Choosing and Adjusting

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Abstract

The paper develops a model of individual decision-making in which cognitive adjustment complements choice of action. Because adjustment is durable, initial positions in consumption ("endowments") matter for future consumption decisions. Thus the theory provides a unified foundation in primitives for the endowment effect, behaviors traditionally attributed to sunk-cost bias, and the effectiveness of marketing practices. Demonstrations of cognitive dissonance in the literature fit a pattern of adjustment effects predicted in the model based on restriction of the individual's choice set. Natural experiments that similarly leverage choice-restricting events could allow identification of endowment effects and other adjustment phenomena in the market.

Keywords individual decision-making; endowment effect; sunk cost fallacy; cognitive dissonance; endogenous preference.

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1 Introduction

When we humans act, we tend to adjust to our actions. We buy a home, choose a spouse, or decide to take a position on a political issue. We then – or sometimes in anticipation – get "psyched up," rehearse the best qualities of our selected course, get acclimated to the inevitability of what we are doing, and rationalize.

Adjustment efforts take varying forms. They may be specific and top-of-mind, or broad-based and ambient. A consumer may actively rationalize the additional expense associated with an all-electric vehicle shortly after purchasing a new Tesla. Meanwhile, over several months and almost without being aware of it, the same individual may find he is "growing into" being a Tesla owner, becoming more accustomed to and accepting of the car's various features and thus enjoying them more. Adjustment may occur as an instantaneous and almost imperceptible process, as when the purchaser of a roll-on quickly assembles an argument for choosing deodorant rather than anti-perspirant. Or it may be extended and obvious to all, as in the case of marital engagement. The desire to adjust optimally to an action may motivate people to seek resources external to themselves, such as friends' advice, information on the Internet, or persuasive images in television commercials. Whether or not they seek external inputs to aid adjustment, individuals invest scarce resoures of attention and energy in the process. And while even the smallest purchases engender a modicum of supportive thinking, the bigger the commitment one makes, the harder one endeavors to learn to love it.[1]

There is a substantial amount of experimental evidence on the complementarity of actions and cognitive processes that alter perceptions of actions. Individuals asked to re-rate alternatives following a decision or in anticipation of one increase their ratings of chosen alternatives and in some cases diminish ratings of non-chosen alternatives (Lieberman *et al.* 2001, Kitayama *et al.* 2004, Sharot *et al.* 2010, Wakslak 2012). Studies using functional magnetic resonance imaging (fMRI) indicate changes in preference-related brain activity contemporaneous with the changes in subjects' subjective rating of stimuli accompanying decisions or actions (Sharot *et al.* 2009, Van Veen *et al.* 2009, Izuma *et al.* 2010, Jarcho *et al.* 2011, Qin *et al.* 2011, Kitayama *et al.* 2013, Tompson *et al.* 2016).

Festinger's (1962) theory of cognitive dissonance explains some of these phenomena conceptually in terms of individuals preferring their actions to be aligned with their beliefs; when they are not aligned,

[1] Adjustment, as I have defined it, is distinct from search, which has the objective of identifying one's best choice. The purpose of adjustment is to increase utility from a choice one has already determined – or is, in parallel, determining – to be one's best.

the theory contends, people may become uncomfortable and so alter their beliefs to restore a sense of comfort. The tri-component model of attitude also reflects the notion that action moves hand-in-hand with adaptive changes in beliefs and feelings (see, e.g., Grimm 2005, Chih *et al.* 2015). This model has been applied extensively to explain consumer behavior and, as such, has formed the basis for a substantial amount of marketing strategy.

Despite the evidence that people adjust to their actions and its general acceptance by psychologists and marketers, the role of adjustment in decision-making has received little attention from economists. This likely stems from the fact that adjustment nominally involves a change in tastes. Most economists are reluctant to consider changing tastes because they tend to upend traditional approaches to identifying preferences and measuring welfare. Yet the idea that choices not only reflect, but also create, preferences continues to gain currency in the social sciences (Ariely & Norton 2008, Acharya *et al.* 2016).

In this paper I offer a theory of individual decision-making in which adjustment complements choice. I circumvent some of the thornier issues associated with taste change by modeling a consumer who obtains utility from an adjustment-augmented commodity. In my framework, adjustive thinking quasi-changes preferences in the sense of increasing the consumer's marginal utility for the product; but in the context in which the consumer operates - rationally choosing both a quantity of the product and quantity of adjustment as complementary inputs to utility – tastes may be said to be fixed. An exogenous parameter representing product quality (or, alternatively, consumer-specific taste for the product) affects the marginal utilities of both the product and adjustment. While product consumption only affects utility contemporaneously, adjustive thinking creates a durable stock of product-specific attitude that affects the utility of future consumption.[2]

People do not always adjust simply to enhance their current and future consumption; sometimes they do so to justify a past decision to themselves. For example, subjects in forced compliance experiments change their beliefs to feel better about actions they took that were incongruent with their beliefs or values at the time (Festinger & Carlsmith 1959). To capture such motivations, I allow regret to play a role in the agent's objective function. To give meaning, in turn, to the notion of regret, I introduce endowments – exogenous actions occurring in the initial period of the model prior to the realization of quality/taste. Endowments can represent choices the individual made under uncertainty or subject

[2]The manner in which thinking in the model creates an enduring stock of adjustment is similar to the mechanism in Becker's (1965, 1985) model in which certain present actions create enduring stocks that are complementary with future actions. Becker's complementary stocks explain diverse behaviors that appear prima facie to represent changing tastes, including addiction and effects stemming from the accumulation of human capital (Stigler & Becker 1977, Becker & Murphy 1988). However, Becker's theory does not admit cognition as a complement to action and so fails to recognize the role played by adjustment in the utility obtained from consumption activities.

to a constraint; they may in the extreme represent actions forced on the individual. Such actions are suboptimal given the realization of quality, or they may appear so in hindsight. The regret-driven individual attempts to use adjustment to directly rationalize the "endowed" decision, in essence rendering it post-hoc optimal.

The model provides a unified foundation in primitives for a number of phenomena that have previously been treated separately. First, it demonstrates that endowments increase consumption of the same good at the margin in future periods – an endowment effect – because they drag adjustment with them. This finding offers an account of the endowment effect more robustly consistent with the evidence than existing explanations (e.g., loss aversion, ownership effects). The effect is unambiguously increased by regret, because the need to justify one's prior action quite generally implicates greater adjustment. Second, the model recognizes escalation of commitment as a pattern of repeated action driven by ongoing regret. The persistently regretful agent harks back continually to his initial "error," seeking again and again to justify it. His rationalization then drives repeated action of the same kind taken initially. The model's predictions in this regard are consistent with experimental evidence that regret fosters repeat purchase behavior by consumers (Mittelstaedt 1969) and more broadly fit with descriptive accounts of escalation of commitment from the literature (e.g., Staw 1976). Third, the model shows that many demonstrations of cognitive dissonance in the literature fit a predictable pattern of adjustment effects arising from restriction in the individual's choice set for agents who are motivated by regret. This recognition paves the way for new public policy approaches to dissonance-related problems. It also reveals in the design of cognitive dissonance experiments a framework for identifying the broader set of adjustment phenomena - including most notably endowment effects - in the market using natural experiments. Fourth, the model provides a robust rational-agent explanation for a range of marketing practices. In particular it offers an understanding of why advertising, price promotions, and free introductory offers are observed *even when they do not serve to increase the consumer's information about a product.*

In a paper relevant to the present effort, Eyster (2002) analyzes sunk-cost effects using a dynamic decision-making framework in which an agent acts in the present to justify past actions. His model is able to explain scenarios such as Thaler's (1980) classic example of a family that decides to go to a basketball game during a snowstorm, the family noting that they would not have gone had they received the tickets for free rather than purchasing them. Strategic complementarity of the initial action (deciding to purchase the tickets) and a subsequent action that justifies the initial one (attending the game) propels observed sunk-cost behavior.

The adjustment-to-choice model offers an advance over Eyster's approach in cases where a cognitive layer is essential to understanding the mechanism of post hoc justification. To demonstrate, consider the scenario, described by Akerlof & Dickens (1982), in which individuals face a dissonance-producing decision of whether to work in a hazardous industry and, subsequently, are given the opportunity to purchase safety equipment. Based purely on the complementarity of actions, one would expect the workers to purchase the safety equipment in some cases when the benefits to improved safety do not exceed equipment costs: the adoption of the equipment renders more prudent in retrospect one's decision to work in the industry. Yet, consistent with the anecdotal evidence on safety-related behavior in a range of situations (e.g., motorcycle helmet adoption, hockey headgear use, AIDS testing), workers in such situations typically *avoid* the equipment even when its isolated net benefit is positive. The reason, as described in the discussion on cognitive dissonance in Section 4 of this paper, is that such scenarios involve a discontinuous choice set that induces inconsistency with continuously variable beliefs; the resultant regret precipitates compensatory adjustment that reduces adoption of future behaviors that might have been rational *but for* the adjustment. The key mathematical result from the model associated with this behavior is that regret reverses the sign of the marginal effect of product quality on adjustment.

In conceiving of an individual who curates his thoughts to minimize his regrets from a past action, the adjustment theory naturally relates to the concept of motivated reasoning (see Epley & Gilovich 2016 for a survey of the literature). The idea behind motivated reasoning is that certain beliefs are desirable and will be held when it is possible for the individual to hold them rationally. The literature's focus is on what forms of reasoning are rationally tenable and under what circumstances; tenability and desirability then jointly govern whether or not the reasoning occurs. Adjustment for the purposes of minimizing regret may be contextualized as a very specific form of motivated reasoning: the use of complementary thinking to rationalize a previously-chosen intensity of an action. Given this scope for adjustment, I assume that tenable rationalizations can always be found – simply that it must be costly to invent them – whereby the key question becomes not whether one adjusts in a given circumstance, but how much. This proves relevant to determining how much ongoing action will occur - a focal question of the present paper.

The rest of this paper is structured as follows. Section 2 lays out a portable model of individual decision-making involving adjustment to choice. Section 3 introduces endowments and regret. Section 4 applies the model to explain the four core phenomena highlighted above: the endowment effect, escalation of commitment, cognitive dissonance reactions, and marketing practices. Section 5 concludes by discussing alternative assumptions and some possibilities for future work. The Appendix contains proofs of all results.

2 Adjustment to Choice

Consider an individual who consumes over an infinite series of periods, indexed t, extending out from an initial period $t = 0$. He must make two decisions each period: how much $x_t \geq 0$ to consume of a good x, and how much "adjustive" thinking $T_t \geq 0$ to engage in in support of that good.[3] Let us initially assume that the thinking decision and the consumption action decision at t are made simultaneously. The good x is not durable: the consumption decision must be renewed each period and the history of previous consumption does not matter directly for current utility. However, thinking creates a durable stock of adjustment, y_t, according to the process

$$\begin{aligned} y_t &= (1-\sigma)\, y_{t-1} + T_t,\ t > 0 \\ y_0 &= T_0 \end{aligned} \tag{1}$$

in which $\sigma \in (0, 1)$. This stock of adjustment is, in turn, complementary with the contemporaneous consumption of x.

I formalize the utility of complementary consumption and adjustment using the household production function approach proposed by Becker (1965) to model the allocation of time. Let instantaneous utility u be a function of a "commodity" z the individual consumes and that is, in turn, "produced" using inputs x and y, to wit, $u(z)$ where $z = z(x, y)$. I assume $u(.)$ and $z(.)$ satisfy the following properties:

(A1) The production function $z(.)$ exhibits diminishing returns to x and y and constant returns to scale; that is, $z_x, z_y > 0$, $z_{xx}, z_{yy} < 0$, with z homogeneous of degree one in x and y.

(A2) Marginal utility is positive and diminishing in z: $u_z > 0$, $u_{zz} < 0$.

(A3) The elasticity of z_x with respect to y is larger in absolute value than the elasticity of u_z with respect to y, and the elasticity of z_y with respect to x is larger in absolute value than

[3] More broadly, good x might be an activity that could be engaged in at varying levels of intensity.

the elasticity of u_z with respect to x; that is, $u_z z_{xy} > -u_{zz} z_x z_y$.[4] This property, when combined with (A1) and (A2), guarantees that consumption and adjustment are mutually complementary in utility, not just in z.

This specification has the intuitive characteristic that the value of adjustment is related to the consumption value attached to the goods in question, so that the first unit of adjustment effort expended on a car would have greater payoff, say, than the first unit of such effort invested in a can of anti-perspirant. Homogeneity of the production function ensures that consumption of and adjustment to the good have the same complementary relationship at the margin when both are scaled up proportionally. Diminishing marginal returns to z ensure that, even as consumption and corresponding adjustment are scaled up proportionally, their combination contributes less and less to utility. For convenience, I will at times express instantaneous consumption utility as the derived function of the quantities of instantaneous activity and thinking, $u(x,T) \equiv u(z(x,y(T)))$.

The consumption problem occurs in the context of a broader economy in which there are many goods and activities in which money and effort can be invested. To focus the analysis, I assume the consumer possesses an invariable, finite supply of effort, $K > 0$, each period. This effort may be allocated to adjust to good x; or to earn labor income, paid in a numeraire commodity, that may be spent on x or on other consumption activities. The price of x, supplied by a competitive industry, is normalized to one. Each unit of numeraire not spent on x garners one unit of utility (via the other consumption activities).

At time $t = \tau$, the consumer's preferences are represented by the quasilinear discounted utility function

$$U(x_\tau, x_{\tau+1}, ...; T_\tau, T_{\tau+1}, ...) = \tag{2}$$
$$\mu u(x_\tau, T_\tau) + K - x_\tau - T_\tau + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau} \left[\mu u(x_t, T_t) + K - x_t - T_t\right]$$

where $\mu > 0$ is the good or activity's quality level and $\delta \in (0,1)$ is the consumer's discount rate. The consumer's problem in $t = \tau$ is therefore

$$\max_{T_\tau, x_\tau} (2) \tag{3}$$

Quality is an intrinsic characteristic of the good and does not vary over time. The variable μ might,

[4] One example of a utility function that satisfies (A1)-(A3) is the extended Cobb-Douglas formulation, $u(z) = z^d$, for $d \in (0,1)$; with $z(x,y) = x^a y^b$, for $a, b \in \left[\frac{1}{3-d}, 1\right)$, $a + b = 1$. See Appendix.

alternatively, represent the consumer's exogenous taste for, or attitude toward, the good (i.e., in the traditional "fixed" sense), whereby μ is intrinsic jointly to the consumer and the good. This latter interpretation conceives of a consumer whose attitude comprises fixed and discretionary components. The consumer begins in the initial period $t = 0$ with only his fixed component set; whereupon, according to the process in (1), he begins to invest in the discretionary component, adjusting to the activity that she is about to commence.

The consumption process arising out of this complementary utility model may be thought of as similar to driving a car on a cold day. One can turn the car on and drive it productively without warming up the engine, taking the "fixed" components of the car as given and obtaining utility from those. But one might get better results - better contemporaneous acceleration, as well as longer life for the engine - if one invests in warming the car up first. The model allows for two extremes with respect to the ongoing process of adjusting to consumption. With $\sigma = 0$, adjustment does not depreciate at all: once the consumer has finished his initial adjustment to an activity, she is "adjusted," and she does not have to do it again. With $\sigma = 1$, adjustment depreciates completely, whence fresh adjustment is required every period. For the values of σ in between, the setup conceives of a consumption activity that requires some ongoing discretionary cognitive effort in order for the consumer to remain primed, or "psyched up." (Marriage provides a good example: you have to invest in it every day if you want optimal results!)

In intertemporal models, it is a common feature of the literature for the consumer to be modeled as a sequence of temporal selves who make choices in a dynamic game with one another (e.g., Pollak 1968, Peleg & Yaari 1973, Goldman 1980, and Laibson 1997). This approach lends itself naturally to modeling dynamically inconsistent preferences whereby, for example, the consumer at t would disagree with the tradeoff decision between consumption in $t+1$ and $t+2$ that a consumer at $t+1$ would make. The preferences given by (2) are not dynamically inconsistent; however, I will use a modified objective that does exhibit dynamically inconsistent preferences when I consider regret in section 3. Therefore I adopt here, and carry forward through subsequent analysis, the approach of modeling the infinite-period consumption problem as an infinite game, with an infinite number of players, or "selves," indexed based on their respective periods of control over the consumption and thinking decisions (Laibson 1997). I seek subgame perfect equilibrium (SPE) strategies of this game. In that context, I will let S_t represent the set of feasible strategies in the game for self t, where a strategy represents a move (x_t, T_t), while $S = \prod_{t=0}^{\infty} S_t$ represents the joint strategy space for all selves.

The following is the main technical result of the core model:

Theorem 1. *The infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve* (2) *has a unique subgame perfect equilibrium strategy, $s^*(\mu) \in S$, characterized by: (i) a steady-state $(x^*(\mu), y^*(\mu))$ such that $(x_0, T_0) = (x^*(\mu), y^*(\mu))$ and $(x_t, T_t) = (x^*(\mu), \sigma y^*(\mu))$ for all $t > 0$; and (ii) $x^*_\mu, y^*_\mu > 0$.*

Theorem 1 confirms that there is a unique equilibrium path for a discounted-future-utility-maximizing consumer who simultaneously chooses action and complementary adjustive thinking every period starting from the very first period. In a world in which product quality is known from the outset, the unique consumption and adjustment path is a steady state in which both depend positively on the level of quality. The consumer engages in adjustive thinking each period at the intensity level needed to maintain steady-state adjustment given its depreciation rate, σ.

3 Endowments

Now consider a tweak to the setup: suppose x_0 is fixed exogenously at $\bar{x}_0 > 0$ rather than being set by the consumer contemporaneously with adjustive thinking at $t = 0$. This might represent a number of things. The consumer might have committed to a level of consumption before knowing the good's quality level (or his taste for it). Subsequently, quality is revealed, placing him in the position of deciding how much adjustment to do given his prior consumption commitment and the quality level. Or the consumer might be exogenously endowed with a level of consumption, having received the good as a gift or bequest. Or the individual might somehow have been compelled to take an action independent of his tastes or beliefs. In all these cases, the level $\bar{x}_0$ presents itself at the start of the game before anything else is determined - an existing "fact" to which the consumer must adapt his adjustment level.

For the case of endowments I offer the following revised equilibrium existence result:

Lemma 1. *The infinite consumption game in which the consumer at each $t > 0$ chooses (x_t, T_t) to solve* (2) *and the consumer at $t = 0$ chooses T_0 given x_0 fixed at $\bar{x}_0$ to solve* (2) *has a unique subgame perfect equilibrium strategy, $s^*(\bar{x}_0, \mu) \in S$.*

3.1 Equilibrium with Regret

In general, the level of endowed actions will not be consistent with the consumer's realized tastes or the realization of quality and so not on the optimizing steady-state path. That is, $\bar{x}_0 \neq x^*(\mu)$. This gives rise to the possibility that the consumer will feel he erred in having taken the action at its endowed level and will regret his error. He may see himself as responsible and may question his own judgment. Where he is clearly not responsible – having merely acted in a way that was chosen for him by someone else – he may experience incongruence between his values and his action. In both cases, the experience is unpleasant. The consumer acts to minimize such displeasure.

One may think of the experience of regret as reframing the consumer's problem. A non-regret-driven consumer chooses and adjusts in order to make the most of his present and future consumption opportunities. A regret-driven consumer instead uses his power to choose and adjust in the present in order to minimize the discrepancy between the path he believes he should have chosen in the past and the one he actually chose. It is possible in general that a consumer may embody a little bit of both the forward-looking and the backward-looking individual, whence his decision process takes a hybrid form. The extent to which he acts in a forward-looking or backward-looking way could vary over time, following his moods, pressures imposed by his social environment, or other relevant aspects of his situation.

To fix ideas, let us assume the consumer's objective at $t = \tau$ takes the following general form:

$$\max_{T_\tau, x_\tau | \bar{x}_0} U^R(x_\tau, T_\tau; \beta_\tau) = (1 - \omega\beta_\tau)\left\{\mu u(x_\tau, T_\tau) + K - x_\tau - T_\tau + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau}\left[\mu u(x_t, T_t) + K - x_t - T_t\right]\right\}$$
$$+ \omega\beta_\tau \left\{\left[\mu u(\bar{x}_0, T_\tau) + K - \bar{x}_0 - T_\tau\right] - \left[\mu u(x_0^*(T_\tau), T_\tau) + K - x_0^*(T_\tau) - T_\tau\right]\right\} \quad (4)$$

This function weights the quasilinear discounted future utility function in (2) with a second term. That second term in effect revisits the consumption decision from $t = 0$ in hindsight. It is a strictly non-positive loss function that is minimized, at a value of zero, when the endowed level of consumption turns out to have been the optimizing level *given adjustment* at $t = \tau$. The consumer endeavors through this term, in effect, to manipulate the instrument that he currently has available ($t = \tau$ adjustment) to render his earlier decision ($t = 0$ consumption) post-hoc optimal, acting *as if* through current adjustment he could turn back the clock and somehow make better his prior consumption decision.

Here, $\omega \in [0, 1)$ parameterizes the degree of regret that the consumer experiences overall, while $\beta_\tau \in [0, 1)$ reflects the degree of regret experienced specifically at $t = \tau$. I will employ the notation

$\beta = (\beta_0, \ldots, \beta_t, \ldots)$ for the infinite vector of date-specific regrets. This form anticipates a consumer who not only experiences immediate displeasure (i.e., in $t = 0$) from having erred, but who continues to experience *lingering* displeasure in later periods and to experience a lingering desire to reduce it through adjustment. It allows for maximum generality in specifying how regret lingers over time, allowing the model to represent, for example, a person who feels substantial regret for his decision one period, a lot less the next, and then a subsequent resurgence in later periods. Non-lingering regret is nested as the special case $\beta_\tau = 0 \; \forall \tau > 0$; and other interesting special cases such as a simple decay path (i.e., $\beta_\tau = \beta^\tau$) could also be represented.

I assume the consumer does not experience meta-regret, that is, regret over experiencing regret. For this reason, because the endowment does not affect utility directly except for the current period, the regret component does not include any terms relating to future periods. Moreover, I assume as a baseline that the consumer conjectures naïvely that his future self will not be affected by regret. There are a number of assumptions one could make about future regret conjectures (see, e.g., Eyster 2002), including that the consumer has perfect foresight about the degree of regret he will experience in every future period. I discuss relaxing the naïve consumer assumption in section 5.

The preferences represented by (4) are, in general, dynamically inconsistent. This occurs because the regret-driven part of the consumer cares only about minimizing regret in the current period (say, t) and not about consumption in any future period. This influences the way the overall consumer evaluates the tradeoff between $t + 1$ and $t + 2$. Because the degree of regret generally varies over time, when $t + 1$ arrives, the tradeoff between $t + 1$ and $t + 2$ is viewed differently by the overall consumer.[5] One may check that this characteristic of regret leads to a marginal rate of substitution between periods $t + 1$ and $t + 2$ from the perspective of the decision-maker at t that does not equal the marginal rate of substitution between those same periods from the perspective of the decision-maker at $t + 1$, consistent with the demonstration of dynamic inconsistency proposed by Laibson (1997).

The equilibrium existence result for the regret context depends on complementary restrictions on the convexity of the regret term and the degree of regret. If the regret term is quite convex, then it must be weighted lightly in the objective for the overall objective to be strictly globally concave. If the regret term is not very convex or is concave, then it may be weighted heavily in the objective. Existence is guaranteed by bounding the convexity above, whence one may obtain an upper bound on the degree of

[5] A naïve consumer experiencing any degree of regret over time - even a constant degree - will perceive his preferences to be dynamically inconsistent as he will believe his future self not to be affected by regret.

regret that guarantees global concavity of the overall objective.

Lemma 2. *Assume $u(z(x,y))$ such that, for any $\bar{x}_0 > 0$, there exists $Z(\bar{x}_0, \mu) < \infty$ where for all T_τ at all $\tau \geq 0$,*

$$\mu u_z z_{yy}(\bar{x}_0, y_\tau) + \mu u_{zz} z_y^2(\bar{x}_0, y_\tau) - \mu u_z z_{yy}(x_0^*(T_\tau), y_\tau)$$
$$-\mu u_{zz} z_y^2(x_0^*(T_\tau), y_\tau) - \mu u_z z_{xy}(x_0^*(T_\tau), y_\tau) x_T^* - \mu u_{zz} z_y z_x(x_0^*(T_\tau), y_\tau) x_T^* < Z(\bar{x}_0, \mu)$$

Then it follows there exists $\bar{\omega}(\beta) > 0$ such that for $\omega \in [0, \bar{\omega})$ the infinite consumption game in which the consumer at each $t > 0$ chooses (x_t, T_t) to solve (4) and the consumer at $t = 0$ chooses T_0 given x_0 fixed at $\bar{x_0}$ to solve (4) has a unique subgame perfect equilibrium strategy, $s^*(\bar{x_0}, \mu, \omega, \beta) \in S$.

3.2 Comparative Statics

With Theorem 1 and Lemmas 1 and 2 as technical underpinning, two comparative static results relating to adjustive thinking in the initial period set the stage for applications of the model.

Proposition 1.

1. *For a consumer who does not experience regret ($\omega = 0$), adjustive thinking in the initial period increases with quality* (complementary adjustment).
2. *With extreme regret ($\omega = 1$), adjustive thinking decreases with quality* (compensatory adjustment).
3. *More generally, $\partial^2 T_0 / \partial\mu\partial\omega < 0$; that is, intensified regret causes adjustive thinking to become increasingly compensatory.*

Proposition 1 establishes that regret reverses the direction of the effect of quality on adjustment in the initial period. The result is intuitive. A higher quality good yields greater benefits to incremental adjustment in consumption utility. Thus, quality "surprises" excite the forward-looking individual, motivating him to invest more in adjusting to the unexpectedly desirable good. I refer to such use of adjustive thinking as *complementary adjustment.* On the other hand, for an individual who is only concerned about minimizing regret, unexpected quality is perceived as a threat. It suggests, viewed in relation to an endowment, that he made a judgmental error and *chose too little* of the good. The regret-afflicted individual is motivated only to reduce that unpleasant perception and to offset the surprise, which he achieves by adjusting *less* when quality is higher. The offset is achieved in that his reduced adjustment lowers in retrospect the corresponding optimal level of consumption to bring it in line with the consumption quantity he previously chose. I refer to this use of adjustive thinking as *compensatory adjustment*. For cases along the continuum between the extremes of pure regret and pure discounted consumption-utility maximization, a greater weight on regret in the objective implies an increase in the compensatory motive.

The second comparative static result concerns the endowment's effect on initial period adjustment.

Proposition 2. *A larger endowment induces increased adjustive thinking in the initial period (i.e., $\partial T_0/\partial \bar{x}_0 > 0$). This tendency increases with the degree of regret (i.e., $\partial^2 T_0/\partial \bar{x}_0 \partial \omega > 0$).*

The more of the good the individual has, the greater the value of adjusting to it. This is true purely from the perspective of consumption utility: if you are going to do more of something, it is all the more important that you love it. Regret implies the same effect: the larger the amount of the endowed good, holding quality constant, the greater the perception that one choose too much of it. This motivates additional compensatory thinking, so that the amount of the good is justified post hoc by the amount of adjustment the individual later committed to it.

To understand why the endowment's effect on adjustive thinking is *greater* the greater one's regret, consider a simple thought experiment. Suppose, to start with, that there is an over-endowment $\bar{x}_0 > x^*$. A regret-free individual will set T_0 higher than what he would choose for x^*. Next, suppose the quality level is increased, ceteris paribus, to the level of quality that would correspond to the endowment if that amount of the good had been chosen with quality known. Based on Proposition 1, the same regret-free individual would set T_0 yet higher. Next, suppose the individual is transformed into one who experiences at least some regret, and simultaneously consider a reduction in the quality level *back* to its previous lower level. Again invoking Proposition 1, one can see that T_0 is either decreased less than it had been previously increased by raising quality, or else increased further still if the degree of regret is great enough. The net observation is that a higher endowment, all else equal, results in greater adjustive thinking the more regretful the individual.

4 Analysis

In the following subsections, I consider several applications of the adjustment-to-choice model.

4.1 The Endowment Effect

When a person who is forward-looking adjusts to his choices, a remarkable thing happens when he is endowed with a significant amount of a good. He learns to love it. If adjustment is sufficiently durable, these feelings may carry over into future periods, inducing increased consumption of the good in future periods. In particular, the larger the endowment and the more durable the adjustment, the longer the consumption effect extends.

Proposition 3. *(Endowment Effect) Let* $(x^*(\mu), y^*(\mu))$ *be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each* $t \geq 0$ *chooses* (x_t, T_t) *to solve* (2)*, and suppose* $\bar{x}_0 > x^*$*. Then: (i) the regret-free consumer chooses* $y_0 > y^*$*; and (ii) if* $\bar{x}_0$ *large enough, as defined implicitly by*

$$\mu u_z z_y \left(\bar{x}_0, \frac{y^*}{(1-\sigma)^\tau}\right) \quad > \quad \mu u_z z_y (x^*, y^*)$$

then $y_t > y^*$ *and* $x_t > x^*$ *for all* $t \leq \tau$.

The endowment effect, under the adjustment model, is a mere possession effect. Merely possessing an object motivates thinking to improve attitude toward the object in hand. Durability of the attitude is all that is needed to motivate persistent behavior going forward. Note that this has nothing to do with the honoring of sunk costs: it follows for a "rational" discounted future utility maximizer from a process by which thinking complements action.

There are two main pre-existing explanations of the endowment effect in the literature. The first, better known in economics, is that individuals have reference-dependent preferences and experience loss aversion relative to their reference point as described by prospect theory (Kahneman *et al.* 1991). That is, the loss of items internally referenced as being in one's possession is registered as more consequential than the gain of the same items not in one's possesion. In this account, reference dependence is taken as primitive, and no attempt is made to answer why parting with something in trade should be viewed as a "loss." The loss aversion account cannot explain ownership effects whereby individuals – whether buyers or sellers, and whether what they are trading is their own or not – value objects more when they own an identical item (Morewedge *et al.* 2009).

The second explanation, developed primarily in the psychological literature, is that individuals value more the things they own simply because they own them (Morewedge *et al.* 2009). In this account, the increased intrinsic valuation by individuals of items they own is taken as primitive. As such, an ownership account cannot explain why there are no endowment effects associated with money (DellaVigna 2009).

The adjustment approach supplants the primitives in both these accounts and, in doing so, sets up a new explanation more robustly consistent with the evidence. The adjustment model is consistent with observations that experienced traders do not exhibit the endowment effect (DellaVigna 2009): experienced traders are likely to treat non-personally-owned, traded items as mere commodities that they do a business in, whence there is nothing, in their eyes, to be "adjusted to."[6] Meanwhile, money, as a pure medium of exchange, does not bear being adjusted to: there is not a material object with respect to which one may, through thinking, improve one's attitude.

Two additional observations about the model's adjustment-based endowment effect deserve special note. First, because adjustment is durable, the endowment effect has the potential to persist even after the individual no longer owns the item. One may say, in a sense, that ownership *imprints* the consumer. The finding contrasts with both reference-dependence and ownership effect predictions for the endowment effect and has far-reaching implications. Empirical measurement of this durability characteristic is possible, as I discuss later. Second, consistent with the numerous interpretations of endowments in the model, the endowment effect may be expected to rear its head in a variety of contexts. A gift or bequest, forced acquisition, or acquisition of a good before quality is known, all imprint the consumer and increase his valuation of the good, potentially forever. This, too, can have far-reaching implications, including *inter alia* for the profitability of various marketing strategies, as I discuss in a later subsection.

The endowment effect is unambiguously strengthened by regret, as the following result indicates:

Proposition 4. *(Sweet Lemons) Let $(x^*(\mu), y^*(\mu))$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve (2), suppose $\bar{x}_0 > x^*$, and consider a consumer characterized by non-lingering regret $\beta_\tau = 0\ \forall \tau > 0$ and $\beta_0 > 0$ for $\omega \in [0,1]$. Define $\bar{x}_{0\tau}(\omega)$ to be the size of endowment needed to induce this consumer to set $x_t > x^*$ for all $t \leq \tau$. Then $\frac{\partial \bar{x}_{0\tau}(\omega)}{\partial \omega} < 0$.*

Put simply, the greater the degree of the consumer's regret, the smaller the endowment it takes to induce an endowment effect in any given future period. The mechanism is through the effect of regret on justification-oriented thinking, which I call the "sweet lemons" effect. A regretful consumer chooses

[6] Kőszegi & Rabin (2006) describe the trader phenomenon in terms of traders' rational expectations that there is a high probability of parting with items they have just acquired. This perspective is fully consistent with the adjustment theory view of a rational adjuster who will not adjust if she does not expect to own the object.

a product that later turns out not to live up to expectations. Rationalizing his decision, he tells himself in effect that the decision must not have been so bad (i.e., the lemon was "sweet"), because, after all, it motivated him enough to adjust considerably. This behavior leads naturally to ongoing consumption of the "lemon." The more strongly a person feels the need to justify a past action (because he more intensely regrets it), the more justification he will engage in, creating a larger stock of adjustment that will carry forward further into the future.

The self-justification motive's amplification of the endowment effect proposed in Proposition 4 is an important testable implication of the overall model. If subjects primed with a need to justify their acquisition of an item report or demonstrate a greater valuation of the item, this could provide support for the adjustment account of the endowment effect in preference to the loss aversion account.

4.2 Escalation of Commitment

A range of psychological evidence suggests that real-life decision-makers exhibit "sunk-cost bias," violating the normative principle that rational agents should not take account of sunk costs in making decisions. In particular, individuals who have previously invested more in a course of action may be more likely to continue it, a phenomenon referred to variously as the "Concorde effect" (Dawkins & Carlisle 1976) or "escalation of commitment" (Staw 1976). Recent behavioral economic models have rationalized such behavior when past actions and present actions are strategic complements. This can occur, as discussed in the introduction, as a consequence of agents exhibiting a taste for consistency of action (Eyster 2002); or, say, if previously-sunk costs provide a signal of the value of projects to forgetful agents who must consider whether to continue investment in the present period (Baliga & Ely 2011).

The adjustment model offers another basis for making sense of escalation-of-commitment-type behaviors, one based more firmly in psychological primitives. They can occur if agents who adjust exhibit a lingering desire to achieve a psychological accommodation of prior errors.

Proposition 5. *(Sweet Lemons Redux) Let $(x^*(\mu), y^*(\mu))$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve (2), suppose $\bar{x}_0 > x^*$, and consider a consumer characterized by lingering regret such that $\beta_\tau > 0$ for some $\tau > 0$ with $\omega \in (0,1]$. Then $x_\tau > x^*$.*

Decision-makers who feel "invested" (in the sunk-cost sense) in a past course of action are motivated to believe that the prior action was worthwhile. This is ulimately something that is achieved at a cognitive level by rationalization - that is, by adjustment. The process proceeds along the lines detailed following Proposition 4. Proposition 5 proposes that this process will be invoked in any future period in which there is some regret.

The lingering desire to accommodate prior errors is functional for under-endowments as well, as recognized by the following proposition:

Proposition 6. *(Sour Grapes) Let* $(x^*(\mu), y^*(\mu))$ *be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each* $t \geq 0$ *chooses* (x_t, T_t) *to solve* (2)*, suppose* $\bar{x}_0 < x^*$*, and consider a consumer characterized by lingering regret such that* $\beta_\tau > 0$ *for some* $\tau > 0$ *with* $\omega \in (0, 1]$*. Then* $x_\tau < x^*$*.*

I call this the "sour grapes" proposition because it reflects the situation of the fox in the Aesop parable who sees grapes that are out of his reach. He rationalizes that they were sour and so not worth his while anyway. Suppose the fox is given the chance the next day to eat the grapes. Would he pass them up? Proposition 6 indicates that this depends on whether he experiences a lingering desire to rationalize not having gotten the grapes yesterday, as only then would he continue to perceive them as sour and act upon that assumption.

Note the asymmetry between under-endowments and over-endowments in the model when there is no lingering regret. An over-endowment - that is, an endowment of an amount of the activity greater than what would have been chosen with quality known - causes an endowment effect. An under-endowment can be rectified immediately the following period - given that the consumer knows quality and is free to choose the amount of the activity - whence there is no future reduction in consumption, that is, no endowment effect. Lingering regret, however, gets in the way of this correction, causing behavior to be biased downward based on the need to continue to rationalize the downside error from the initial period.

4.3 Cognitive Dissonance

4.3.1 The Model and General Principles

Cognitive dissonance has been defined as what occurs when an individual simultaneously holds two beliefs that are inconsistent (Festinger 1962, Aronson 2004). The term has been used to characterize a range of observed behavioral phenomena typically involving the individual changing his beliefs in response to being compelled to take an action or being faced with a difficult decision. For example, in the scenario proposed by Akerlof & Dickens (1982) discussed in the introduction, workers find their decision to work in a hazardous industry unsettling and so adjust their beliefs about the industry's hazards. Subsequently, as a result of their changed beliefs, they choose not to purchase safety equipment. Other economic manifestations of cognitive dissonance referred to in the literature include, among others, failure to set aside optimal amounts for retirement by individuals who prefer not to think about their earning ability being diminished by old age (Akerlof & Dickens 1982); and failure of individuals to adjust effort levels to accomplish a desired task, once it becomes clear that the task will not be as easy as expected (Aronson 2004).

The adjustment model offers an advance in terms of understanding these phenomena and their implications. Cognitive dissonance phenomena generally may be characterized in the model's parlance as involving an endowment, corresponding regret, and a corresponding adjustment reaction. The consequential recognition is that, in most experimentally-induced and many naturally-occurring cognitive dissonance situations, the endowed action that conflicts with the individual's initial preferences and so propels the cognitive dissonance reaction may be framed in terms of a *restriction of the choice set*. Such choice set restrictions have predictable behavioral effects through the adjustment framework.

To fix ideas, consider in the adjustment model an individual whose objective includes a regret term. Let us suppose the individual may over a sequence of periods take a quantity of the action that is constrained to lie on the unit internal; thus, for the initial period, $x_0 \in [0, 1]$. Suppose also that quality μ is known at the outset and is such that the individual's optimizing action is an interior solution $x_0 \in (0, 1)$. Let us define the set $\mathbb{C} \subseteq [0, 1]$ of choice options actually available to the decision-maker, with the remaining options on $[0, 1]$ being unavailable. In the case where $\mathbb{C} = [0, 1]$, the individual chooses x_0^* and sets T_0 to optimally adjust to x_0^*. In this situation, he experiences no regret. Put another way, there is no dissonance between the individual's action and his preferences.

Now consider the case of forced compliance, as per the experiments of Festinger & Carlsmith (1959). This case may be represented in the model by an individual being compelled to take a greater quantity of action than he would have chosen given its quality level μ. Formally, $\mathbb{C} = [\underline{x}, 1]$ where $1 \geq \underline{x} > x_0^*$. This situation creates the potential for regret: the quantity $\underline{x}$ is "too high" and must be rationalized. To reduce his misgivings, the individual adjusts with greater intensity (as per Proposition 2) to support the endowed action. Given the durability of adjustment, his initial adjustive thinking may decrease misgivings about taking the action in the future, increasing his tendency to engage in it again. Both the hypothesized attitudinal adjustment to the endowed action and increased propensity to take it again are consistent with outcomes observed by Festinger & Carlsmith (pp. 203 and 208). The critical characteristic of the scenario that set in motion the cognitive dissonance reaction was a restriction in the choice set; had the individual been able to choose his action freely, there would have been no dissonance, no adjustment, and no altered future behavior.

A variation of the phenomenon is provided by the scenario discussed previously of the workers in the hazardous industry. Suppose a particular worker judges the dangers of the industry a priori as neither extreme nor non-existent, such that he would like to participate in the industry workforce but only tentatively or partially (i.e., $x_0^* \in (0,1)$). Of course, life does not normally permit a worker to be tentative: you have to take a job, or else pass it up. Here, then, $\mathbb{C} = \{0,1\}$. The choice set exhibits a restriction - more precisely, a discontinuity - jumping from 0 to 1. If the worker judges that taking the job is his best option of the two available and he takes it, he experiences dissonance: the job is more dangerous than he is comfortable with for a full-time commitment. His response is to engage in adjustive thinking to support his decision to take the job: he convinces himself the job is not as hazardous as people say. Such attitudes tend to persist, whence the dynamic problem arises. If, in the future, the individual is given the opportunity to purchase safety equipment, the option to do so does not look as attractive as it might have before he started the job.

A choice-set restriction is also at the heart of a "crime and punishment" cognitive dissonance reaction described by Dickens (1986). In an oft-repeated experiment, children are warned not to play with a desirable toy. One group is threatened with a severe punishment for disobedience, while another is told to expect a mild punishment. Much later, the children are again put in the room with the toy, but this time without the threat of punishment. It is observed that the children previously threatened with the severe punishment are more likely to play with the toy than those threatened with the mild punishment. The standard interpretation of these studies is that those threatened with mild punishment had to justify to themselves their decision not to play with the toy. The adjustment model clarifies the mechanism of this process. The children in the experiment are, in effect, faced with a binary decision: do not play at all, or play (at all) and be subject to a punishment. Thus the choice set is perceived as discontinuous. (Were a child to "dabble" in playing with the toy, this would not prevent him from being punished, so it is fair to say that "dabbling" is dominated by playing "all out" and does not represent an intermediate option worthy of consideration.) A child threatened with a severe punishment does not need an intermediate option: he views playing with the toy as "low quality" activity, whence his decision not to play at all is relatively consistent with his preferences. But in the case of the child threatened with a mild punishment, not playing creates significant dissonance and requires adjustment. Hence the subsequent changes in the child's behavior after he chooses initially not to play.

Note that the occurrence of a cognitive dissonance reaction hinges on what the adjustment model characterizes as regret. And the critical role of regret, as established in Proposition 1, is that it reverses the direction of the marginal effect of activity quality on engagement in adjustive thinking. Consider again the worker in the hazardous industry, and suppose that worker does not exhibit regret per the model's parlance. The more hazardous the industry, the less such a worker would engage in adjustive thinking to support it (i.e., because hazards reduce the perceived "quality" of working in the industry, there is less benefit to be had from being "psyched up" to work in it). Meanwhile the regret-burdened worker engages in *more* adjustive thinking the more hazardous the industry is. When safety equipment is later introduced, a non-regret-laden worker sees clearly the low quality of the industry he works in and purchases the equipment; the regret-laden worker does not and therefore fails to see the value of safety equipment.

4.3.2 Public Policy Implications

The adverse effect on social welfare of behaviors that stem from cognitive dissonance is well recognized, particularly in areas relating to public health. Policy analysts understand that attitudes can pose problems when one is trying to get people to take socially or even privately desirable actions. Too often, however, policy approaches take the individual's attitude as a *fait accompli*, whence the logical approach is to compel action despite attitude. Requiring motorcycle riders to wear helmets, workers to purchase safety equipment, and those at risk for disease to get tested or treated - all represent examples of this policy approach.

What may get lost in the conventional policy discussion is the interplay between the choice set and the formation of the attitude. In recognizing that attitudes are naturally formed during the initial decision process, the adjustment theory broadens the scope of available policy tools to include manipulating the choice set. One area where such a tool has already received some recognition is in crime interdiction. When one alters the publicized punishment for a crime, one in effect alters the choice set open to the would-be criminal. By proper manipulation of the punishments, one may manipulate attitudes - and indeed *employ* cognitive dissonance - in such a way that crime is reduced. This recognition can lead to better policy proposals not just for dealing with serious crimes, but minor infractions of routine social rules as well. (See, e.g., Gneezy & Rustichini 2000.)

Since choice-set discontinuities can give rise to cognitive dissonance, policy problems arising out of dissonance may sometimes be solved by offering people continuous choices or otherwise reframing the choice set as continuous. One area where this approach might be useful is the problem of attaining a consensus on politically polarized social issues. Take the example of climate change. It is fair to say that the climate change debate is currently framed in binary terms: accept that humans are causing a significant change in the earth's climate, or deny this proposition categorically. In effect, a restricted choice set compels people to "take a side." After choosing which side they are on, individuals then work to reduce their dissonance, adjusting their beliefs to be consistent with their chosen side, and filtering new information as it arrives according to its alignment with their desired mental position.[7] If instead a continuum of positions were tenable, people could select the position most consistent, in their judgment, with the facts known to them.[8] As new information came to light, suggesting that the human role in climate change was either greater or not as great as had previously been believed, individuals would feel free to adjust their position on the issue and their corresponding attitudes. Making such a regime possible would necessitate developing social norms for being receptive to other people's perspectives and encouraging discussion of the issue with greater openness. Adopting new language for speaking about the issue that does not cast choices in binary or otherwise restrictive terms might play a role.[9] At a societal level, progress toward positions aligned with the facts could potentially be greatly facilitated, bringing with it more harmonious relations between people with differing views.

4.3.3 Empirical Testing Implications

The recognition that many experimental demonstrations of cognitive dissonance involve choice-set restrictions points to the possibility of demonstrating the broader set of adjustment behaviors out in the market using *natural* experiments that involve choice-set restrictions. Consider again the unrestricted set of possibilities for action $x_0 \in [0, 1]$, and now let us suppose the propensity to take the action in question depends monotonely on an observable individual characteristic s, which is a continuous variable.[10] Without loss of generality assume $\frac{\partial x_0^*}{\partial s} > 0$. Suppose that the stock of adjustment y_0 complements x_0, and suppose further that y_0 is measurable in terms of observable actions by the individual (or, alterna-

[7] For evidence of individuals behaving in this way, see Sunstein *et al.* (2017)

[8] While people who have been engaged in the debate on climate for years most likely have entrenched positions, issue framing could make a difference in the positions of those (younger) individuals who are new to the issue. Apropos of this, Akerlof & Dickens (1982) note that innovation in a society often originates with outsiders, for whom resistance to changing established beliefs is not an obstacle.

[9] See, for example, Tabuchi, Hiroko, "In America's Heartland, Discussing Climate Change Without Saying 'Climate Change,'" The New York Times, January 28, 2017.

[10] More generally, of course, s could be a vector of individual characteristics.

tively, by a set of attitudes revealed through answers to questions that one could ask the individual).[11] If the de facto choice set is restricted - say, to $\mathbb{C} = \{0, 1\}$ - then one has a natural experiment. Focus on those individuals whose levels of s place them close to the transitional value of x_0^* at which the individual would switch from choosing 0 to 1, given the restriction. If those persons choosing 1 exhibit significantly different values of y_0 from those choosing 0, then one has demonstrated conclusively that the decision concerning x_0 has *caused* the change in attitude and any resultant behaviors. The analysis may be conducted formally using regression discontinuity designs.[12]

Another approach uses a temporal restriction of the choice set. Consider the announcement of a new product feature. A consumer's decision to purchase a product not possessing the innovated feature, if made prior to the announcement of the new feature, may be viewed as an endowment. The model predicts increased adjustment by endowed consumers in support of the unimproved product, leading to an increased propensity going forward to purchase products that do not have the innovated feature. Properly controlling for all other relevant factors, it should be possible to test whether such an endowment effect is present in this case. In fact, one could potentially demonstrate persistence of an endowment effect well beyond the period of ownership of the initial product. Along these lines, one might hypothesize - and also test - that consumers who purchased their first cell phone prior to the introduction of smartphones would, all else equal, be more likely to purchase a "flip phone" going forward. Indeed, intuition and casual observation suggest such patterns of habituated consumer behavior.

Relaxations of choice-set restrictions of various kinds may pose similiar temporal natural experiments for measuring adjustment. Examples include the entry of a new brand into the market, or a loosening of regulations that opens new options for action by decision-makers.

4.4 Marketing Practices

Traditional economic theories have conceived of two roles for advertising: to provide information about a product, and to persuade consumers to prefer the product. Both propositions have limitations. The information theory cannot explain advertisers' costly efforts devoted to crafting message and image in ads otherwise devoid of informational content.[13] The persuasion theory offers no explanation as to why

[11] Note that the variables used to measure y_0 are distinguished from the characteristic s in that they are endogenous while s must be convincingly exogenous.

[12] For a fuller discussion of the regression discontinuity approach, see, e.g., Angrist & Pischke (2008).

[13] The theory *has* offered an explanation of non-substantive advertising as providing a signal that the product is of sufficient quality to warrant a costly advertising expenditure (Nelson 1974). But if the purpose is just to show that money is being spent and the ad is not in some measure intended to be persuasive, why should message and image details matter?

advertising should elicit a response at all from a rational consumer.

The adjustment theory posits a rational consumer who seeks to increase the utility she obtains from the products she chooses. From this emerges a new primitives-based explanation of persuasive advertising as *facilitating self-persuasion.* One posits advertising formally in the adjustment framework as an expenditure that reduces consumers' adjustment costs with respect to the advertised product. Intuitively, advertisements provide "fodder" - in the form of helpful arguments, gut appeals, and seductive images - for a consumer who is trying to become as enamored of a product as possible. Through this conception, there is the potential for purely persuasive advertising both to influence demand and to serve an efficient, rather than wasteful, purpose. Such a view breaks with traditional economics, which has perceived only informative advertising as welfare-enhancing.[14]

The adjustment model also makes way for a more coherent economic understanding of other marketing practices. The practice of giving free product (e.g., samples, trial-basis services, short-term memberships) has traditionally been viewed in economics as providing an incentive to consumers to learn more about the product.[15] Thus giveaways can be an efficient aid to search in a world of consumers with strictly fixed preferences. But if learning about a product is the only motivation for such practices, then why are further coupons and discounts offered to people who have already redeemed previous offers? Viewed through the lens of adjustment, giveaways play another role: they build an *endowment.* The consumer who receives a free one-month membership to a fitness club rationally adjusts to use of the club facilities; because such adjustment is durable, her marginal utility from use of the club is increased in future periods and she is more likely to see the benefit of purchasing a long-term membership. Coupons, discounts, and price promotions all function in a similar way: they create incentives for the consumer to establish an endowment that will lead, through the endowment effect, to future (full-price) purchases. Marketers have long recognized these sorts of benefits and, in fact, are more prone to talk in terms of dynamic preference-smithing effects on consumers (e.g., creating a habit, building a relationship) than they are to think in terms of simply providing the consumer with more information as a basis for making an optimal decision.

[14] Akerlof & Dickens (1982) suggest that advertising may help consumers by furnishing them with an "external justification" for believing that a purchased product meets their needs (p. 317). This idea closely aligns with my notion. For a full analysis of the effects of advertising as self-persuasion in a competitive context, see Nagler (2017).

[15] See, for example, the discussion in the introduction of Sun (2011).

5 Conclusion

This paper has presented a generally applicable theory of individual decision-making based on adjustment to choice. Rather than reiterate the findings of the model, I will use this section to discuss the effect of alternative assumptions about the foreseeability of regret and to propose some promising directions for future research focused on applications of the framework.

My results relating to the outcomes for a regret-driven consumer were based on the assumption that the consumer naïvely conjectures that he will not experience regret in the future. Suppose instead that the consumer experiences regret in one future period $t = \tau > 0$ and that the $t = 0$ consumer - more specifically, his non-regret-driven self - accurately predicts the extent of his regret in that period. (More generally the consumer might experience - and his $t = 0$ self might accurately ancitipate - regret in multiple future periods. But the results for the single-period future regret case are equivalent and provide a simpler illustration.) It is intuitive that such a consumer would wish to set adjustment to compensate for what he considered the errors of his future regret-driven self. The nature of his strategic adjustment behavior would depend on whether $x_0 < x^*$ or $x_0 > x^*$. In the case of $x_0 < x^*$, the $t = 0$ consumer believes his future regret-driven self would set adjustment too low. Because adjustment is durable, he may compensate fully by choosing y_0 large enough to induce a sufficiently large y_τ for the relevant $t = \tau$. Balancing his relative concerns for utility in $t = 0$ and $t = \tau$, he would set current adjustive thinking higher than if he did not anticipate future regret, but in general lower than the amount needed to fully compensate for that regret. In the case of $x_0 > x^*$, the $t = 0$ consumer believes his future regret-driven self would set adjustment too high. However, there is nothing he can do about this. If the $t = 0$ consumer were even to set $y_0 = 0$, his future regret-driven self would set adjustive thinking sufficiently high to induce the level dictated by his regret, regardless. Thus the asymmetry between under- and over-endowments shows up in the fact that a strategic non-regret-driven self is able to manage the principal-agent problem arising with his regret-driven future self in the former case, but not the latter - in essence, because he cannot "push on a string." More broadly, one can see that relaxing my assumption about naïve regret conjectures does not change directionally the results discussed in section 4. The theory, as modified by the alternative assumption, still predicts that regret will lead to a strengthened endowment effect and escalation of commitment.

Allowing future consumption in the adjustment model to depend directly on present consumption decisions could provide new insights into self-control problems.[16] The traditional approach assumes that differences in the degree to which individuals procrastinate or seek immediate gratification flow mainly from differences in general attitudes toward the future, reflected typically in discount rates.[17] But this notion of the role of general time-preference is inconsistent with evidence that self-control problems are *specifically* treatable: for example, individuals who are put through a regimen of goal visualization are more likely to achieve their visualized goals (Cheema & Bagchi 2011). Such findings suggest an alternative model in which complementary adjustment to future goals in the present increases the valuation of those goals, tipping the balance in favor of present decisions that support achievement of the goals (e.g., saving now, staying in school, spending more time studying). Thus, individuals who find such adjustment easier are the ones more likely to exhibit successful self-control.

The present paper's analysis of adjustment was restricted to a partial equilibrium context involving focus on a single product. Extending the adjustment model's approach to a multi-product context would allow for the analysis of *lifestyle* choice - that is, the simultaneous choice of attitudes towards multiple, potentially related product or activity choices (*ensembles*). A general equilibrium model is the proper structure for considering the broader effects of advertising (i.e., in fostering lifestyles, rather than simply stimulating demand for specific products) and other inter-market effects of relevance.

Equilibrium in differentiated product markets has traditionally been modeled with spatial frameworks such as the seminal "beach" proposed by Hotelling (1929). These models represent product offerings as fixed or endogenous locations along a continuum, and consumers' heterogeneous fixed preferences are similarly represented by location. A given consumer will generally not find an ideal choice among the available products, in that there will not be a product offering at his precise location, whence his utility loss from consuming something less-than-ideal is represented by a "transportation cost" from his location to the location of the nearest product. A model of competition in differentiated products with adjustment to choice could provide for improved predictions regarding the price and market share outcomes accruing to various relevant exogenous factors. Introducing adjustment into the standard spatial framework implies a consumer whose location is not fixed, but who "moves closer" to his chosen product, trading off transportation cost against adjustment cost. Nagler (2016) offers an exploratory treatment, but there is more work to be done.

Finally, the empirical measurement of adjustment, along the lines of Subsection 4.3.3, could provide

[16] I thank Hanna Halaburda for pointing out this application.

[17] O'Donoghue & Rabin (1999) observe that whether individuals are naïve or sophisticated about their self-control problems can also exert a distinct influence on their behavior.

a proof of concept for the adjustment theory, particularly as a unifying framework. It could, moreover, expand our ability to recognize instances of the range of phenomena that I have proposed here as manifestations of adjustment. I offer two examples. First, as demonstrations of cognitive dissonance have previously only been made using lab experiments, future research that leverages natural experiments could expand the scope for recognizing manifestations of dissonance. Second, past experimental evidence of the endowment effect has been limited to instances in which subjects currently own the relevant good. Using appropriate natural experiments, such as the temporal restriction example offered in Subsection 4.3.3, it might be possible to show that endowment effects, consistent with the theory presented in this paper, arise not only out of loss aversion or current ownership, but as the broader "have-you-ever-owned-it" effects of a durable cognitive adjustment process.

A Appendix

Satisfaction of (A1)-(A3) for Extended Cobb-Douglas Formulation. Consider the extended Cobb-Douglas utility example proposed in footnote 4: let $u(z) = z^d$, for $d \in (0,1)$; and $z(x,y) = x^a y^b$, where $a, b \in \left[\frac{1}{3-d}, 1\right)$ and $a + b = 1$. One may easily confirm that this setup satisfies (A1) and (A2). Moreover, $u_z = dz^{d-1}$, $u_{zz} = d(d-1)z^{d-2}$, $z_x = ax^{a-1}y^b$, $z_y = bx^a y^{b-1}$, and $z_{xy} = abx^{a-1}y^{b-1}$; thus

$$\begin{aligned} u_z z_{xy} &= dx^{a(d-1)}y^{b(d-1)}abx^{a-1}y^{b-1} \\ &= abdx^{a(a-1)(d-1)}y^{b(b-1)(d-1)} \end{aligned}$$

$$\begin{aligned} -u_{zz}z_y z_x &= -d(d-1)z^{d-2}bx^a y^{b-1}ax^{a-1}y^b \\ &= abd(1-d)x^{a(d-2)}y^{b(d-2)}y^{2b-1}x^{2a-1} \\ &= abd(1-d)x^{a(2a-1)(d-2)}y^{b(2b-1)(d-2)} \\ &\le abd(1-d)x^{a(a-1)(d-1)}y^{b(b-1)(d-1)} < u_z z_{xy} \end{aligned}$$

where the first inequality in the last line follows from $a, b \ge \frac{1}{3-d}$. Thus this setup satisfies (A3).

Proof of Theorem 1. The optimization problem is the same every period, so fix $t = \tau$ and consider the problem for self τ of solving (3). This player recognizes that his move (x_τ, T_τ) will influence the subsequent moves of future selves. Specifically, his choice of T_τ will influence future decisions because it affects the future stock of adjustment which in turn influences the preferences of future selves; his choice of x_τ has no bearing on future selves' preferences or decisions. A subgame perfect strategy takes account of how the choice of T_τ affects responses in all $t > \tau$ by treating T_t as a function $T_t(T_\tau)$ for all $t > \tau$ when choosing the optimal T_τ.

Observe from (2) that self τ's objective function nests the objective function of each $t > \tau$. One may rewrite (2) accordingly as

$$U_\tau \equiv \mu u(x_\tau, T_\tau) + K - x_\tau - T_\tau + \delta U_{\tau+1} \tag{A.1}$$

whence the nesting of future selves' objectives for $t > \tau + 1$ follows by induction. Observe also that T_t for $t > \tau$ appears in self τ's objective function only within U_t. It follows that one may write

$$\frac{\partial U_\tau}{\partial T_\tau} = \left.\frac{\partial U_\tau}{\partial T_\tau}\right|_D + \sum_{t=1}^{\infty} \frac{\partial U_\tau}{\partial U_{\tau+t}} \left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D \frac{\partial T_{\tau+t}}{\partial T_\tau} \tag{A.2}$$

where the "D" indicates the direct effect, not through the choice of T by another self.

For an interior solution $\left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D = 0$ for all $t \geq 1$ in (A.2). Thus by the envelope theorem $\frac{\partial U_\tau}{\partial T_\tau} = \left.\frac{\partial U_\tau}{\partial T_\tau}\right|_D$, that is, the overall effect of T_τ on utility is equal to the direct effect, ignoring effects of T_τ through the $T_{\tau+t}$ that are already being optimized by future selves.

With this in mind, the first-order conditions of (3) may be written

$$\begin{aligned} \mu u_z z_x &= 1 \\ \mu u_z z_y &= 1 \end{aligned}$$

As the problem is the same in every period, the first-order conditions yield the same $(x^*(\mu), y^*(\mu))$ interior solution in all periods $t \geq 0$. It follows that $T_0 = y^*(\mu)$ and $T_t = \sigma y^*(\mu)$ for all $t > 0$. Note that the constant marginal cost of adjustment implies the consumer engages in adjustment in each period t until the level $y^*(\mu)$ is reached, regardless of y_{t-1}. It is moreover easy to see from the first-order conditions that $x^*_\mu, y^*_\mu > 0$.

One now must show that this interior solution is the unique maximum. The derivatives of (A.1) needed to evaluate the Hessian with respect to this objective, reflecting the relevant values of (x, y) at which they are evaluated, are

$$\frac{\partial U_\tau}{\partial x_\tau} = \mu u_z z_x (x_\tau, y_\tau) - 1; \; \frac{\partial^2 U_\tau}{\partial x_\tau^2} = \mu \left(u_z z_{xx} (x_\tau, y_\tau) + u_{zz} z_x^2 (x_\tau, y_\tau) \right)$$
$$\frac{\partial^2 U_\tau}{\partial x_\tau \partial T_\tau} = \mu \left(u_{zz} z_x z_y (x_\tau, y_\tau) + u_z z_{xy} (x_\tau, y_\tau) \right)$$
$$\frac{\partial U_\tau}{\partial T_\tau} = \mu u_z z_y (x_\tau, y_\tau) + \delta (1-\sigma) \mu u_z z_y (x_{\tau+1}, y_{\tau+1}) + \ldots - 1$$
$$\frac{\partial^2 U_\tau}{\partial T_\tau^2} = \mu \left(u_z z_{yy} (x_\tau, y_\tau) + u_{zz} z_y^2 (x_\tau, y_\tau) \right) + \delta (1-\sigma)^2 \mu \left(u_z z_{yy} (x_{\tau+1}, y_{\tau+1}) + u_{zz} z_y^2 (x_{\tau+1}, y_{\tau+1}) \right) + \ldots \quad \text{(A.3)}$$

To simplify notation, I drop the arguments for $t = \tau$ and show only arguments for $t > \tau$. The Hessian is

$$\begin{aligned}
|H| &= \mu^2 \begin{vmatrix} u_z z_{xx} + u_{zz} z_x^2 & u_{zz} z_x z_y + u_z z_{xy} \\ u_{zz} z_x z_y + u_z z_{xy} & u_z z_{yy} + u_{zz} z_y^2 + \\ & \delta (1-\sigma)^2 \mu \left(u_z z_{yy} (x_{\tau+1}, y_{\tau+1}) + u_{zz} z_y^2 (x_{\tau+1}, y_{\tau+1}) \right) + \ldots \end{vmatrix} \quad \text{(A.4)} \\
&= \left(u_z z_{xx} + u_{zz} z_x^2 \right) \left(u_z z_{yy} + u_{zz} z_y^2 \right) - \left(u_{zz} z_x z_y + u_z z_{xy} \right) \left(u_{zz} z_x z_y + u_z z_{xy} \right) \\
&\quad + \left(u_z z_{xx} + u_{zz} z_x^2 \right) \delta (1-\sigma)^2 \mu \left(u_z z_{yy} (x_{\tau+1}, y_{\tau+1}) + u_{zz} z_y^2 (x_{\tau+1}, y_{\tau+1}) \right) + \ldots \\
&> \left(u_z z_{xx} + u_{zz} z_x^2 \right) \left(u_z z_{yy} + u_{zz} z_y^2 \right) - \left(u_{zz} z_x z_y + u_z z_{xy} \right) \left(u_{zz} z_x z_y + u_z z_{xy} \right) \\
&= u_z^2 z_{xx} z_{yy} + u_z u_{zz} z_x^2 z_{yy} + u_z u_{zz} z_{xx} z_y^2 + u_{zz}^2 z_x^2 z_y^2 - u_{zz}^2 z_x^2 z_y^2 - 2 u_z u_{zz} z_{xy} z_x z_y - u_z^2 z_{xy}^2
\end{aligned}$$

Given that z is homogenenous of degree one in (x, y), Euler's formula yields $z_x x + z_y y = z$. Differentiating this identity with respect to x yields $z_{xx} x + z_x + z_{xy} y = z_x$, and similarly with respect to y one obtains $z_{xy} x + z_{yy} y + z_y = z_y$. Thus, ${}^{z_{xx}}/_{z_{xy}} = -{}^{y}/_{x} = {}^{z_{xy}}/_{z_{yy}}$, whence $z_{xx} z_{yy} = z_{xy}^2$. Thus the last line of (A.4) simplifies to $u_z u_{zz} z_x^2 z_{yy} + u_z u_{zz} z_{xx} z_y^2 - 2 u_z u_{zz} z_{xy} z_x z_y > 0$. Since $\frac{\partial^2 U}{\partial x_\tau^2} < 0$ and $\frac{\partial^2 U}{\partial T_\tau^2} < 0$, $U_\tau (x_\tau, T_\tau)$ is strictly concave. This establishes the solution (x_τ^*, y_τ^*) to the first-order conditions as the unique interior absolute maximum for τ whence, by induction, this is true for all $\tau \geq 0$.

To dispense with the possibility of a corner solution: it was noted previously that the unique interior solution is $(x_t^*, y_t^*) = (x^* (\mu), y^* (\mu))$ for all $t \geq 0$, whence $T_t^* = \sigma y^* (\mu)$ for all $t > 0$. A corner solution of $T_1 = 0$ could result if and only if $(1-\sigma) y_0^* (\mu) > y^* (\mu)$. But this would imply $y_0^* (\mu) > y^* (\mu)$, which is a contradiction of the unique interior solution for $t = 0$. Thus no corner solution will result in $t = 1$. By similar argument, a corner solution in any $t = \tau$ requires $y_{\tau-1}^* (\mu) > y^* (\mu)$, a contradiction of the interior solution for the prior period. By induction, the interior solution is the only solution for all periods.

This confirms $(x_0, T_0) = (x^*(\mu), y^*(\mu))$ and $(x_t, T_t) = (x^*(\mu), \sigma y^*(\mu))$ for all $t > 0$ constitutes the unique subgame perfect equilibrium strategy for the infinite game.

Proof of Lemma 1. Let (x^*, y^*) represent the steady-state optimum under the unique subgame perfect equilibrium with no endowment, and let $T_0(\bar{x}_0)$ represent the level of T_0 arising from the first-order condition for T based on $\bar{x}_0$. Because $\bar{x}_0$ can vary relative to the level x^*, $y_0(\bar{x}_0) = T_0(\bar{x}_0)$ varies relative to y^* such that it is possible that $(1-\sigma) y_0(\bar{x}_0) > y^*$ and, more generally, that $(1-\sigma)^t y_0(\bar{x}_0) > y^*$. In each period $t \geq 1$, if $(1-\sigma)^t y_0(\bar{x}_0)$ is below the steady-state optimum y^*, then the individual chooses $T_t > 0$, an interior solution. The proof of Theorem 1 establishes that this solution is unique. If $(1-\sigma)^t y_0(\bar{x}_0)$ is at or above that optimal level, there is a corner solution in individual t's optimization at $T_t = 0$.[18] One must account for this possibility in the analysis.

Again consider (A.2). Now, in each term following the first term of this expression, the derivatives $\left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D$ and $\frac{\partial T_{\tau+t}}{\partial T_\tau}$ for $t \geq 1$ exhibit complementary slackness. Consider again the two cases with respect to $(1-\sigma)^t y_0(\bar{x}_0)$ referenced above. In the former case, $\left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D = 0$ and $\frac{\partial T_{\tau+t}}{\partial T_\tau} = -(1-\sigma)^t < 0$; in the latter, $\left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D < 0$ and $\frac{\partial T_{\tau+t}}{\partial T_\tau} = 0$. In either case, all the terms except the first in (A.2) are equal to zero. Thus the effect of T_τ on U_τ reduces to T_τ's direct effect.

In view of this, a trivial variation on the proof of Theorem 1 yields the existence of a unique subgame perfect equilibrium strategy given $\bar{x}_0$. If $(1-\sigma) T_0^*(\bar{x}_0) \leq y^*$, then there is a unique subgame perfect equilibrium strategy for all $t \geq 0$ and it is an interior solution. If instead $(1-\sigma)^\tau T_0^*(\bar{x}_0) > y^*$ for all $0 < \tau \leq t$ for some t, then there will be a corner solution $T_\tau^* = 0$ (with corresponding y_τ^*) for all $0 < \tau \leq t$ and it is unique, while for all $\tau > t$ there is an interior solution and, as demonstrated in the proof of Theorem 1, it is unique. Collectively these designate a unique subgame perfect equilibrium strategy for the infinite consumption game.

[18] Setting $T_t < 0$ to arrive at E^* does not make sense, because it would be costly to reduce T and only would reduce the value of complementary consumption.

Proof of Lemma 2. The derivatives of (4) needed to evaluate the Hessian with respect to this objective are:

$$\frac{\partial U^R}{\partial x_\tau} = (1-\omega\beta_\tau)\left(\mu u_z z_x\left(x_\tau, y_\tau\right) - 1\right); \ \frac{\partial^2 U^R}{\partial x_\tau^2} = (1-\omega\beta_\tau)\,\mu\left(u_z z_{xx}\left(x_\tau, y_\tau\right) + u_{zz} z_x^2\left(x_\tau, y_\tau\right)\right)$$

$$\frac{\partial^2 U^R}{\partial x_\tau \partial T_\tau} = (1-\omega\beta_\tau)\,\mu\left(u_{zz} z_x z_y\left(x_\tau, y_\tau\right) + u_z z_{xy}\left(x_\tau, y_\tau\right)\right)$$

$$\begin{aligned}\frac{\partial U^R}{\partial T_\tau} = (1-\omega\beta_\tau)\Big(\mu u_z z_y\left(x_\tau, y_\tau\right) - 1 + \delta\Big\{\Big[(1-\sigma) + T'_{\tau+1}\left(T_\tau\right)\Big]\mu u_z z_y\left(x_{\tau+1}, y_{\tau+1}\right) - T'_{\tau+1}\left(T_\tau\right)\Big\} \\ +\delta^2\Big\{\Big[(1-\sigma)^2 + (1-\sigma)\,T'_{\tau+1}\left(T_\tau\right) + T'_{\tau+2}\left(T_\tau\right)\Big]\mu u_z z_y\left(x_{\tau+2}, y_{\tau+2}\right) - T'_{\tau+2}\left(T_\tau\right)\Big\} + \dots\Big) \\ + \omega\beta_\tau\left(\mu u_z z_y\left(\bar{x}_0, y_\tau\right) - \mu u_z z_y\left(x_0^*\left(T_\tau\right), y_\tau\right)\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 U^R}{\partial T_\tau^2} = (1-\omega\beta_\tau)\Bigg(\mu u_z z_{yy}\left(x_\tau, y_\tau\right) + \mu u_{zz} z_y^2\left(x_\tau, y_\tau\right) \\ + \delta\left\{\Big[(1-\sigma) + T'_{\tau+1}\Big]^2\left[\mu u_z z_{yy}\left(x_{\tau+1}, y_{\tau+1}\right) + \mu u_{zz} z_y^2\left(x_{\tau+1}, y_{\tau+1}\right)\right]\right\} \\ +\delta^2\left\{\Big[(1-\sigma)^2 + (1-\sigma)\,T'_{\tau+1} + T'_{\tau+2}\Big]^2\left[\mu u_z z_{yy}\left(x_{\tau+2}, y_{\tau+2}\right) + \mu u_{zz} z_y^2\left(x_{\tau+2}, y_{\tau+2}\right)\right]\right\} + \dots\Bigg) \\ + \omega\beta_\tau\left[\mu u_z z_{yy}\left(\bar{x}_0, y_\tau\right) + \mu u_{zz} z_y^2\left(\bar{x}_0, y_\tau\right) - \mu u_z z_{yy}\left(x_0^*\left(T_\tau\right), y_\tau\right)\right. \\ \left. -\mu u_{zz} z_y^2\left(x_0^*\left(T_\tau\right), y_\tau\right) - \mu u_z z_{xy}\left(x_0^*\left(T_\tau\right), y_\tau\right) x_T^* - \mu u_{zz} z_y z_x\left(x_0^*\left(T_\tau\right), y_\tau\right) x_T^*\right] \quad \text{(A.5)}\end{aligned}$$

where $x_0^*\left(T_0\right)$ arises implicitly as the solution to $\mu u_z z_x\left(x_0^*, y_0\right) = 1$. (Thus terms $\mu u_z z_x\left(x_0^*\left(T_0\right), y_0\right) x_T^* - x_T^*$ drop out of the parentheses on the bottom line of $\frac{\partial U^R}{\partial T_\tau}$.)

Suppose first that $(1-\sigma)\,y_{\tau-1} < y_\tau^*$, such that the solution arising from the first-order condition is an interior solution and candidate unique solution. Note, in this case, that all the T'_t terms in (A.5) are constants that either take a value of zero or $-(1-\sigma)^{t-\tau}$. The value taken by each of these terms follows from the form of the state equation (1) and depends specifically on the size of the base value of T_τ, which determines in turn whether each T_t is a corner or interior solution. Suppose first that T_τ is relatively small; then the depreciated value of adjustment $(1-\sigma)\,y_\tau$ will be less than the optimizing value of $y_{\tau+1}$, whence there will be an interior solution for $T_{\tau+1}$ such that $T'_{\tau+1}\left(T_\tau\right) = -(1-\sigma)$. It follows that $T'_{\tau+t}\left(T_\tau\right) = 0$ for all $t > 1$. If, however, T_τ is a bit larger but not too large, $(1-\sigma)\,y_\tau$ will be large enough to push $T_{\tau+1}$ to a corner solution at zero; T_τ would then be large enough that changes in its value would influence the choice of T in the following period, $\tau+2$, whence $T'_{\tau+2}\left(T_\tau\right) = -(1-\sigma)^2$

and $T'_{\tau+t}(T_\tau) = 0$ for all $t > 2$. And so on. It can be shown therefore that each squared expression in square brackets in $\frac{\partial^2 U^R}{\partial T_\tau^2}$ is nonnegative; more precisely, they take the value $(1-\sigma)^t$ up to a threshold t and zero thereafter. Overall, the sum of the curly bracketed terms within the non-regret component of $\frac{\partial^2 U^R}{\partial T_\tau^2}$ (i.e., the portion that is weighted by $1-\omega\beta_\tau$), which I shall refer to as "NR" for convenience, is unambiguously negative. The regret component (weighted by $\omega\beta_\tau$), which I shall refer to as "R", is ambiguously signed.

Dropping the arguments for $t = \tau$ to simplify notation, the Hessian may be written

$$\begin{aligned}
|H| &= \mu^2 \begin{vmatrix} (1-\omega\beta_\tau)\left(u_z z_{xx} + u_{zz} z_x^2\right) & (1-\omega\beta_\tau)\left(u_{zz} z_x z_y + u_z z_{xy}\right) \\ (1-\omega\beta_\tau)\left(u_{zz} z_x z_y + u_z z_{xy}\right) & (1-\omega\beta_\tau)\left[\mu u_z z_{yy} + \mu u_{zz} z_y^2 + NR\right] + \omega\beta_\tau R \end{vmatrix} \\
&= \mu^2 (1-\omega\beta_\tau)^2 \Big[\left(u_z z_{xx} + u_{zz} z_x^2\right)\left(\mu u_z z_{yy} + \mu u_{zz} z_y^2\right) - \left(u_{zz} z_x z_y + u_z z_{xy}\right)^2 + \left(u_z z_{xx} + u_{zz} z_x^2\right) NR\Big] \\
&\quad + \mu^2 (1-\omega\beta_\tau)\, \omega\beta_\tau \left(u_z z_{xx} + u_{zz} z_x^2\right) R
\end{aligned}$$

This takes the sign of

$$\begin{aligned}
&(1-\omega\beta_\tau)\Big[\left(u_z z_{xx} + u_{zz} z_x^2\right)\left(\mu u_z z_{yy} + \mu u_{zz} z_y^2\right) - \\
&\left(u_{zz} z_x z_y + u_z z_{xy}\right)^2 + \left(u_z z_{xx} + u_{zz} z_x^2\right) NR\Big] \\
&+ \omega\beta_\tau \left(u_z z_{xx} + u_{zz} z_x^2\right) R
\end{aligned} \tag{A.6}$$

The proof of the Theorem showed that $\left(u_z z_{xx} + u_{zz} z_x^2\right)\left(\mu u_z z_{yy} + \mu u_{zz} z_y^2\right) - \left(u_{zz} z_x z_y + u_z z_{xy}\right)^2 > 0$, whence it follows that the entire portion of (A.6) weighted by $1-\omega\beta_\tau$ is positive. It is therefore sufficient for signing the entire expression positive, given ω sufficiently small, that R be bounded above.

Proof of Proposition 1. Begin by writing the version of (4) for $\tau = 0$:

$$\begin{aligned}
\max_{T_0|\bar{x}_0} U^R(\bar{x}_0, T_0) = (1-\omega\beta_0)&\left\{\mu u(\bar{x}_0, T_0) + K - \bar{x}_0 - T_0 + \sum_{t=1}^{\infty} \delta^t \left[\mu u(x_t, T_t) + K - x_t - T_t\right]\right\} \\
&+ \omega\beta_0 \left\{\left[\mu u(\bar{x}_0, T_0) + K - \bar{x}_0 - T_0\right] - \left[\mu u(x_0^*(T_0), T_0) + K - x_0^*(T_0) - T_0\right]\right\}
\end{aligned} \tag{A.7}$$

Take the first-order condition with respect to T_0, using (A.3):

$$(1-\omega\beta_0)\left(\mu u_z z_y(\bar{x}_0,y_0)-1+\delta\left\{\left[(1-\sigma)+T_1'(T_0)\right]\mu u_z z_y(x_1,y_1)-T_1'(T_0)\right\}\right.$$
$$\left.+\delta^2\left\{\left[(1-\sigma)^2+(1-\sigma)T_1'(T_0)+T_2'(T_0)\right]\mu u_z z_y(x_2,y_2)-T_2'(T_0)\right\}+\dots\right)$$
$$+\,\omega\beta_0\left\{\mu u_z z_y(\bar{x}_0,y_0)-\mu u_z z_y(x_0^*(T_0),y_0)\right\}=0 \quad \text{(A.8)}$$

where, as mentioned in the proof of Lemma 2, $x_0^*(T_0)$ arises implicitly as the solution to $\mu u_z z_x(x_0^*,y_0)=1$. (Thus terms $\mu u_z z_x(x_0^*(T_0),y_0)x_T^*-x_T^*$ drop out of the last line.) Recall, from the proof of Lemma 2, that all terms $T_\tau'(T_0)$ are constants. Totally differentiating (A.8) and using D_1, D_2, N_{11}, N_{12}, N_{21}, and N_{22} to represent respective curly bracketed terms yields

$$\left[(1-\omega\beta_0)\left\{\mu u_z z_{yy}(\bar{x}_0,y_0)+\mu u_{zz}z_y^2(\bar{x}_0,y_0)+\delta\left[(1-\sigma)+T_1'(T_0)\right]^2\left[\mu u_z z_{yy}(x_1,y_1)+\mu u_{zz}z_y^2(x_1,y_1)\right]+\dots\right\}\right.$$
$$+\,\omega\beta_0\left\{\mu u_z z_{yy}(\bar{x}_0,y_0)+\mu u_{zz}z_y^2(\bar{x}_0,y_0)\right.$$
$$\left.\left.-\mu u_z z_{yy}(x_0^*(T_0),y_0)-\mu u_{zz}z_y^2(x_0^*(T_0),y_0)-\mu u_z z_{xy}(x_0^*(T_0),y_0)x_T^*-\mu u_{zz}z_y z_x(x_0^*(T_0),y_0)x_T^*\right\}\right]dT_0$$
$$=-\left[(1-\omega\beta_0)\left\{u_z z_y(\bar{x}_0,y_0)+\delta\left[(1-\sigma)+T_1'(T_0)\right]u_z z_y(x_1,y_1)+\dots\right\}\right.$$
$$\left.+\,\omega\beta_0\left\{u_z z_y(\bar{x}_0,y_0)-u_z z_y(x_0^*(T_0),y_0)\right\}\right]d\mu$$
$$-\left[(1-\omega\beta_0)\left\{\mu u_z z_{xy}(\bar{x}_0,y_0)+\mu u_{zz}z_x z_y(\bar{x}_0,y_0)\right\}+\omega\beta_0\left\{\mu u_z z_{xy}(\bar{x}_0,y_0)+\mu u_{zz}z_x z_y(\bar{x}_0,y_0)\right\}\right]dx_0$$
$$\Longleftrightarrow\left[(1-\omega\beta_0)D_1+\omega\beta_0 D_2\right]dT_0=\left[-(1-\omega\beta_0)N_{11}-\omega\beta_0 N_{12}\right]d\mu+\left[-(1-\omega\beta_0)N_{21}-\omega\beta_0 N_{22}\right]dx_0$$
$$\text{(A.9)}$$

This yields, using Cramer's rule, for $\omega=0$:

$$\frac{\partial T_0}{\partial\mu}=-\frac{N_{11}}{D_1}=-\frac{u_z z_y(\bar{x}_0,y_0)+\delta\left[(1-\sigma)+T_1'(T_0)\right]u_z z_y(x_1,y_1)+\dots}{\mu u_z z_{yy}(\bar{x}_0,E_0)+\mu u_{zz}z_y^2(\bar{x}_0,y_0)+\delta\left[(1-\sigma)+T_1'(T_0)\right]^2\cdot\left[\mu u_z z_{yy}(x_1,y_1)+\mu u_{zz}z_y^2(x_1,y_1)\right]+\dots}>0 \quad \text{(A.10)}$$

For $\omega=1$ and $\beta_0=1$ - the case of extreme regret in the initial period - y_0^* is set for given $\bar{x}_0$ by implicit solution to $\mu u_z z_x(\bar{x}_0,y_0^*)=1$ as the "justifying" value of y_0. This allows one to perform comparative static analysis for this case by totally differentiating $\mu u_z z_x(\bar{x}_0, y_0^*) = 1$:

$$[\mu u_z z_{xy}(\bar{x}_0, y_0^*) + \mu u_{zz} z_x z_y(\bar{x}_0, y_0^*)]\, dT_0$$
$$= -[u_z z_x(\bar{x}_0, y_0^*)]\, d\mu - \left[u_z z_{xx}(\bar{x}_0, y_0^*) + u_{zz} z_x^2(\bar{x}_0, y_0^*)\right] d\bar{x}_0 \quad \text{(A.11)}$$

This yields

$$\frac{\partial T_0}{\partial \mu} = -\frac{u_z z_x(\bar{x}_0, y_0^*)}{\mu u_z z_{xy}(\bar{x}_0, y_0^*) + \mu u_{zz} z_x z_y(\bar{x}_0, y_0^*)} < 0$$

recalling that, by assumption, $u_z z_{xy} > -u_{zz} z_y z_x$. To understand what happens as ω varies between 0 and 1, one applies Cramer's rule to (A.9) for general ω and then differentiates with respect to ω:

$$\begin{aligned} \frac{\partial T_0}{\partial \mu} &= -\frac{(1-\omega\beta_0) N_{11} + \omega\beta_0 N_{12}}{(1-\omega\beta_0) D_1 + \omega\beta_0 D_2} \\ \to \frac{\partial^2 T_0}{\partial \mu \partial \omega} &= -\beta_0 \frac{[(1-\omega\beta_0)D_1+\omega\beta_0 D_2](N_{12}-N_{11})-[(1-\omega\beta_0)N_{11}+\omega\beta_0 N_{12}](D_2-D_1)}{[(1-\omega\beta_0)D_1+\omega\beta_0 D_2]^2} \end{aligned} \quad \text{(A.12)}$$

Recognizing that (A.12) takes the sign of the numerator, let us evaluate the numerator. This can be simplified as follows, prior to substitutions:

$$\begin{aligned} &-[(1-\omega\beta_0) D_1 + \omega\beta_0 D_2](N_{12} - N_{11}) + [(1-\omega\beta_0) N_{11} + \omega\beta_0 N_{12}](D_2 - D_1) \\ &= -(1-\omega\beta_0) D_1 N_{12} + (1-\omega\beta_0) D_1 N_{11} - \omega\beta_0 D_2 N_{12} + \omega\beta_0 D_2 N_{11} \\ &\quad - (1-\omega\beta_0) D_1 N_{11} + (1-\omega\beta_0) D_2 N_{11} - \omega\beta_0 D_1 N_{12} + \omega\beta_0 D_2 N_{12} \\ &= -(1-\omega\beta_0) D_1 N_{12} + \omega\beta_0 D_2 N_{11} + (1-\omega\beta_0) D_2 N_{11} - \omega\beta_0 D_1 N_{12} \\ &= -D_1 N_{12} + D_2 N_{11} \end{aligned}$$

Substitution back yields

$$\begin{aligned} &-\left\{\mu u_z z_{yy}(\bar{x}_0, y_0) + \mu u_{zz} z_y^2(\bar{x}_0, y_0) + \delta\left[(1-\sigma) + T_1'(T_0)\right]^2 [\mu u_z z_{yy}(x_1, y_1) + \mu u_{zz} z_y^2(x_1, y_1)] + \dots\right\} \\ &\cdot \{u_z z_y(\bar{x}_0, y_0) - u_z z_y(x_0^*(T_0), y_0)\} + \{\mu u_z z_{yy}(\bar{x}_0, y_0) + \mu u_{zz} z_y^2(\bar{x}_0, y_0) - \mu u_z z_{yy}(x_0^*(T_0), y_0) \\ &-\mu u_{zz} z_y^2(x_0^*(T_0), y_0) - \mu u_z z_{xy}(x_0^*(T_0), y_0)\, x_T^* - \mu u_{zz} z_y z_x(x_0^*(T_0), y_0)\, x_T^*\} \\ &\cdot \left\{u_z z_y(\bar{x}_0, y_0) + \delta\left[(1-\sigma) + T_1'(T_0)\right] u_z z_y(x_1, y_1) + \dots\right\} \end{aligned}$$

Now evaluate this expression at $\mu = \bar{\mu}$, the value of μ at which the endowment $\bar{x}_0$ would have resulted if $(\bar{x}_0, T_0)$ were chosen simultaenously with μ known, that is, the value of μ for which $x_0^*(T_0) = \bar{x}_0$. This is the appropriate value at which to evaluate $\partial^2 T_0 / \partial\mu\partial\omega$, as the expression is then interpreted as showing how regret influences the individual's response to a quality surprise relative to endowment-based expectations. At this value, the second term in curly brackets is zero. This leaves

$$\begin{aligned}
&\mu u_z \left\{u_z z_{yy}(\bar{x}_0, y_0) + u_{zz} z_y^2(\bar{x}_0, y_0) - u_z z_{yy}(x_0^*(T_0), y_0)\right. \\
&\qquad \left. - u_{zz} z_y^2(x_0^*(T_0), y_0) - u_z z_{xy}(x_0^*(T_0), y_0)\, x_T^* - u_{zz} z_y z_x(x_0^*(T_0), y_0)\, x_T^*\right\} \\
&\qquad\qquad \cdot \left\{z_y(\bar{x}_0, y_0) + \delta\left[(1-\sigma) + T_1'(T_0)\right] z_y(x_1, y_1) + \ldots\right\}
\end{aligned}$$

The second and fourth terms cancel at $x_0^*(T_0) = \bar{x}_0$, as do the first and third, leaving

$$\begin{aligned}
&\mu u_z \left\{- u_z z_{xy}(x_0^*(T_0), y_0)\, x_T^* - u_{zz} z_y z_x(x_0^*(T_0), y_0)\, x_T^*\right\} \\
&\qquad \cdot \left\{z_y(\bar{x}_0, y_0) + \delta\left[(1-\sigma) + T_1'(T_0)\right] z_y(x_1, y_1) + \ldots\right\} \qquad (A.13)
\end{aligned}$$

An expression for x_T^* comes from totally differentiating $\mu u_z z_x(x_0^*, y_0) = 1$:

$$\begin{aligned}
\left[\mu u_{zz} z_x^2(x_0^*, y_0) + \mu u_z z_{xx}(x_0^*, y_0)\right] dx_0 &= -\left[\mu u_z z_{xy}(x_0^*, y_0) + \mu u_{zz} z_x z_y(x_0^*, y_0)\right] dT_0 \\
\rightarrow x_T^* &= -\frac{u_z z_{xy}(x_0^*, y_0) + u_{zz} z_x z_y(x_0^*, y_0)}{u_z z_{xx}(x_0^*, y_0) + u_{zz} z_x^2(x_0^*, y_0)} > 0 \qquad (A.14)
\end{aligned}$$

Thus, given $u_z z_{xy} > -u_{zz} z_y z_x$, one can sign (A.13) as negative. It follows that $\partial^2 T_0 / \partial\mu\partial\omega < 0$.

Proof of Proposition 2. Applying Cramer's rule to (A.9) yields, for $\omega = 0$:

$$\frac{\partial T_0}{\partial \bar{x}_0} = -\frac{N_{21}}{D_1} = -\frac{\mu u_z z_{xy}(\bar{x}_0, y_0) + \mu u_{zz} z_x z_y(\bar{x}_0, y_0)}{\begin{array}{r}\mu u_z z_{yy}(\bar{x}_0, y_0) + \mu u_{zz} z_y^2(\bar{x}_0, y_0) + \delta\left[(1-\sigma) + T_1'(T_0)\right]^2 \cdot \\ \left[\mu u_z z_{yy}(x_1, y_1) + \mu u_{zz} z_y^2(x_1, y_1)\right] + \ldots\end{array}} > 0$$

because $u_z z_{xy} > -u_{zz} z_y z_x$. For $\omega = 1$ and $\beta_0 = 1$ - the case of extreme regret in the initial period - one may apply Cramer's rule to (A.11), yielding

$$\frac{\partial T_0}{\partial \bar{x}_0} = -\frac{u_z z_{xx}(\bar{x}_0, y_0^*) + u_{zz} z_x^2(\bar{x}_0, y_0^*)}{\mu u_z z_{xy}(\bar{x}_0, y_0^*) + \mu u_{zz} z_x z_y(\bar{x}_0, y_0^*)} > 0$$

To understand what happens as ω varies between 0 and 1, apply Cramer's rule to (A.9) for general ω and then differentiate with respect to ω:

$$\begin{aligned} \frac{\partial T_0}{\partial x_0} &= -\frac{(1-\omega\beta_0)\,N_{21}+\omega\beta_0 N_{22}}{(1-\omega\beta_0)\,D_1+\omega\beta_0 D_2} \\ \rightarrow \frac{\partial^2 T_0}{\partial x_0 \partial\omega} &= -\beta_0 \frac{[(1-\omega\beta_0)D_1+\omega\beta_0 D_2](N_{22}-N_{21})-[(1-\omega\beta_0)N_{21}+\omega\beta_0 N_{22}](D_2-D_1)}{[(1-\omega\beta_0)D_1+\omega\beta_0 D_2]^2} \end{aligned} \tag{A.15}$$

The sign of (A.15) takes on the sign of the numerator. Simplifying along analogous lines to the proof for Proposition 1:

$$-\left[(1-\omega\beta_0)\,D_1+\omega\beta_0 D_2\right](N_{22}-N_{21})+\left[(1-\omega\beta_0)\,N_{21}+\omega\beta_0 N_{22}\right](D_2-D_1)=-D_1 N_{22}+D_2 N_{21}$$

Substitution back yields

$$\begin{aligned} &-\left\{\mu u_z z_{yy}\left(\bar{x}_0,y_0\right)+\mu u_{zz} z_y^2\left(\bar{x}_0,y_0\right)+\delta\left[(1-\sigma)+T_1^{'}\left(T_0\right)\right]^2\left[\mu u_z z_{yy}\left(x_1,y_1\right)+\mu u_{zz} z_y^2\left(x_1,y_1\right)\right]+\ldots\right\} \\ &\cdot\left\{\mu u_z z_{xy}\left(\bar{x}_0,y_0\right)+\mu u_{zz} z_x z_y\left(\bar{x}_0,y_0\right)\right\}+\left\{\mu u_z z_{yy}\left(\bar{x}_0,y_0\right)+\mu u_{zz} z_y^2\left(\bar{x}_0,y_0\right)-\mu u_z z_{yy}\left(x_0^*\left(T_0\right),y_0\right)\right. \\ &\left.-\mu u_{zz} z_y^2\left(x_0^*\left(T_0\right),y_0\right)-\mu u_z z_{xy}\left(x_0^*\left(T_0\right),y_0\right)x_T^*-\mu u_{zz} z_y z_x\left(x_0^*\left(T_0\right),y_0\right)x_T^*\right\} \\ &\cdot\left\{\mu u_z z_{xy}\left(\bar{x}_0,y_0\right)+\mu u_{zz} z_x z_y\left(\bar{x}_0,y_0\right)\right\} \end{aligned}$$

The second and fourth bracketed expressions - N_{22} and N_{21} - are equal and clearly positive. Summing the terms that multiply them and simplifying yields

$$\begin{aligned} &-\mu u_z z_{xy}\left(x_0^*\left(T_0\right),y_0\right)x_T^*-\mu u_{zz} z_y z_x\left(x_0^*\left(T_0\right),y_0\right)x_T^*-\mu u_z z_{yy}\left(x_0^*\left(T_0\right),y_0\right) \\ &-\mu u_{zz} z_y^2\left(x_0^*\left(T_0\right),y_0\right)-\delta\left[(1-\sigma)+T_1^{'}\left(T_0\right)\right]^2\left[\mu u_z z_{yy}\left(x_1,y_1\right)+\mu u_{zz} z_y^2\left(x_1,y_1\right)\right]+\ldots \end{aligned}$$

Evaluate just the first four terms, substituting (A.14):

$$
\begin{aligned}
&-\left[\mu u_z z_{xy}\left(x_0^*\left(T_0\right), y_0\right)+\mu u_{zz} z_y z_x\left(x_0^*\left(T_0\right), y_0\right)\right]\left[-\frac{u_z z_{xy}\left(x_0^*\left(T_0\right), y_0\right)+u_{zz} z_x z_y\left(x_0^*\left(T_0\right), y_0\right)}{u_z z_{xx}\left(x_0^*\left(T_0\right), y_0\right)+u_{zz} z_x^2\left(x_0^*\left(T_0\right), y_0\right)}\right] \\
&\qquad -\mu u_z z_{yy}\left(x_0^*\left(T_0\right), y_0\right)-\mu u_{zz} z_y^2\left(x_0^*\left(T_0\right), y_0\right) \\
&=\mu\left[u_z z_{xy}\left(x_0^*\left(T_0\right), y_0\right)+u_{zz} z_y z_x\left(x_0^*\left(T_0\right), y_0\right)\right]^2\left[\frac{1}{u_z z_{xx}\left(x_0^*\left(T_0\right), y_0\right)+u_{zz} z_x^2\left(x_0^*\left(T_0\right), y_0\right)}\right] \\
&\qquad -\mu u_z z_{yy}\left(x_0^*\left(T_0\right), y_0\right)-\mu u_{zz} z_y^2\left(x_0^*\left(T_0\right), y_0\right)
\end{aligned}
$$

Multiply through by $-u_z z_{xx}\left(x_0^*\left(T_0\right), y_0\right)-u_{zz} z_x^2\left(x_0^*\left(T_0\right), y_0\right)$ and the sign is the same. Removing the arguments to save space:

$$
\begin{aligned}
&-\mu\left[u_z z_{xy}+u_{zz} z_y z_x\right]^2+\mu u_z^2 z_{xx} z_{yy}+\mu u_z u_{zz} z_x^2 z_{yy}+\mu u_z u_{zz} z_{xx} z_y^2+\mu u_{zz}^2 z_x^2 z_y^2 \\
&=-\mu\left[u_z^2 z_{xy}^2+u_{zz}^2 z_y^2 z_x^2+2 u_z u_{zz} z_{xy} z_y z_x\right]+\mu u_z^2 z_{xx} z_{yy}+\mu u_z u_{zz} z_x^2 z_{yy}+\mu u_z u_{zz} z_{xx} z_y^2+\mu u_{zz}^2 z_x^2 z_y^2 \\
&=\mu\left[-u_z^2 z_{xy}^2-u_{zz}^2 z_y^2 z_x^2-2 u_z u_{zz} z_{xy} z_y z_x+u_z^2 z_{xx} z_{yy}+u_z u_{zz} z_x^2 z_{yy}+u_z u_{zz} z_{xx} z_y^2+u_{zz}^2 z_x^2 z_y^2\right] \\
&=\mu\left[-2 u_z u_{zz} z_{xy} z_y z_x+u_z u_{zz} z_x^2 z_{yy}+u_z u_{zz} z_{xx} z_y^2\right] \\
&=-\mu u_z u_{zz}\left[2 z_{xy} z_y z_x-z_x^2 z_{yy}-z_{xx} z_y^2\right]=-\mu u_z u_{zz}\left[2 \sqrt{z_{xx} z_{yy}} z_y z_x-z_x^2 z_{yy}-z_{xx} z_y^2\right] \\
&=-\mu u_z u_{zz}\left[z_x \sqrt{-z_{yy}}+z_y \sqrt{-z_{xx}}\right]^2>0
\end{aligned}
$$

because, due to homogeneity of z, $z_{xx} z_{yy}=z_{xy}^2$. Thus $\partial^2 T_0 / \partial \bar{x}_0 \partial \omega>0$.

Proof of Proposition 3. Use the first-order conditions without regret in the case of an endowment $\bar{x}_0$. As noted in the proof of Lemma 1, $(1-\sigma) y_0\left(\bar{x}_0^1\right)=y^*$ defines the threshold of a corner solution for adjustive thinking in $t=1$, where $\bar{x}_0^1$ gives the level of the endowment that corresponds to this threshold. At the threshold itself, the unique interior solution that obtains for $t>0$ remains unaffected, thus $\left(x_t, y_t\right)=\left(x^*, y^*\right)$ for $t \geq 1$ even when $\left(x_0, y_0\right)=\left(\bar{x}_0^1, \frac{y^*}{1-\sigma}\right)$. $\bar{x}_0^1$ is defined implicitly by the first-order condition with respect to T,

$$
\begin{aligned}
\mu u_z z_y\left(\bar{x}_0^1, \frac{y^*}{1-\sigma}\right)+\delta(1-\sigma) \mu u_z z_y\left(x_1, y_1\right)+\ldots &= 1 \\
\Rightarrow \mu u_z z_y\left(\bar{x}_0^1, \frac{y^*}{1-\sigma}\right)+\delta(1-\sigma) \mu u_z z_y\left(x^*, y^*\right)+\ldots &= 1
\end{aligned}
$$

All the terms of the first-order condition except the first are identical when $(x_0, y_0) = (x^*, y^*)$, whence one can write

$$\mu u_z z_y \left(\bar{x}_0^1, \frac{y^*}{1-\sigma}\right) = \mu u_z z_y (x^*, y^*) \tag{A.16}$$

Now consider the threshold of a corner solution for adjustive thinking in $t = 2$, defined by corresponding expression $(1-\sigma)^2 y_0 \left(\bar{x}_0^2\right) = y^*$. Given the depreciation rate of adjustment, $y_1 = \frac{y^*}{1-\sigma}$. Therefore, because the process is the same every period, there is an expression identical to (A.16) that defines a hypothetical $t = 1$ threshold level of consumption $\bar{x}_1^2$, such that $\bar{x}_1^2 = \bar{x}_0^1$. Using the first-order condition with respect to T, then,

$$\begin{aligned} \mu u_z z_y \left(\bar{x}_0^2, \frac{y^*}{(1-\sigma)^2}\right) + \delta (1-\sigma) \mu u_z z_y (x_1, y_1) + \ldots &= 1 \\ \Longleftrightarrow \mu u_z z_y \left(\bar{x}_0^2, \frac{y^*}{(1-\sigma)^2}\right) + \delta (1-\sigma) \mu u_z z_y (x^*, y^*) + \ldots &= 1 \end{aligned}$$

whence one can write

$$\mu u_z z_y \left(\bar{x}_0^2, \frac{y^*}{(1-\sigma)^2}\right) = \mu u_z z_y (x^*, y^*) \tag{A.17}$$

By induction, the level of the endowment that corresponds to the threshold of a corner solution for adjustive thinking in $t = \tau$ is defined implicitly by

$$\mu u_z z_y \left(\bar{x}_0^\tau, \frac{y^*}{(1-\sigma)^\tau}\right) = \mu u_z z_y (x^*, y^*)$$

Properties (A1)-(A3) imply complementarity of x and y in utility. Thus, $\bar{x}_0 > \bar{x}_0^\tau$ implies $y_t > y^*$ for all $t \leq \tau$, which in turn implies $x_t > x^*$, which is what we sought to show.

Proof of Proposition 4. The proof of Proposition 3 establishes that for the no-regret case there exists a threshold endowment level for every τ that is defined implicitly by $(1-\sigma)^\tau y_0 (\bar{x}_0^\tau) = y^*$ and by the no-regret first-order condition with respect to T. Proposition 2 establishes the manner in which the first-order condition for T changes with regret, whereby $\partial^2 T_0 / \partial \bar{x}_0 \partial \omega > 0$. From the tautology $(1-\sigma)^\tau y_0 (\bar{x}_0^\tau) = y^*$ it then follows from $\partial^2 T_0 / \partial \bar{x}_0 \partial \omega > 0$ that $\frac{\partial \bar{x}_{0\tau}(\omega)}{\partial \omega} < 0$.

Proof of Proposition 5. Consider first the extreme regret case $\beta_\tau = \omega = 1$. It is clear from analysis of (4) that $\bar{x}_0 > x^*$ implies $y_\tau > y^*$. Now instead suppose $\beta_\tau > 0$ with $\omega \in (0, 1]$. We consider two cases. Suppose first that $(1-\sigma)^\tau y_0(\bar{x}_0) > y^*$. Then $y_\tau > y^*$ for $\beta_\tau = 0$, whence by complementarity of x and y, it follows that $x_\tau > x^*$ for $\beta_\tau = 0$. As it has already been established that $y_\tau > y^*$ in the extreme regret case, then $y_\tau > y^*$ also when $\beta_\tau > 0$ with $\omega \in (0, 1]$, and we are done. Suppose instead that $(1-\sigma)^\tau y_0(\bar{x}_0) \leq y^*$. Then $y_\tau = y^*$ for $\beta_\tau = 0$. But since $y_\tau > y^*$ in the extreme regret case, it follows also that $y_\tau > y^*$ for any $\beta_\tau > 0$, whence by complementarity of x and y, $x_\tau > x^*$, and we are done.

Proof of Proposition 6. Consider first the extreme regret case $\beta_\tau = \omega = 1$. It is clear from analysis of (4) that $\bar{x}_0 < x^*$ implies $y_\tau < y^*$. Now instead suppose $\beta_\tau > 0$ with $\omega \in (0, 1]$. It is obvious that $(1-\sigma)^\tau y_0(\bar{x}_0) < y^*$. This means $y_\tau = y^*$ for $\beta_\tau = 0$. But since $y_\tau < y^*$ in the extreme regret case, it follows also that $y_\tau < y^*$ for any $\beta_\tau > 0$, whence by complementarity of x and y, $x_\tau < x^*$, and we are done.

References

Acharya, Avidit, Blackwell, Matthew, & Sen, Maya. 2016. Explaining Preferences from Behavior: A Cognitive Dissonance Approach. *Manuscript, Stanford University.*

Akerlof, George A, & Dickens, William T. 1982. The economic consequences of cognitive dissonance. *The American economic review*, **72**(3), 307–319.

Angrist, Joshua D, & Pischke, Jörn-Steffen. 2008. *Mostly harmless econometrics: An empiricist's companion.* Princeton university press.

Ariely, Dan, & Norton, Michael I. 2008. How actions create–not just reveal–preferences. *Trends in Cognitive Sciences*, **12**(1), 13–16.

Aronson, Elliot. 2004. *The social animal.* 9th edn. Worth Publishers.

Baliga, Sandeep, & Ely, Jeffrey C. 2011. Mnemonomics: the sunk cost fallacy as a memory kludge. *American Economic Journal: Microeconomics*, **3**(4), 35–67.

Becker, Gary S. 1965. A Theory of the Allocation of Time. *The economic journal*, 493–517.

Becker, Gary S. 1985. Human Capital, Effort, and the Sexual Division of Labor. *Journal of Labor Economics*, S33–S58.

Becker, Gary S, & Murphy, Kevin M. 1988. A theory of rational addiction. *Journal of political Economy*, **96**(4), 675–700.

Cheema, Amar, & Bagchi, Rajesh. 2011. The effect of goal visualization on goal pursuit: Implications for consumers and managers. *Journal of Marketing*, **75**(2), 109–123.

Chih, Wen-Hai, Liou, Dah-Kwei, & Hsu, Li-Chun. 2015. From positive and negative cognition perspectives to explore e-shoppers' real purchase behavior: an application of tricomponent attitude model. *Information Systems and e-Business Management*, **13**(3), 495–526.

Dawkins, R, & Carlisle, T R. 1976. Parental investment, mate desertion and a fallacy. *Nature*, **262**, 131–132.

DellaVigna, Stefano. 2009. Psychology and economics: Evidence from the field. *Journal of Economic literature*, **47**(2), 315–372.

Dickens, William T. 1986. Crime and punishment again: the economic approach with a psychological twist. *Journal of Public Economics*, **30**(1), 97–107.

Epley, Nicholas, & Gilovich, Thomas. 2016. The Mechanics of Motivated Reasoning. *The Journal of Economic Perspectives*, **30**(3), 133–140.

Eyster, Erik. 2002. Rationalizing the past: A taste for consistency. *Nuffield College Mimeograph*.

Festinger, Leon. 1962. *A theory of cognitive dissonance*. Vol. 2. Stanford university press.

Festinger, Leon, & Carlsmith, James M. 1959. Cognitive consequences of forced compliance. *The Journal of Abnormal and Social Psychology*, **58**(2), 203.

Gneezy, Uri, & Rustichini, Aldo. 2000. A fine is a price. *The Journal of Legal Studies*, **29**(1), 1–17.

Goldman, Steven M. 1980. Consistent plans. *The Review of Economic Studies*, **47**(3), 533–537.

Grimm, Pamela E. 2005. Ab components' impact on brand preference. *Journal of Business Research*, **58**(4), 508–517.

Hotelling, Harold. 1929. Stability in Competition. *Economic Journal*, **39**(153), 41–57.

Izuma, Keise, Matsumoto, Madoka, Murayama, Kou, Samejima, Kazuyuki, Sadato, Norihiro, & Matsumoto, Kenji. 2010. Neural correlates of cognitive dissonance and choice-induced preference change. *Proceedings of the National Academy of Sciences*, **107**(51), 22014–22019.

Jarcho, Johanna M, Berkman, Elliot T, & Lieberman, Matthew D. 2011. The neural basis of rationalization: Cognitive dissonance reduction during decision-making. *Social Cognitive and Affective Neuroscience*, **6**(4), 460–467.

Kahneman, Daniel, Knetsch, Jack L, & Thaler, Richard H. 1991. Anomalies: The endowment effect, loss aversion, and status quo bias. *The journal of economic perspectives*, **5**(1), 193–206.

Kitayama, Shinobu, Snibbe, Alana Conner, Markus, Hazel Rose, & Suzuki, Tomoko. 2004. Is there any 'free' choice? Self and dissonance in two cultures. *Psychological Science*, **15**(8), 527–533.

Kitayama, Shinobu, Chua, Hannah Faye, Tompson, Steven, & Han, Shihui. 2013. Neural mechanisms of dissonance: An fMRI investigation of choice justification. *Neuroimage*, **69**, 206–212.

Kőszegi, Botond, & Rabin, Matthew. 2006. A model of reference-dependent preferences. *Quarterly Journal of Economics*, 1133–1165.

Laibson, David. 1997. Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, **112**(2), 443–478.

Lieberman, Matthew D, Ochsner, Kevin N, Gilbert, Daniel T, & Schacter, Daniel L. 2001. Do amnesics exhibit cognitive dissonance reduction? The role of explicit memory and attention in attitude change. *Psychological Science*, **12**(2), 135–140.

Mittelstaedt, Robert. 1969. A dissonance approach to repeat purchasing behavior. *Journal of Marketing Research*, 444–446.

Morewedge, Carey K, Shu, Lisa L, Gilbert, Daniel T, & Wilson, Timothy D. 2009. Bad riddance or good rubbish? Ownership and not loss aversion causes the endowment effect. *Journal of Experimental Social Psychology*, **45**(4), 947–951.

Nagler, Matthew G. 2016. Differentiated Product Equilibrium with Adjustment to Choice. *City College of New York working paper*.

Nagler, Matthew G. 2017. Assisted Self-Persuasion: Advertising with Consumer Adjustment to Choice. *City College of New York working paper.*

Nelson, Phillip. 1974. Advertising as information. *Journal of political economy*, **82**(4), 729–754.

O'Donoghue, Ted, & Rabin, Matthew. 1999. Doing it now or later. *American Economic Review*, 103–124.

Peleg, Bezalel, & Yaari, Menahem E. 1973. On the existence of a consistent course of action when tastes are changing. *The Review of Economic Studies*, **40**(3), 391–401.

Pollak, Robert A. 1968. Consistent planning. *The Review of Economic Studies*, **35**(2), 201–208.

Qin, Jungang, Kimel, Sasha, Kitayama, Shinobu, Wang, Xiaoying, Yang, Xuedong, & Han, Shihui. 2011. How choice modifies preference: Neural correlates of choice justification. *NeuroImage*, **55**(1), 240–246.

Sharot, Tali, De Martino, Benedetto, & Dolan, Raymond J. 2009. How choice reveals and shapes expected hedonic outcome. *Journal of Neuroscience*, **29**(12), 3760–3765.

Sharot, Tali, Velasquez, Cristina M, & Dolan, Raymond J. 2010. Do decisions shape preference? Evidence from blind choice. *Psychological Science*, **21**(9), 1231–1235.

Staw, Barry M. 1976. Knee-deep in the big muddy: A study of escalating commitment to a chosen course of action. *Organizational behavior and human performance*, **16**(1), 27–44.

Stigler, George J, & Becker, Gary S. 1977. De gustibus non est disputandum. *The american economic review*, **67**(2), 76–90.

Sun, Monic. 2011. Disclosing multiple product attributes. *Journal of Economics & Management Strategy*, **20**(1), 195–224.

Sunstein, Cass R, Bobadilla-Suarez, Sebastian, Lazzaro, Stephanie C, & Sharot, Tali. 2017. How people update beliefs about climate change: Good news and bad news. *Cornell Law Review*, **102**(6), 1431–1443.

Thaler, Richard. 1980. Toward a positive theory of consumer choice. *Journal of Economic Behavior & Organization*, **1**(1), 39–60.

Tompson, Steven, Chua, Hannah Faye, & Kitayama, Shinobu. 2016. Connectivity between mPFC and PCC predicts post-choice attitude change: The self-referential processing hypothesis of choice justification. *Human brain mapping*, **37**(11), 3810–3820.

Van Veen, Vincent, Krug, Marie K, Schooler, Jonathan W, & Carter, Cameron S. 2009. Neural activity predicts attitude change in cognitive dissonance. *Nature Neuroscience*, **12**(11), 1469–1474.

Wakslak, Cheryl J. 2012. The experience of cognitive dissonance in important and trivial domains: A construal-level theory approach. *Journal of Experimental Social Psychology*, **48**(6), 1361–1364.