁶.

5.1 Calibration

The parameter a is calibrated to match the share of formal labor observed in the data (0.51). The steady state level of debt, $\bar{D}$, is calibrated to match the average net foreign debt to GDP ratio for Colombia, taken from the updated version of the dataset developed by [Lane and Milesi-Ferretti \(2007\)](#). The parameters of the AR(1) process for z_t^F , ρ^F and σ^F , are calibrated to match the first order autocorrelation and volatility of Colombia's yearly GDP. The discount rate β is calibrated to be $1/R^*$ so the model is stationary. The parameters σ is taken from the SMOEs literature. The yearly depreciation rate, δ , is taken from the Colombian series of aggregate capital stock developed by [Fernández Martin et al. \(2017\)](#). Parameters α_I and α_F are taken from the microdata. The mean of z_t^F is set to 1.19, and the value of z_t^I is set to one. In this way, the relative TFP is normalized to match the one observed in the data. Tax rates for Colombia in the same year, 2016, are obtained from

¹⁵On the Appendix I provide the system of equilibrium equations in steady state.

¹⁶The fact that the simulation based-results yield the same as the non-stochastic steady state implies that the role of uncertainty is negligible. However, even if the purpose of the model is to analyze static moments, the stochastic process is useful if one wants to match important business cycle moments. Indeed, as it is standard in emerging small open economies (e.g. [Neumeyer and Perri 2005](#), [Uribe and Yue 2006](#), [Aguiar and Gopinath 2007](#)) the model yields a countercyclical trade balance, and when only the formal sector is measured, consumption is more volatile than output, in line with the results of [Restrepo-Echavarria \(2014\)](#).

Rincón-Castro et al. (2017). The value of σ in the utility function is set to 2, which is standard in the business cycle literature for SMOEs. Parameters ω , and R^* are set equal to 1.4, and 1.0145, a and α are taken from Fernández and Meza (2015). The parameter ρ is set to match an intratemporal elasticity of substitution of 0.875, which is standard for this type of economies in the literature.

I assume that Law enforcement follows a the simple functional form which ensures that $H(.)$ has the properties described in subsection 4.1.1: $H(k_t^I, g_t) = 1 - e^{-\gamma(k_t^I + g_t)}$, where γ can be interpreted as the exogenous efficiency of Law enforcement (e.g. a measure of institutional quality). The parameter γ is calibrated to match, in steady state, the fact that $H(.) = 0.27$ in Colombia, as observed in the cross-country data. Table 2 summarizes the parametrization of the model.

Table 2: Calibration

Parameter	Value	Source
a	0.6	Calibration
ρ	1-1/0.875	SMOE literature
τ^y	0.092	Data
α_I	0.29	Data
α_F	0.15	Data
τ^w	0.104	Data
τ_r	0.062	Data
$\bar{D}$	0.062	Calibration
ω	1.4	Fernandez and Meza (2015)
R^*	1.0145	Fernandez and Meza (2015)
σ	2	SMOE Literature
ρ^F	0.78	Calibration
σ^F	0.007	Calibration
δ	0.035	Data
$\bar{z}_F$	1.19	Data
$\bar{z}_I$	1	Data
β	0.9857	Calibration
γ	1.3908	Data

Notes: The table shows, for each parameter in the model (first column), its value (second column), and whether it is calibrated, taken from the data, or taken from the literature (third column).

5.2 Results

[Table 3](#) shows how the model performs at matching both ratios, $\frac{k_t^F}{p_t k_t^I}$ and $\frac{y_t^F}{p_t y_t^I}$, as well as the value of aggregate output generated by the model, all described in the first column. Column 2 in the table shows the observed ratios. The last column shows the results of the model, namely the baseline model. In terms of capital stock, which is our main focus, the model generates a relative formal capital stock of 2.83, compared to 2.94 in the data (see [Table 1](#)), meaning it explains 96% of the relative size of the informal sector in terms of capital stock. With respect to relative formal output, the model yields a value of 1.85, compared to 2.1 in the data, explaining 88% of the relative size of the informal sector in terms of output.

Alternatively, the third column in [Table 3](#) shows the results of a model without Law enforcement ($H = 0$). This framework explains 65% of the relative formal capital stock (1.92 in the model compared to 2.94 in the data). This is not a poor performance, albeit it overstates the size of capital stock, but in terms of output this simple model yields a relative formal output equal to 0.76. In other words, the model absent of Law enforcement considerably overstates the output of the informal sector, such that it is higher than in the formal sector, which is counterfactual. In [subsection 5.4](#) I discuss the results of alternative specifications of Law enforcement.

5.3 Inspecting the mechanism

The underlying mechanism behind the baseline model's ability to better match the relative size of the informal sector than a model without Law enforcement is crucially explained by [Equations 24](#) and [27](#). From the former equation, we can observe that Law enforcement creates an additional wedge that reduces the MPL of the informal sector, therefore reducing the expected wage. The higher the capital stock in this sector, the lower the labor supply to it, negatively affecting informal output. From [Equation 27](#) we have that Law enforcement also creates a wedge that reduces the expected MPK in the informal sector, as well as generating an additional loss in informal output due to the marginal increase in Law enforcement for

Table 3: Baseline model vs Alternative models vs Data

		Alternative model 1	Alternative model 2	Alternative model 3	Baseline model
Unmatched variable	Data	$H = 0$	$H = \theta$	$H = H(k^I)$	$H = H(k^I, g)$
$\frac{k^F}{pk^I}$	2.94	1.92	2.11	2.32	2.83
$\frac{y^F}{py^I}$	2.10	0.76	1.05	1.39	1.85
y		0.31	0.14	0.28	0.35

Notes: The table shows for each formal to informal sector ratio (first column) how the results in different specifications of the model match the data observed in table 2 (second column). The last column shows the results of our baseline model with Law enforcement endogenous to informal capital stock and government expenditure. The third, fourth and fifth column show the results of, respectively: A model without Law enforcement, a model with constant Law enforcement, a model with Law enforcement only depending on informal capital stock. The table also shows the value of output (last row) generated by each model. All results correspond to the steady state.

any future accumulation of capital in that sector. Given the nature of the small open economy, such that in steady state the interest rate is equal to the foreign interest rate (see [Equation 14](#)), the only way to offset the decrease in MPK generated by the aforementioned wedge, is by reducing the capital stock in the informal sector. Together with lower labor supply to this sector, this result in turn lowers the relative size of the informal sector in terms of capital stock as well as in terms of output, compared to a framework where there is no Law enforcement (i.e. $H(\cdot) = 0$). The fact that Law enforcement depends on informal capital stock creates an upper bound level of informal capital to be accumulated so the probability of being audited operating informally does not become so high that the benefits of operating informally are offset by the cost. Likewise, the wedge on informal MPL and MPK created by the presence of Law enforcement is exacerbated by the fact that it is proportional to the government expenditure. The higher the tax collection, the lower the net MPL and MPK will be. This additional punishment generated by government expenditure positively affecting Law enforcement is necessary to bring the model closer to the data, such that without Law enforcement being endogenous to that variable the model would not perform as good as the baseline model in matching the relative size of the informal sector. I explain this last issue

on [subsection 5.4](#) below. Finally, notice that since Law enforcement depends on government expenditure, and there is a balanced budget rule, such that when increasing τ_y , there are less incentives to operate in the formal sector but also the higher revenue would lower the incentives to operate in the informal sector due to more Law enforcement. This non-linearity is critical to explain the puzzles observed in stylized facts 2 and 3. I focus on this feature of the model in [subsection 6.1](#).

5.4 Alternative models

In this section I explore two alternative specifications of Law enforcement. First, simply assuming that Law enforcement is constant, such that $H(.) = \theta$. The fourth column in [Table 3](#) shows the results for this framework. Compared to the model without Law enforcement, this model performs better in matching the relative size of the informal sector, both in terms of capital stock and output. However, it predicts a relative formal to informal output that is close to 1, roughly half the one observed in the data. Moreover, this setup generates an output of 0.14 (third row), compared to 0.31 in the framework without Law enforcement and higher informality. Based on the first stylized fact described in [section 2](#), this is counterfactual. Second, as in [Leal Ordonez \(2014\)](#), I assume that $H(.) = H(k^I)$. This setup yields better results than the previous alternative model, as shown in the fifth column of [Table 3](#). The reason is that now the upper bound on informal capital stock appears to limit the capital accumulation of the informal sector to create an increasing wedge between in the informal MPK and its rate of return. However, this setup has the issue that generates lower output than the framework without Law enforcement. This happens because the reductions in informal output and capital stock occur through lower supply of both inputs, which due to wealth effects is not compensated by higher supply of labor and capital to the formal sector. Again, lower informality with lower aggregate output is counterfactual. Moreover, in spite of the fact that this alternative model performs well in matching the relative size of the informal sector, the absence of government expenditure in Law enforcement does not

provide a mechanism through which we will have a trade off between tax rates and Law enforcement, so the puzzles documented by stylized facts 2 and 3 cannot be solved. Providing a rationale for such puzzles is the purpose of the next section.

6 Law enforcement, tax rates, and informality

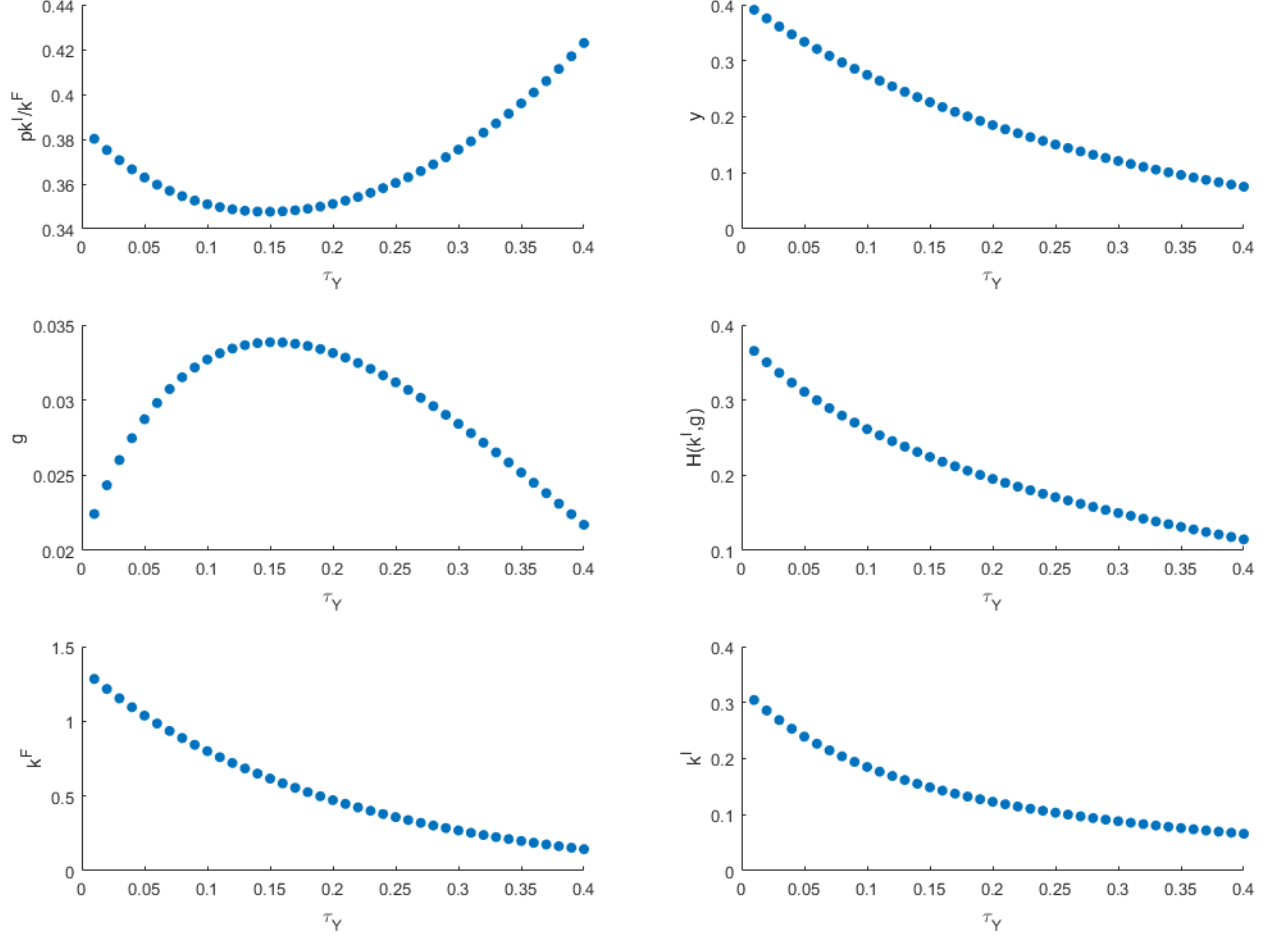
In this section I show how different tax rates affect the relative size of the informal sector, and through what channels we can obtain a negative relation between tax rates and informality, as well as a positive relation between Law enforcement and informality. Such relationships are the puzzling stylized facts 2 and 3 documented in [section 2](#). Considering that in the data the clear negative relation between taxes and informality occurs with respect to the tax on income, namely τ^Y in the model, I explore how such higher rates of that tax can generate, at least for a region, lower informality in terms of capital stock. This due to the fact that the fine is proportional to the income tax, such that there is a non-linearity when increasing taxes: lower incentives to operate in the formal sector, but also higher Law enforcement due to more government's revenue. I also explore how changes in the remaining tax rates, and the parameter of the Law enforcement function (i.e. Institutional quality) affect the relative size of the informal sector.

6.1 Changes in τ^Y

The value of the personal income tax, τ^y , is 0.092 in the data. I experiment with values between 0.01 and 0.3, separated equally by one percentage point. The left-hand side of the first row in [Figure 5](#) shows the relative size of the informal sector (vertical axis) for different values of τ^y (horizontal axis). The dots represent steady state values for each tax rate. The right-hand side of the first row maps the steady output with the same set of values for the tax. The second row shows, in its first plot, the government expenditure (and revenue) on the vertical axis for different values of τ^y . The second plot of the same row shows $H(k^I, g)$

in the vertical axis for different values of τ^y . The last two figures show, respectively, the values of formal and informal capital stock for different values of the tax on formal income.

Figure 5: Experiment 1 - Changes in τ^y



Notes: The figure shows the steady state values of different key variables for different tax rates on income, τ^Y , shown in the horizontal axis. The first row shows respectively the relative size of the informal sector in terms of capital stock, and GDP. The second figure shows respectively government's expenditure (and revenue), and Law enforcement. The last row shows respectively formal and informal capital stocks.

Here we observe the most interesting result among the policy experiments: There is a range of the income tax for which its changes have ambiguous effects over the size of the informal sector. That range are all tax rates in the neighborhood of 0.15. Increasing the

tax rate from the baseline value, 0.09, to 0.15 minimizes the size of the informal sector; however increasing it more leads to the standard result of the literature where higher taxes imply a higher size of the informal sector. On the other hand, decreasing the tax rate to a level lower than the baseline value will increase the size of the informal sector. This result relates directly to the left plot in [Figure 2](#), where the personal income tax rate seems to have a counter intuitive negative correlation with informality. Here we can see that, for any country where income the tax rate is constrained by some lower and upper bounds, it is possible to reduce such tax rate and increase the size of the informal sector. In other words, this result is able to match the fact that, in the data, there is an inverse relation between personal income taxes and the size of the informal sector. However, were those taxes to be higher than certain threshold (0.15 in our case), this relationship would invert and become as expected: higher taxes imply higher informality.

From the last two plots in [Figure 5](#) notice that the capital stock in both sectors falls, however it does it faster in the informal sector than in the formal for a range of tax rates between 0.01 and 0.15. This result occurs because a higher tax on formal income reduces the MPK of the informal sector, such that capital stock in that sector decreases to offset such effect completely because the interest rate does not adjust given the small open economy. Lower informal capital stock reduces the wage in the informal sector. Due to the wealth effect over labor supply as well as higher taxes on income, supply of labor also decreases in the formal sector, which reduces the MPK on it. To offset that reduction so the interest rate remains equal to the foreign interest rate, joint with the fact that higher tax on formal income reduces incentives to save in the formal sector, the supply of formal capital decreases. As a result, GDP falls quasi monotonically for higher tax rates. This means that the fact that higher informality implies lower output, as described in the stylized Fact 1, is a phenomenon that is true above the threshold tax rate.

Finally, the second row of [Figure 5](#) is illustrative of what happens with government expenditure (and revenue). The first plot in this row is a Laffer curve, where increasing

taxes generates more tax collection in spite of lower output; however for tax rates above 0.15, the decrease in output is so large that the total tax collection becomes decreasing in the tax rate. Even though the Law enforcement is decreasing in the tax rate, due to lower informal capital decreasing, the increased government expenditure up to the threshold keeps the Law enforcement higher than what it would be if it did not depend on g (In the appendix I explore the results of this experiment when we assume that g is constant, and show how and why the mechanism reverts). This mechanism contributes to generate a sharper fall in the informal capital stock than in the formal for tax rates between 0.01 and 0.15. For rates beyond this last value g decreases and the mechanism reverts. The reason why Law enforcement decreases with higher tax rates is because there is lower informal capital stock. This channel generates a positive correlation between Law enforcement and informality within the range of tax rates in which higher income tax rates imply lower informality, as documented in stylized fact 3. Thus, the model not only matches better the relative size of the informal sector than a framework without endogenous Law enforcement, but it also provides a rationale for the apparent counter intuitive relations between income tax rates, Law enforcement, and the size of the informal sector.

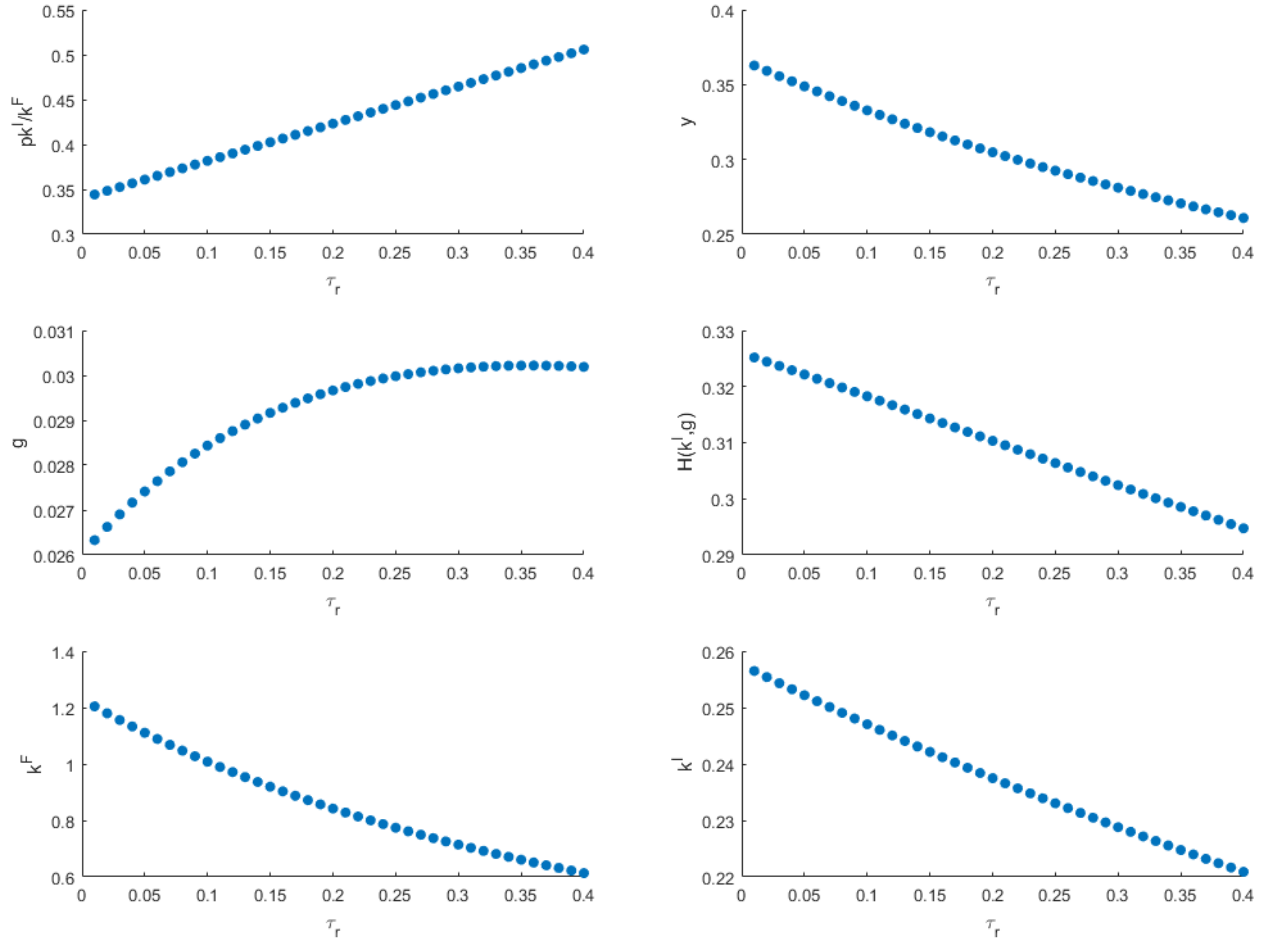
The main implication here is that Law enforcement depending on both informal capital and government expenditure induces a non-linearity that could have different policy implications than the traditional view. For instance, in the Colombian case there seems to be room for an income-tax increase that fosters formality; however, such increase should not be of more than approximately six percentage points if the objective is to minimize the size of the informal sector.

6.2 Changes in τ^r

The value of τ^r is 0.062 in the data. I use the same grid than in the previous two experiments. The structure of [Figure 6](#) is the same as [Figure 5](#), but now the horizontal axis has τ^r .

As a policy, reducing the tax rate on capital paid by firms is the most effective in re-

Figure 6: Experiment 2 - Changes in τ^r



Notes: The figure shows the steady state values of different key variables for different tax rates on income, τ^r , shown in the horizontal axis. The first row shows respectively the relative size of the informal sector in terms of capital stock, and GDP. The second figure shows respectively government's expenditure (and revenue), and Law enforcement. The last row shows respectively formal and informal capital stocks.

ducing informality and increasing output. This policy does not bring a trade-off with Law enforcement. The reason why this happens is that τ^r only affects the formal firm. By reducing it, the demand for formal capital increases, and with a higher level of formal capital stock comes a higher level of formal labor. Production increases considerably such that tax collections are higher than before and enforcement does not decrease in this case. In the

Colombian case it is clear that this policy, regardless of whether there is a restricted domain in which the tax can be modified, it is always better to decrease the size of the tax rate to its exogenously assumed lower bound.

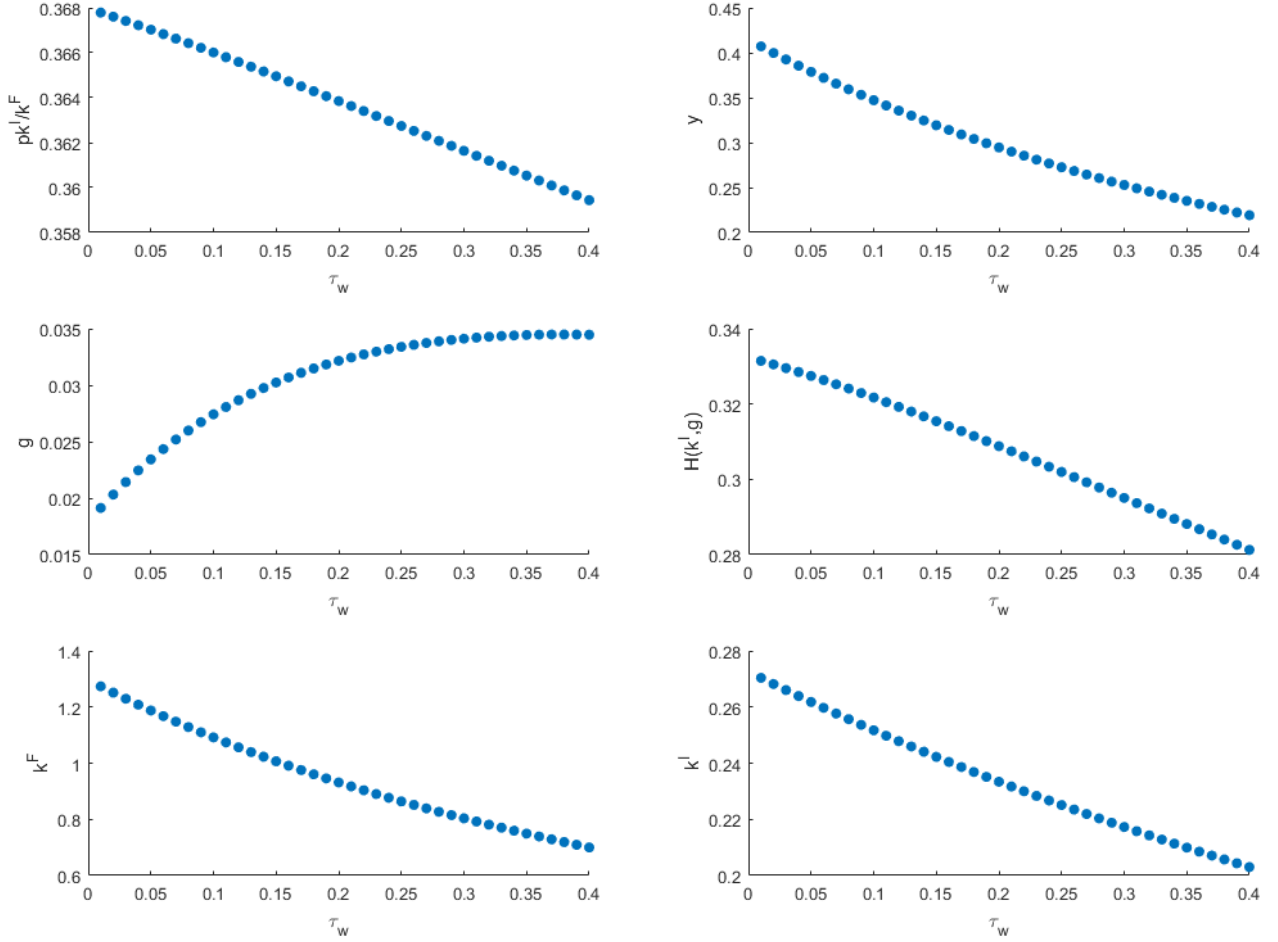
6.3 Changes in τ^w

I also use the same grid for τ^w than the one for τ^y . [Figure 7](#) shows the results for this experiment in the same order than Experiment 1, except that now the horizontal axis has τ^w instead of τ^y .

Even though it seems that the relative size of the informal sector decreases with higher tax rates on labor paid by firms, zooming into the vertical axis of the first plot in [Figure 7](#) one can notice that the quantitative different across points is almost nil. In other words informality practically does not react to changes in τ^w , although GDP does as predicted in the literature. The reason why this happens is that when such tax decreases, the formal wage increases, implying an increase in labor supply to the formal firm. Here, a higher tax on wages decreases wages in the formal sector and, as a result, labor supply to it. For the MPLs to equilibrate, labor supply in the informal sector also decreases. Total income is less, which reduces savings and capital supply in both sectors. This leads to a reduction in Law enforcement, although not considerable since the higher tax rates increase government's revenue up to an upper bound.

Here the main implication is that a reduction in the tax rate on labor paid by firms should not have, at least for this calibration, major effects over decreasing informality. In the case of Colombia recent research by [Bernal et al. \(2017\)](#), [Fernández and Villar \(2017\)](#), [Kugler et al. \(2017\)](#) and [Morales and Medina \(2017\)](#) show the positive impact over reducing informality of a tax reform in 2012 consisting on, among others, reducing the payroll contributions made by firms. In the way taxes are mapped here, a fraction of those payroll contributions is captured by τ^y ; so there could be room for further reductions in that type of tax. However, the results of this experiment suggest that in this environment if such reductions were to be

Figure 7: Experiment 3 - Changes in τ^w



Notes: The figure shows the steady state values of different key variables for different tax rates on income, τ^w , shown in the horizontal axis. The first row shows respectively the relative size of the informal sector in terms of capital stock, and GDP. The second figure shows respectively government's expenditure (and revenue), and Law enforcement. The last row shows respectively formal and informal capital stocks.

continued, they would not have further significant impact over reducing informality.

6.4 Changes in γ

I interpret γ as the efficiency of Law enforcement. Since the functional form of Law enforcement is akin to an exponential distribution, then a higher value of this

parameter implies a lower expected value of informal capital stock¹⁷. Nevertheless, it is not easy to quantitatively interpret an increase in γ . For this reason, I create a grid for this parameter between 0.5 and 3 (the baseline value is 1.39), and map it into the value of $H(k^I, g)$ in the stochastic steady state. This results in almost monotonic increases in Law enforcement between 0.27 and 0.5. Figure 8 shows the graphs with the same structure as before but now the policy variable in the horizontal axis is γ . The second plot of the second row in the Figure shows that every increase in γ of 0.5 is approximately an increase of ten percentage points in $H(k^I, g)$.

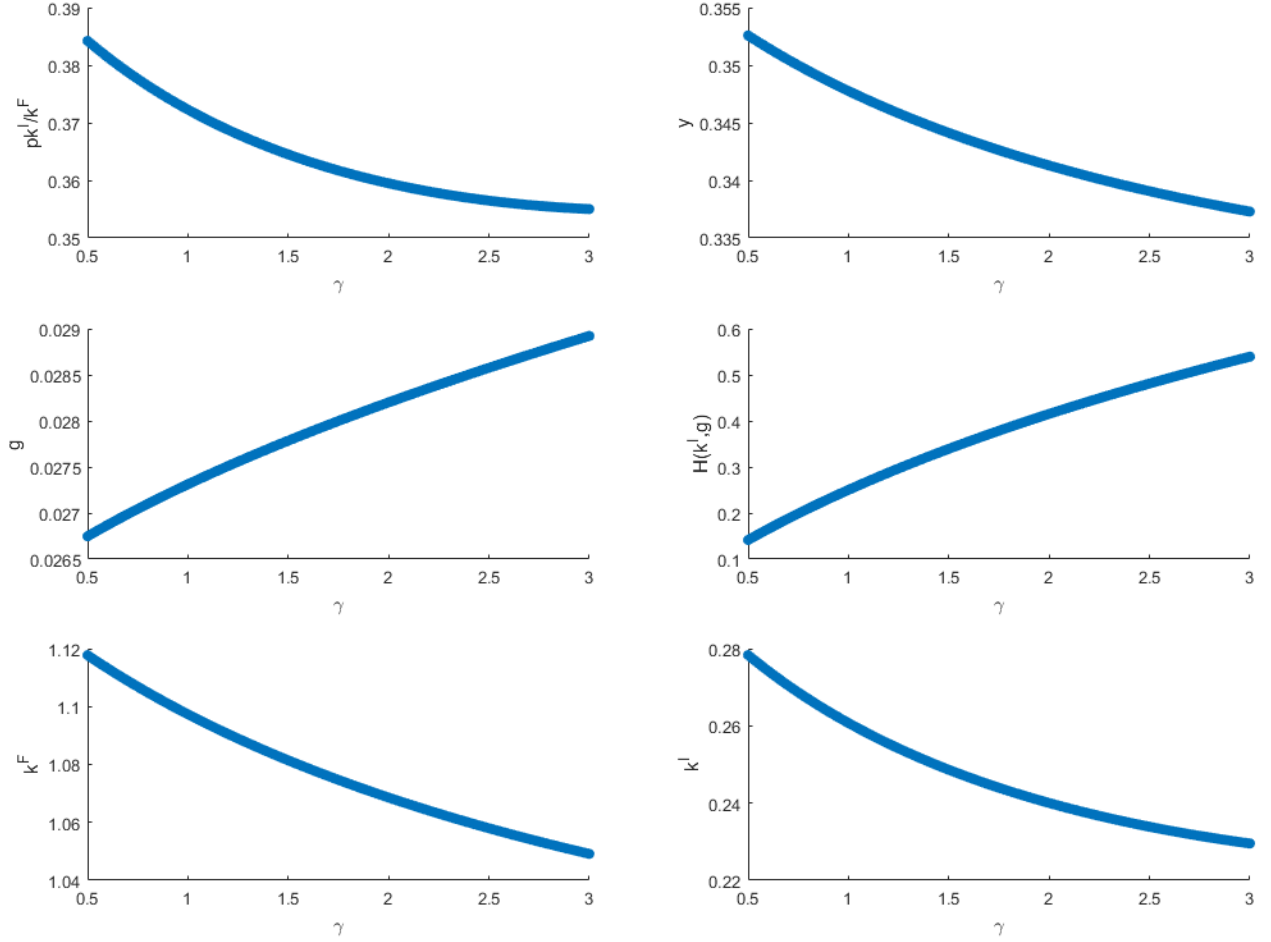
Increasing the quality of institutions mildly decreases the size of the informal sector, as shown in the first plot of Figure 8. However, it has negative effects on aggregate output. The reason is that a higher γ implies lower MPL and MPK. The lower MPK is offset by a reduction in informal capital stock, and the lower MPL lowers labor supply to the informal sector. For wages to equilibrate, the wealth effect dominates, and the labor supply in the formal sector also decreases. As a result the MPK in the formal sector decreases, which is offset by a decrease in the MPK of the informal sector. Altogether implies lower output in both sectors, hence lower total output. This results implies that in the new equilibrium taxes are too high given a higher level of enforcement for each scenario, which deter capital accumulation in the formal sector as well as labor supply to it. This combination hinders aggregate output given the set of taxes. The main takeaway of this experiment is that an improvement in the quality of institutions that leads to more efficiency of Law enforcement must be accompanied with an according reduction in taxes.

7 Discussion

There are three main takeaways from the results above: First, the Baseline model is able to match better the relative size of the formal sector than a model absent of Law enforcement,

¹⁷Recall that for any standard exponential distribution with parameter γ , the expected value of the random variable is $1/\gamma$.

Figure 8: Experiment 4: Changes in γ



Notes: The figure shows the steady state values of different key variables for different tax rates on income, γ , shown in the horizontal axis. The first row shows respectively the relative size of the informal sector in terms of capital stock, and GDP. The second figure shows respectively government's expenditure (and revenue), and Law enforcement. The last row shows respectively formal and informal capital stocks.

up to a point that it explains a considerable fraction of relative capital stock (about 0.96) and relative output (about 0.88). Second, the Baseline model is not able to fully rationalize the data, which is in line with the literature in the sense that there are other factors contributing to determine the size of the informal sector such as enforcement of contracts (Quintin, 2008), provision of public goods (Loayza, 1999), entry, exit costs and financial frictions (Hamann

[et al., 2011](#)). Third, in terms of optimal policy, decreasing tax rates is not always optimal since there is a trade-off with enforcement, creating as a result a bounded range within which (income) tax rates increases are optimal to minimize informality.

In the present model, the observed relative TFP is not able to capture all those other characteristics of the economy that would fully contribute to match the relative size of the informal sector; meaning that one could augment the model to consider for those that are more appealing in the data. In spite of the limitations of the Baseline model, it is an important tool for positive analysis because it shows that Law enforcement plays an important role in determining the relative size of the informal sector, both in terms of capital stock and output. Mainly, informal firms have incentives to remain small due to lower probability of being detected, and taxes contribute to increase such probability by funding government expenditure that is positively related to the degree of Law enforcement.

In terms of normative analysis, the model serves as a point of reference to measure the impact of different fiscal and institutional policies, given a bounded (and positive) set of tax rates. The rationale provided by the model to explain the puzzles observed in the stylized facts, highlights two important venues of research: First, the literature where more government expenditure provides a benefit for the formal sector, usually through the provision of public goods (see [Bardey and Mejía 2019](#), [Posada and Mejía 2012](#), and [Loayza 1999](#)), in the context of Law enforcement, usually assumes only labor in the production function. Embedding such framework with Law enforcement depending on capital stock could be promising in deriving Ramsey-optimal policies. Moreover, considering the evidence from [Leal Ordonez \(2014\)](#) where informal capital stock is an important source of missallocation, and monitoring plays an important role on that distortion, our framework is a step forward that can be implemented to show how the government can directly rationalize such missallocation outcomes, by weighting the trade-offs between the benefits of increasing taxes (more enforcement) against its costs. Second, since this work shows how imperfect Law enforcement distorts the marginal productivity of capital and labor in the informal sector, and through

general equilibrium effects it affects the supply of labor and capital also in the formal sector, it is appealing to study the role of imperfect enforcement in a more granular way. For instance, not all firms may have a symmetric reaction to Law enforcement, and more importantly, there could be a mismatch between firms' productivity and enforcement (such that firms with higher informal capital stock, could not necessarily be more productive, implying lower levels of informal and aggregate output).

8 Conclusions

I build a two-sector (formal and informal) small open economy DSGE model augmented with Law enforcement that is endogenous to informal capital stock and government expenditure. I show that the model performs better in matching the relative size of the informal sector in terms of capital stock and output than a framework without Law enforcement. I highlight the fact that incorporating this type of Law enforcement contributes to match the relative stock of capital because accumulation of capital in the informal sector provides incentives to remain small in order to reduce the probability of being detected operating in that sector. Simultaneously, the government's role in Law enforcement is active, giving taxes a non-linearity in the form of a Laffer curve which generates the normative implications for policy makers that they should not assume that tax reductions will automatically decrease the size of the informal sector. The model is able to explain standard Facts related to informality based on cross-country data, specially the counter intuitive findings showing how income tax rates and informality are negatively correlated, and how Law enforcement and informality are positively correlated.

Policy experiments suggest that there is a range for personal income taxes for which the relation between informality and tax rates is decreasing; in contrast with the standard view. However, the policy that guarantees lower informality and higher output is reducing the corporate tax rate. The model suggests that the quality of institutions plays an important

role in determining optimal tax policies. However, it remains ambiguous to understand what policies determine the quality of intuitions and what do exogenous changes to them mean. Moreover, in a context where other factors that determine the size of the informal sector are not considered in the economy, the quality of institutions could be explained by other approaches developed in previous works, such as the endogenous degree of contract enforcement, or the provision of public goods. Embedding those factors into a dynamic framework would contribute substantially to a better understanding of the determinants of informality and the design of optimal policies to reduce it without hindering output and welfare.

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Appendix

A.1 Cross-country Data

A.1.1 Size of the informal sector

To measure the size of the informal sector, I follow [Benton and Portes \(1989\)](#), [Pratap and Quintin \(2006a\)](#) and [Medina and Schneider \(2018\)](#) to define informality: “Informal activities are a process of income-generation that is unregulated by the institutions of society, in a legal and social environment in which similar activities are regulated.”

Measures of the size of the informal sector in each country are usually either estimates of informal output as share of total GDP or estimates of the share of labor that is informal which usually are taken from household’s survey data. The former can be estimated through direct or indirect methods. However, due to the unobserved nature of the informal sector,

cross-country data that is comparable using direct methods may yield spurious relations. On the other hand, indirect methods, such as the Multiple Indicators Multiple Causes (MIMIC) relies on standardized observable variables across countries to estimate the size of the informal sector, which is a latent variable. In this direction, [Medina and Schneider \(2018\)](#) developed an improved MIMIC procedure that uses light intensity as one of the determinants of the informal sector instead of the commonly used aggregate GDP, contributing to obtain a cleaner estimation of the size of informal activity¹⁸.

Thus, as measures of the size of the informal sector I use two: First, the aforementioned estimation from [Medina and Schneider \(2018\)](#), which provides for a set of 158 countries estimates of informal output as share of total GDP. Second, the standard measure of informal labor as share of total labor force, which I take from the World Bank Development Indicators (WDI) database ¹⁹. This last indicator however is restricted to a subset of 39 countries. For both indicators of informality I calculated the size of the informal sector for the year 2016 as well as the average between the years 1991 and 2016.

A.1.2 Aggregate economic activity

Aggregate economic activity is measured by per-capita real GDP in PPP terms. The source is also the WDI database. Those values are calculated for the year 2016 and the average between 1991 and 2016.

A.1.3 Law enforcement, quality of institutions and tax rates

Traditional institutional measures of the quality of institutions or effectiveness of Law enforcement are useful to make qualitative comparisons across countries through rankings of different categories (such as corruption perception, efficiency of the justice system, enforce-

¹⁸Recent relevant literature such as [Schneider and Enste \(2002\)](#), [Schneider \(2004\)](#), [Feld and Schneider \(2010\)](#), [Schneider and Buehn \(2017\)](#) discusses where we are in terms of measuring the size of the informal sector. See also [Pratap and Quintin \(2006a\)](#) for a discussion on different approaches on how to measure the informal sector.

¹⁹Usually, this data is estimated by using monthly survey data of the labor market. Informal workers are those who do not contribute by any means to social security (specifically, health and pension).

ment of property rights, etc). However, measures of quantitative nature that can later be mapped into a model are hard to obtain. Therefore, I follow [Ihrig and Moe \(2004\)](#) and [Restrepo-Echavarria \(2014\)](#) to develop a simple definition Law enforcement that allows me to obtain a cross-country comparable measure to be mapped into the model: Law enforcement is the probability that any firm or establishment is audited and fined (by assumption, there is now bribery in this definition).

Following this, I gather data from the World Bank Enterprise Survey (WBES) which has data firm-level data of 104 countries for one to three years²⁰. Among the questions in the survey, aside from the standard ones such as income, number of workers, and others, I focus on the one that asks whether that year the firm was audited or not by a government official. Then, defining Ins as a dummy that takes the value of 1 when the firm was inspected and 0 when not, the unconditional probability, H , of firm i being inspected in a given country j at year t is:

$$H_{i,j,t} = \frac{\sum_{i=1}^{N_{j,t}} Ins_{i,j,t}}{N_{j,t}}$$

Where $N_{j,t}$ is the total number of firms surveyed in country j at year t . Then, for each country, our measure of Law enforcement is the average of $H_{i,j,t}$ across the available years for such country:

$$H_{i,j} = \frac{\sum_{t=1}^T H_{i,j,t}}{T}$$

This H is our measure of Law enforcement, which follows the definition aforementioned in this section. Thus, this variable and Law enforcement are assumed to be isomorphic. Additionally, I obtain data to measure the quality of institutions that will allow me to untangle the relationship between such quality and informality and the one between Law enforcement and informality. These measures are the judicial efficiency index and control of

²⁰The time periods are not the same for each country, for example Colombian firms appear on the years 2014 and 2017 but Swedish firms only appear in the year 2014.

corruption index, which I take from the World Economic Forum for a subset of 42 countries in 2016.

Finally, to get the relationship between tax policies, the size of the informal sector and Law enforcement, I obtain cross country data on corporate and income tax rates. I use the data set constructed by [Vegh and Vuletin \(2015\)](#), who develop a data set for tax rates between 2009 and 2013 that has corporate taxes, personal income taxes and value added taxes. Since I am not studying the effects of value added taxes in distorting consumption decisions from households, I only focus on the first two. I use the tax rates of the year 2013.

A.3 Equilibrium and non-stochastic steady state in a model without Law enforcement

A.3.1 Equilibrium equations in a model without Law enforcement

The model with no Law enforcement has the following 21 endogenous variables (state and controls): $c_t^F, c_t^I, c_t, n_t^F, n_t^I, n_t, d_{t+1}, k_{t+1}^F, k_{t+1}^I, p_t, r_t, w_t, q_t, g_t, i_t^F, i_t^I, y_t^F, y_t^I, y_t, nx_t, \lambda_t$.

Using the functional forms described in [section 4](#), the full system of 21 equilibrium equations in the standard model is the following:

$$c_t = [a(c_t^F)^\rho + (1 - a)(c_t^I)^\rho]^{1/\rho}$$

$$n_t = n_t^F + n_t^I$$

$$(1 - \tau^y)(w_t n_t^F + r_t k_t^F) + p_t y_t^I + g_t + q_t d_{t+1} = d_t + c_t^F + p_t c_t^I + i_t^F + p_t i_t^I$$

$$y_t^I = z_t^I (k_t^I)^{\alpha_I} (n_t^I)^{1-\alpha_I}$$

$$k_{t+1}^F = i_t^F + (1 - \delta)k_t^F$$

$$k_{t+1}^I = i_t^I + (1 - \delta)k_t^I$$

$$(c_t - (n_t)^\omega)^{-\sigma} [a(c_t^F)^\rho + (1 - a)(c_t^I)^\rho]^{1/\rho-1} a(c_t^F)^{\rho-1} = \lambda_t$$

$$p_t = \left(\frac{1-a}{a}\right) \left(\frac{c_t^F}{c_t^I}\right)^{1-\rho}$$

$$(c_t - (n_t)^\omega)^{-\sigma} \omega(n_t^F + n_t^I)^{\omega-1} = \lambda_t(1 - \tau^y)w_t$$

$$(c_t - (n_t)^\omega)^{-\sigma} \omega(n_t^F + n_t^I)^{\omega-1} = p_t \lambda_t (1 - \alpha_I) z_t^I (k_t^I)^{\alpha_I} (n_t^I)^{-\alpha_I}$$

$$\lambda_t q_t = \beta E_t(\lambda_{t+1})$$

$$\lambda_t = \beta E_t\{\lambda_{t+1}[(1 - \tau^y)r_{t+1} + 1 - \delta]\}$$

$$\lambda_t p_t = \beta E_t\{p_{t+1} \lambda_{t+1} [\alpha_I z_{t+1}^I (k_{t+1}^I)^{\alpha_I-1} (k_{t+1}^I)^{1-\alpha_I} + 1 - \delta]\}$$

$$y_t^F = z_t^F (k_t^F)^{\alpha_F} (n_t^F)^{1-\alpha_F}$$

$$\alpha_F z_t^F (k_t^F)^{\alpha_F-1} (n_t^F)^{1-\alpha_F} = (1 + \tau^r)r_t$$

$$(1 - \alpha_F)z_t^F (k_t^F)^{\alpha_F} (n_t^F)^{-\alpha_F} = (1 + \tau^r)w_t$$

$$\tau^w w_t n_t^F + \tau^r r_t k_t^F + \tau^y (w_t n_t^F + r_t k_t^F) = g_t$$

$$\frac{1}{q_t} = R^* + \psi[e^{d_{t+1} - \bar{d}} - 1]$$

$$y_t^I = c_t^I + i_t^I$$

$$tb_t = \frac{y_t^F - c_t^F - i_t^F - g_t}{y_t^F}$$

$$y_t = y_t^F + p_t y_t^I$$

The competitive equilibrium, as defined by Definition 1, is a set of policy functions for the described endogenous variables such that given prices, p_t , r_t and w_t , and the stochastic process for z_t^F , it solves the system of equations above and the no-Ponzi constraint described by Equation 9.

A.3.2 Non-stochastic steady state in a model without Law enforcement

I drop the time subscript and the expectations operator from all variables to find the non-stochastic steady state in the standard model. Since solving this non-linear system of equations requires the use of standard numerical methods, I collapse the system of equations into a 16-variable system (which is the minimum number of variable needed to solve the whole system) such that its solution is computationally more efficient. The 16 variables to solve in the non-stochastic steady state are: c^F , c^I , n^F , n^I , d , k^F , k^I , p , r , w , q , g , i^F , i^I , y^F , y^I .

The system of 16 equations is the following:

$$(1 - \tau^y)(wn^F + rk^F) + py^I + g + (q - 1)d = c^F + pc^I + i^F + pi^I$$

$$y^I = z^I(k^I)^{\alpha_I}(n^I)^{1-\alpha_I}$$

$$k^F = \delta k^F$$

$$k^I = \delta k^I$$

$$p = (\frac{1-a}{a})(\frac{c^F}{c^I})^{1-\rho}$$

$$\omega(n^F + n^I)^{\omega-1} = (c)^{1-\rho}a(c^F)^{\rho-1}(1 - \tau^y)w$$

$$\omega(n^F + n^I)^{\omega-1} = (c)^{1-\rho}(1-a)(c^I)^{\rho-1}(1 - \tau^y)(1 - \alpha_I)z^I(k^I)^{\alpha_I}(n^I)^{-\alpha_I}$$

$$q = \beta$$

$$1 = \beta[(1 - \tau^y)r + 1 - \delta]$$

$$1 = \beta[\alpha_I z^I(k^I)^{\alpha_I-1}(n^I)^{1-\alpha_I} + 1 - \delta]$$

$$y^F = z^F(k^F)^{\alpha_F}(n^F)^{1-\alpha_F}$$

$$\alpha_F z^F (k^F)^{\alpha_F - 1} (n^F)^{1 - \alpha_F} = (1 + \tau^r) r$$

$$(1 - \alpha_F) z^F (k^F)^{\alpha_F} (n^F)^{-\alpha_F} = (1 + \tau^w) w$$

$$\tau^w w n^F + \tau^r r k^F + \tau^y (w n^F + r k^F) = g$$

$$d = \bar{d}$$

$$y^I = c^I + i^I$$

With this system of equations we can directly obtain the non-stochastic steady state values by simply plugging the values found here in the rest of the non-stochastic steady state equations of the equilibrium system described in the previous section.

A.4 Equilibrium and non-stochastic steady state in the Baseline model

A.4.1 Equilibrium equations - Baseline model

Recall that $H(k_t^I, g_t) = 1 - e^{\gamma(k_t^I + g_t)}$. By using this functional form directly in the model, the Baseline model has the same 21 endogenous variables (state and controls) than the model without Law enforcement: $c_t^F, c_t^I, c_t, n_t^F, n_t^I, n_t, d_{t+1}, k_{t+1}^F, k_{t+1}^I, p_t, r_t, w_t, q_t, g_t, i_t^F, i_t^I, y_t^F, y_t^I, y_t, tb_t, \lambda_t$. However, even though the *c.d.f* of Law enforcement is exogenous, the probability of being caught and fined operating informally is endogenous; so whenever we want to find this probability, we consider $H(k_t^I, g_t)$ as endogenous and add the equation of its functional form to find it. The 21-variable system of equilibrium equations is then:

$$c_t = [a(c_t^F)^{1/\rho} + (1-a)(c_t^I)^{1/\rho}]^\rho$$

$$n_t = n_t^F + n_t^I$$

$$(1 - \tau^y)(w_t n_t^F + r_t k_t^F) + [1 - \tau^y \phi(1 - e^{\gamma(k_t^F + g_t)})] p_t y_t^I + g_t + q_t d_{t+1} = d_t + c_t^F + p_t c_t^I + i_t^F + p_t i_t^I$$

$$y_t^I = z_t^I (k_t^I)^{\alpha_I} (n_t^I)^{1-\alpha_I}$$

$$k_{t+1}^F = i_t^F + (1 - \delta) k_t^F$$

$$k_{t+1}^I = i_t^I + (1 - \delta) k_t^I$$

$$(c_t - (n_t)^\omega)^{-\sigma} [a(c_t^F)^\rho + (1-a)(c_t^I)^\rho]^{1/\rho-1} a(c_t^F)^{\rho-1} = \lambda_t$$

$$p_t = \left(\frac{1-a}{a}\right) \left(\frac{c_t^F}{c_t^I}\right)^{1-\rho}$$

$$(c_t - (n_t)^\omega)^{-\sigma} \omega (n_t^F + n_t^I)^{\omega-1} = \lambda_t (1 - \tau^y) w_t$$

$$(c_t - (n_t)^\omega)^{-\sigma} \omega (n_t^F + n_t^I)^{\omega-1} = p_t \lambda_t [1 - \tau^y \phi(1 - e^{-\gamma(k_t^I + g_t)})] (1 - \alpha_I) z_t^I (k_t^I)^{\alpha_I} (n_t^I)^{-\alpha_I}$$

$$\lambda_t q_t = \beta E_t(\lambda_{t+1})$$

$$\lambda_t = \beta E_t\{\lambda_{t+1}[(1 - \tau^y)r_{t+1} + 1 - \delta]\}$$

$$\begin{aligned} p_t \lambda_t = \beta E_t\{p_{t+1} \lambda_{t+1} [1 - \tau^y \phi(1 - e^{-\gamma(k_{t+1}^I + g_{t+1})})] \alpha_I z_{t+1}^I (k_{t+1}^I)^{\alpha_I - 1} (h_{t+1}^I)^{1 - \alpha_I} \\ - z_{t+1}^I (k_{t+1}^I)^{\alpha_I} (h_{t+1}^I)^{1 - \alpha_I} \gamma e^{-\gamma(k_{t+1}^I + g_{t+1})} + 1 - \delta\} \end{aligned}$$

$$y_t^F = z_t^F (k_t^F)^{\alpha_F} (n_t^F)^{1 - \alpha_F}$$

$$\alpha_F z_t^F (k_t^F)^{\alpha_F - 1} (n_t^F)^{1 - \alpha_F} = (1 + \tau^r) r_t$$

$$(1 - \alpha_F) z_t^F (k_t^F)^{\alpha_F} (n_t^F)^{-\alpha_F} = (1 + \tau^r) w_t$$

$$\tau^w w_t n_t^F + \tau^r r_t k_t^F + \tau^y (w_t n_t^F + r_t k_t^F) + \tau^y \phi(1 - e^{-\gamma(k_t^I + g_t)}) p_t y_t^I = g_t$$

$$\frac{1}{q_t} = R^* + \psi[e^{d_{t+1} - \bar{d}} - 1]$$

$$y_t^I = c_t^I + i_t^I + \tau^y \phi(1 - e^{-\gamma(k_t^I + g_t)}) y_t^I$$

$$tb_t = \frac{y_t^F - c_t^F - i_t^F - g_t}{y_t^F}$$

$$y_t = y_t^F + p_t y_t^I$$

The competitive equilibrium in the Baseline model follows Definition 1.

A.4.2 Non-stochastic steady state - Baseline model

Again, for the non-stochastic steady state I drop the time subscripts and the expectations operators, and collapse the system to the following 16 endogenous variables: c^F , c^I , n^F , n^I , d , k^F , k^I , p , r , w , q , g , i^F , i^I , y^F , y^I . The system of equations that I solve using numerical methods is:

$$(wn^F + rk^F)(1 - \tau^y) + [1 - \tau^y \phi(1 - e^{-\gamma(k^I + g)})]py + g + (q - 1)d = c^F + pc^I + i^F + pi^I$$

$$y^I = z^I (k^I)^{\alpha_I} (n^I)^{1-\alpha_I}$$

$$k^F = \delta k^F$$

$$k^I = \delta k^I$$

$$p = \left(\frac{1-a}{a}\right) \left(\frac{c^F}{c^I}\right)^{1-\rho}$$

$$\omega(n^F + n^I)^{\omega-1} = (c)^{1-\rho} a (c^F)^{\rho-1} (1 - \tau^y) w$$

$$\omega(n^F + n^I)^{\omega-1} = (c)^{1-\rho}(1-a)(c^I)^{\rho-1}[1 - \tau^y \phi(1 - e^{-\gamma(k^I+g)})](1 - \alpha_I)z^I(k^I)^{\alpha_I}(n^I)^{-\alpha_I}$$

$$q = \beta$$

$$1 = \beta[(1 - \tau^y)r + 1 - \delta]$$

$$1 = \beta\{[1-\tau^y\phi(1-e^{-\gamma(k^I+g)})]\alpha_I z^I(k^I)^{\alpha_I-1}(n^I)^{1-\alpha_I}-\tau^y\phi\gamma(1-e^{-\gamma(k^I+g)})z^I(k^I)^{\alpha_I}(n^I)^{1-\alpha_I}+1-\delta\}$$

$$y^F = z^F(k^F)^{\alpha_F}(n^F)^{1-\alpha_F}$$

$$\alpha_F z^F(k^F)^{\alpha_F-1}(n^F)^{1-\alpha_F} = (1 + \tau^r)r$$

$$(1 - \alpha_F)z^F(k^F)^{\alpha_F}(n^F)^{-\alpha_F} = (1 + \tau^w)w$$

$$\tau^w w n^F + \tau^r r k^F + \tau^y (w n^F + r k^F) + \tau^y \phi(1 - e^{-\gamma(k^I+g)}) p y^I = g$$

$$d=\bar{d}$$

$$y^I = c^I + i^I + \tau^y \phi(1 - e^{-\gamma(k^I+g)})y^I$$

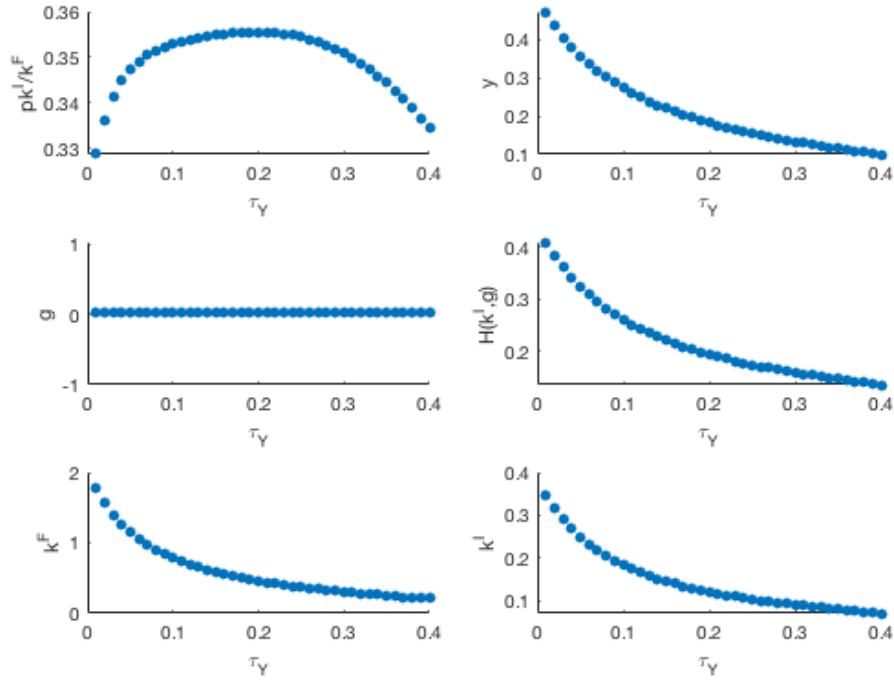
Once this system is solved, the non-stochastic steady state solution for the rest of the variables can be obtained by plugging directly into the deterministic steady-state version of the rest of the equations shown in the previous section.

A.5 Changes in τ^Y when g is constant

To understand better the role of endogenous government expenditure in affecting Law enforcement and the size of the informal sector, here I assume that $g = \bar{g}$. Figure 9 shows the result of changes in τ^Y . The structure of the figure is the same as in Figure 5. Now, the relationship between the income tax rate and the size of the informal sector inverts, and it is positive for the plausible range of taxes according to the data, which is counterfactual. The reason why this happens is that whenever the tax increases, revenue is not increasing, meaning that now government expenditure does not act as a break that curbs the reduction in Law enforcement due to lower informal capital stock.

Table 4 quantifies the differences between the results in Figure 5 and Figure 9. When g is endogenous, for the range of income tax rates between 0.01 and 0.15, the table shows in the second column that the fall in informal capital stock is larger than the fall in formal capital stock (56.07% vs 51.96%). This yields a fall in the relative size of the informal sector of 8.55%. This happens because, even though Law enforcement decreases 39% due to lower informal capital stock, g increases 51%, curbing the reduction in H . For the range above the tax rate that maximizes revenue (third column), 0.15, government expenditure falls and formal capital stock now falls more than informal capital stock, which reverts the relationship between income tax rates and informality to the one in the standard view. On the other hand, when g is exogenous, government expenditure does not change, thus the fall in H shown in the fourth column for the range of τ^Y between 0.01 and 0.15 is larger than when g is endogenous. Therefore, the decrease in informal capital stock is now lower than in the formal, which increases the relative size of the informal sector. After a tax rate of 0.15, the relation reverts.

Figure 9: Changes in τ^y with constant g



Notes: The figure shows the steady state values of different key variables for different tax rates on income, τ^Y , shown in the horizontal axis. The first row shows respectively the relative size of the informal sector in terms of capital stock, and GDP. The second figure shows respectively government's expenditure (and revenue), and Law enforcement. The last row shows respectively formal and informal capital stocks.

Table 4: The effects of changes in τ^Y with endogenous and exogenous g

Change in	Endogenous g		Constant g	
τ^Y	0.01-0.15	0.15-0.4	0.01-0.15	0.15-0.4
k^I	-56.07%	-72%	-64.42%	-67%
k^F	-51.96%	-77%	-67.05%	-65%
pk^I/k^F	-8.55%	22%	7.98%	-6%
H	-39%	-49%	-46%	-39%
g	51%	-36%	0%	0%

The table shows for each variable from the second to last row in the first column, what is the percentage change of its level between the two level of taxes displayed in the second row. Columns two and three show the case of endogenous government expenditure, and columns four and five show the case for constant government expenditure.