*Building Credit History with Heterogeneously-Informed Lenders**

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Abstract

This paper examines a novel mechanism of credit-history building as a way of aggregating information across multiple lenders. We build a dynamic model with multiple competing lenders, who have heterogeneous private information about a consumer's creditworthiness, and extend credit over multiple stages. Acquiring a loan at an early stage serves as a positive signal—it allows the borrower to convey to other lenders the existence of a positively informed lender (advancing that early loan)—thereby convincing other lenders to extend further credit in future stages. This signaling is costly to the least risky borrowers for two reasons. First, taking on an early loan may involve cross subsidization from the least risky borrowers to more risky borrowers. Second, the least risky borrowers may take inefficiently large loans relative to the symmetric-information benchmark. We demonstrate that, despite these two possible costs, the least risky borrowers actually prefer these equilibria to those without information aggregation. Our analysis offers an interesting and novel insight into debt dilution. Contrary to the conventional wisdom, we show that in our model, the original lender is more likely to be repaid if his loan is diluted by more (i.e., borrowers who take on larger additional loans are more likely to repay than ones with smaller additional loans). The reason is that a larger additional loan implies that other lenders have positive signals about the borrower. In other words, while a borrower of a given riskiness is more likely to default on a larger loan, in our model, due to information aggregation, larger loans are only given to the least risky borrowers.

Keywords: Credit History, Information Aggregation, Debt Dilution

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1 Introduction

Credit histories play an essential role in determining individuals' access to credit. It is important to understand not only how credit histories affect lending, but also how borrowers can affect their credit histories. Existing literature treats credit histories as merely a way of keeping track of *public* information regarding an individual's risk profile. In contrast, we highlight the role of credit histories in aggregating *disperse* information among multiple (prospective) lenders. Specifically, we think of borrowers building their credit histories by taking on loans. The ability to qualify for a loan from one lender conveys that lender's positive information to other lenders. This mechanism is complementary to the more conventional story of signaling one's type via repayment of existing loans. Ours is the first paper to explicitly model how borrowers may affect this information aggregation through sequential borrowing.[1]

We build a dynamic model with multiple competing lenders, who have heterogeneous private information about a consumer's creditworthiness.[2] We explore how this private information is aggregated through lending that takes place over multiple stages. There are two key forces at play. Acquiring a loan at an early stage serves as a positive signal—it allows the borrower to convey to other lenders the existence of a positively informed lender (advancing that early loan)—thereby convincing other lenders to extend further credit in future stages. On the other hand, this signaling is costly to the least risky borrowers for two reasons. First, taking on an early loan may involve cross subsidization from the least risky borrowers (those with all positive signals) to more risky borrowers (those with mixed signals). Second, either the presence of cross subsidization or the threat of it makes the least risky borrowers may take inefficiently large loans relative to the symmetric-information benchmark. We demonstrate that despite these two possible costs, the least risky borrowers actually prefer these equilibria to those without information aggregation. We interpret the mechanism of taking an early loan to signal their credit-worthiness to other lenders as *building a credit history*. It captures conventional wisdom present in consumer credit

[1] Chatterjee et al. (2016) also analyze the dynamics of credit scores, but while they consider how *repayment* behavior affects a borrower's *reputation*, we think of borrowing itself as being a signal of the borrower's credit-worthiness.

[2] One way to interpret the assumption of heterogeneous information is to think about lenders observing the same credit history of the consumer, but employing different, imperfectly correlated, models of credit risk to evaluate it. Alternatively, one can imagine lenders collecting information about the consumer in addition to that contained in the credit report.

markets, but absent in the academic discourse, that one way to quickly build a positive credit history is to take on a loan.

Our analysis offers interesting insights into debt dilution, where further lending dilutes existing loans by increasing the consumer's probability of default, and hence decreasing the probability that the consumer repays their original lender. Contrary to conventional wisdom, we show that in our model, consumers who take on larger additional loans are more likely to repay their incumbent lenders than those who take on smaller additional loans. Our analysis provides novel (potentially) testable implications on lending and repayment behavior in the presence of non-exclusivity in consumer credit markets.

In order to study these issues, we build a parsimonious dynamic model of consumer credit with heterogeneously informed lenders. Our model features risk-averse borrowers and competing, risk-neutral lenders. There are two periods, and the first period has two stages. Each borrower has zero income in the first period, and uncertain income in the second period. In the beginning of the first period, lenders receive private signals about the distribution of the borrower's income in the second period. For simplicity, we assume that signals are binary—positive or negative. Lenders can offer loan contracts—described by the loan size and price—to the borrower over the two stages of the first period. Lenders do not observe each others' contracts, but they observe the contract that the borrower accepts.

We analyze Perfect Bayesian Equilibria in this environment. There can be multiple equilibria, and to make the model predictions applicable for empirical testing, it is desirable to have a unique equilibrium outcome. To achieve that, we use the intuitive criterion by Cho and Kreps (1987) (which is a standard way to refine the set of equilibria in signaling games). We show that the unique equilibrium outcome has a property that (the least risky) borrowers choose to build credit history by taking out early loans, to signal their creditworthiness to lenders.

The mechanism of credit-history building is as follows. Borrowers who see offers from lenders in the first stage conclude that these lenders have positive signals about them since negatively-informed lenders do not make offers in the first stage. To transmit this information to other lenders, these borrowers accept an offer—i.e. take out a loan—from a positively informed lender in the first stage. Lenders who see that a borrower accepted an offer conclude that this offer came from a lender with a positive signal, update their belief about the borrower's creditworthiness upwards, and offer better contract terms in the second stage. Importantly, the signaling of the borrower's creditworthiness comes at a cost. First, if the early loan offer is accepted by borrowers with different realizations of signals (by borrowers with all positive signals and by borrowers with some positive and some negative signals), then the price of the loan is such that the least risky borrowers (those with all positive signals) cross-subsidize the more risky borrowers (those some positive and some negative signals). Finally, either because of cross subsidization or in order to avoid it, the least risky borrowers may take inefficiently large loans relative to the symmetric-information benchmark. We show that despite these costs, the least risky borrowers find it optimal to take on early loans to build a credit history. More specifically, the intuitive criterion picks the unique equilibrium (outcome) that is most preferred by the least risky borrowers, and it features credit-history building.

It is important to distinguish credit-history building from improving a credit score. Credit scores are meant to be a summary statistic for borrowers' probability of default. Building a credit history in our model may actually lower a borrower's credit score. Borrowers who take on early loans successfully communicate that they have a lower default probability *for a given loan size*, but they also end up with a higher default probability in equilibrium due to taking on a *larger* loan.[3]

While credit-history building increases the default probability in our model, surprisingly, the same may not be true regarding the extent of loan dilution. Our model yields a striking result that when the original lender faces uncertainty about how much his early loan will be diluted, he is actually *more* likely to be repaid when the borrower takes a *larger* additional loan. The reason is that large (additional) loans are only given to the least risky borrowers in equilibrium. And while taking out a larger loan—for a given quality borrower—increases the risk of default, it turns out that the least risky borrower is still more likely to repay a large loan than the more risky borrower is to repay a medium-size loan. We refer to this finding at the "more-dilution-is-better" result. It provides a new insight into the issue of debt dilution, where more dilution (a larger additional loan) is typically considered to decrease the probability of repayment on the original loan (as in, e.g., Bizer and DeMarzo, 1992). Our mechanism has this "dilution effect" as well: for a borrower of a given risk/quality, a larger loan increases the probability of default. But there is also an additional, "selection effect": less risky/better quality borrowers take out

[3]Correspondingly, the mechanism we are highlighting is distinct and complementary to the idea of doctoring one's credit score, as in, for example, Hu et al. (2017).

larger loans. This selection effect dominates the dilution effect. Importantly, information aggregation is key for this result: a larger top-up loan conveys positive information of the diluting lender.

We also explore how equilibrium outcomes and the costs of credit-history building change as we vary model parameters, specifically, the lenders' signal quality. We find that cross subsidization occurs for low enough and high enough values of the signal precision, and excessive borrowing occurs for intermediate values of the signal precision. The two costs can be present simultaneously, or one at a time. We also illustrate that the "more-dilution-is-better" result is relevant for high enough values of the signal precision.

Our paper offers a new way of interpreting some findings of a growing empirical literature, including Liberman et al. (2017), who look at the effects of taking a payday loan on financial outcomes. The mechanism we are highlighting may help explain why taking on an additional (payday) loan does not lead to any additional financial distress for the borrowers with the lowest ex-ante credit scores.

A key feature of our model is non-exclusivity of relations between borrowers and lenders. Although a large literature has examined consumer credit markets, it has typically assumed exclusivity of debt contract—see, e.g., Chatterjee et al. (2007), Livshits et al. (2007), and surveys by Athreya (2005) and Livshits (2015). While debt dilution is a prominent feature of recent papers on defaultable debt in international finance—see, e.g., Chatterjee and Eyigungor (2012, forthcoming), and Arellano and Ramanarayanan (2012)—the questions studied in that literature are very different from those in the consumer credit literature. The idea of information aggregation among lenders is new to either literature and constitutes our central contribution.

Our paper also provides a theory of why borrowers take loans from multiple lenders. This important feature is absent, for example, from a seminal paper by Bizer and DeMarzo (1992), which shows that the anticipation of debt dilution leads to a too large loan at a too large interest rate, but the whole loan can as well be originated by a single lender. Parlour and Rajan (2001) provide a theory of borrowing from multiple lenders, but in their model borrowing is not sequential, and there is no credit-history building, which is the focus of our paper.

2 The Model

The Environment

There are two periods, I and II, and period I consists of two stages, 1 and 2. We study the interaction between a single borrower and multiple ($2 \times K$, $K \geq 2$) competing lenders.[4] The borrower has no endowment in period I.[5] Her endowment e in period II is stochastic, drawn from a finite support $\{e_\ell, e_m, e_h\}$, where $0 < e_\ell < e_m < e_h$. The probability distribution over these endowment realizations depends on the borrower's unobservable "quality" state $s \in \{g, b\}$. Let $\pi(e, s)$ denote the probability that a borrower with quality s receives endowment e in period II. We assume that the endowment distribution of the g-borrowers first-order stochastically dominates that of the b-borrowers. The ex-ante probability that a borrower's quality is g (and the share of g-borrowers in the population) is $\alpha \in (0, 1)$.

Each borrower (consumer) is risk averse and derives utility from consumption in each of the two periods according to the per-period utility function u: $[0, +\infty) \to \bar{\mathbb{R}}$. The function u is continuous, strictly increasing, and strictly concave. The borrower discounts period-II utility with the discount factor $\beta \in (0, 1)$. Note that there is no discounting across stages within period I. Lenders are risk neutral, have deep pockets, and discount period-II payoffs with the discount factor $\bar{q} = (1 + \bar{r})^{-1}$, where $\bar{r}$ is the risk-free rate of interest.

The only financial instrument available in the economy is a non-contingent defaultable bond payable in period II. If the borrower defaults (fails to pay the full amount owed), she suffers a loss of fraction φ of her endowment. This cost of default is a dead-weight loss, as the lost portion of endowment is destroyed and not transferred to the lenders.

At the beginning of period I, each lender receives a private signal, σ, about the borrower's ex-ante quality. The signals are binary, with support $\{P, N\}$. There are two equal-size classes of lenders, which differ only in the realization of the signal they receive.[6] Within each class, lenders observe the same signal, while signals across the two classes are conditionally independent. For concreteness, we assume that the signal a lender receives is drawn from a distribution that depends on the unobservable quality state of the borrower: $\Pr(\sigma = P | s = g) = \Pr(\sigma = N | s = b) = (1 + \rho)/2$. We refer to $\rho \in (0, 1)$ as the *precision* of the signal.

[4] We can equivalently assume that there are many borrowers.

[5] The assumption of zero endowment is for expositional simplicity only. All of our analysis and results extend if we assume that the borrower has a positive but (relatively) small endowment in period 1.

[6] The assumption of equal sizes of the two classes is only for concreteness. What is important is that there are at least two lenders in each class, and hence they will be competing.

Timing, Information, Actions, and Payoffs

In this section, we describe the interaction between the borrower and lenders as an extensive form game. In each stage of period I, lenders simultaneously offer contracts to the borrower. A contract is a pair (x, q), where x is the face value of the loan (equivalently, the amount of bonds the borrower sells) and q is the price. That is, a borrower who accepts a contract (x, q) from a given lender in a given stage of period I, receives qx from this lender in period I, and has a (defaultable) obligation to repay x to this lender in period II.

Let $O_t = \{(x_t^k, q_t^k, k)\}_k$ denote the set of offered contracts together with the identities of lenders offering these contracts in stage $t \in \{1, 2\}$. (A lender who is not offering a contract can be thought of making an offer $(0, 0)$.) After observing the set of offered contracts in a given stage, the borrower accepts at most one contract in that stage.[7] That is, within a stage, contracts are exclusive. All lenders observe the terms of any contract accepted by the borrower in stage 1 as long as the loan size is no smaller than a minimal threshold $\underline{x}$. All lenders also observe the identity of the lender whose contract was accepted. Thus, the public history in the beginning of stage 2 is the borrower's stage-1 accepted contract, if any, and the identity of the lender whose contract was accepted. We will refer to this public history as the *credit history* of the borrower. Formally, the (public) credit history is $h_2^P = (x_1, q_1, j_1)$ if a contract (x_1, q_1) from lender j_1 was accepted in stage 1, and $h_2^P = (0, 0, 0)$ if no contract was accepted.

Suppose the consumer borrows x_1 at q_1 in stage 1 and x_2 at q_2 in stage 2. The borrower's consumption in period I is then $q_1 x_1 + q_2 x_2$, and the total loan balance carried into period II is $X := x_1 + x_2$. In period II, after observing the realized endowment, e, the borrower chooses whether to repay or default on her debt obligations. Repaying anything less than X is equivalent to defaulting, and results in a dead-weight loss of fraction φ of the endowment. Implicitly, this way of modeling consumer default ensures that partial default is never optimal for the borrower.

If the borrower defaults in period II, her consumption in that period is $(1 - \varphi)e$, and that of her lenders is 0. If the borrower repays X, she consumes $e - X$, and the lenders who lent in stages 1 and 2 consume x_1 and x_2, respectively. It follows immediately that

[7] We assume that if the borrower is indifferent between multiple offers, she accepts each of these offers with equal probability.

the borrower will repay if and only if

$$e - y \geq (1 - \varphi)e. \tag{1}$$

This implies that the borrower's payoff is

$$\pi^B = u(q_1 x_1 + q_2 x_2) + \beta u\left(\max\left\{e - x_1 - x_2, (1 - \varphi)e\right\}\right), \tag{2}$$

and the payoff to a lender associated with a contract (x, q) that he offers and that the borrower accepts in (one of the stages of) period I is

$$\pi^L = -qx + \bar{q}x\mathbb{1}_{[\phi e \geq X]}. \tag{3}$$

Equilibrium Definition

We will study Perfect Bayesian Equilibria of the game described above. In order to facilitate characterization of these equilibria, we define the sequence of problems faced by each agent in the order implied by backward induction.

In the middle of stage 2, after lenders have made their stage-2 offers, the borrower has observed two sets of offers, O_1 and O_2, and her own credit history $h_2^P = (x_1, q_1, j_1)$. Let $h_2^B = (O_1, h_2^P, O_2)$ denote this information set of the borrower. The borrower's stage-2 action is to choose an offer from O_2 (or possibly reject all offers). She does so based in part on her posterior beliefs about her own quality state induced by the history (and her understanding of lenders' strategies). We denote $\theta_2^B(e|h_2^B)$ the probability the borrower assigns in stage 2 to receiving endowment e in the second period. Note that this probability is a convolution of the posterior belief of the borrower regarding her underlying quality s and the probability distribution over outcomes implied by this quality. Of course, the borrower forms her posterior about her underlying quality based on public and private histories, as well as her understanding of lenders' equilibrium strategies—on the equilibrium path it is obtained using Bayes' rule. The borrower's stage-2 action maximizes her expected payoff under θ_2^B and so solves

$$\begin{aligned} V_2\left(h_2^B\right) = \max_{(x_2, q_2, j) \in O_2 \bigcup \{(0,0,0)\}} & u(q_1 x_1 + q_2 x_2) \\ & + \beta \sum_e \theta_2^B(e|h_2^B) u(\max\{e - x_1 - x_2, (1 - \varphi)e\}). \end{aligned} \tag{4}$$

At the beginning of the stage 2, everyone has observed the public credit history of the borrower $h_2^P = (x_1, q_1, j_1)$. Additionally, each lender k knows his private signal about the borrower's state, σ_k, and his offer to the borrower in the first stage, (x_1^k, q_1^k). Thus, the private history of the lender k is $h_2^k = (h_2^P, \sigma_k, (x_1^k, q_1^k))$. When choosing his second-stage offer, the kth lender forms expectations of other lenders' offers. Similar to the borrower, the lender forms his posterior belief $\mu_2(\sigma_{-k})$ regarding the other class's signal based in part on his understanding of equilibrium strategies. Equilibrium strategies imply a mapping from the vector of realized signals and the observed public history into an offer set O_2, which will be faced by the borrower. For any (x, q) offered by the kth lender, denote by δ_2^k the probability of that offer being accepted (as perceived by the kth lender given equilibrium strategies of the borrower and the other lenders).[8] Then, the optimal offer made by lender k solves the following maximization problem:

$$W_2^k\left(h_2^k\right) = \max_{(x,q)} \sum_{\sigma_{-k}} \mu_2(\sigma_{-k}|h_2^k)\, \delta_2^k\,(x, q) \\ \times \left[-qx - q_1 x_1 \mathbb{1}_{j_1=k} + \bar{q}\,(x + x_1 \mathbb{1}_{j_1=k}) \sum_{e} \theta_2^L(e|h_2^k, j_2 = k) \mathbb{1}_{[\varphi e \geq x_1 + x]} \right], \quad (5)$$

where $\theta_2^L(e|.)$ is the lender's posterior probability that the borrower will receive endowment e conditional on the lender's information at the beginning of stage 2 and the fact that her offer was accepted by the borrower.

In stage 1, the borrower chooses among offers in the set O_1 (and the option of rejecting all offers) to maximize

$$V_1(O_1) = \max_{(x,q,k) \in O_1 \bigcup \{(0,0,0)\}} \mathbb{E} V_2(O_1, (x, q, k), O_2(x, q, k)). \quad (6)$$

Note that the borrower understands that her choice of (x, q) influences not only her payoffs in V_2 directly, but also the set of offers she will receive in stage 2, O_2.

Similarly, lenders in stage 1 understand that the offer they make, if accepted, may influence the posteriors of other lenders in the second stage.[9] Having observed their signal,

[8] To be more precise, $\delta_2^k = \delta_2^k\left((x, q, k)|(x_1^k, q_1^k, k), O_1^{-k}(\sigma_k, \sigma_{-k}), h_2^p, O_2^{-k}(\sigma_k, \sigma_{-k}, h_2^P)\right)$, where σ_k is the signal observed by the k-th lender, σ_{-k} is the signal observed by lenders of the other class, and O_i^{-k} is the offer set excluding the offer made by the k-th lender in stage $i = 1, 2$.

[9] In our setting, an individual lender's deviation does not change the borrower's posterior, since the borrower is facing many lenders. But it may affect other lenders' posterior, since lenders do not observe

they make an offer that maximizes their expected profits:

$$W_1^k(\sigma_k) = \max_{(x,q)} \sum_{\sigma_{-k}} \mu_1(\sigma_{-k}|\sigma_k) \quad \left[\delta_1^k\,(x,q)\,W_2^k\,((x,q,k),\sigma_k,(x,q)) \right.$$
$$\left. + \left(1-\delta_1^k\,(x,q)\right) W_2^k\,((x_{-k},q_{-k},-k),\sigma_k,(x,q))\right], \quad (7)$$

where δ_1^k and θ_1^L are defined similar to their stage-2 counterparts. Note that, if accepted, the lender's offer influenced her payoffs not only directly, but also by affecting the offer set O_2 in the subsequent stage.

Definition 1. *A* Perfect Bayesian Equilibrium *consists of offer strategies for the lenders, acceptance strategies for the borrower, and posterior beliefs (for the lenders and the borrower) such that the lenders' and borrower's strategies are optimal and posterior beliefs satisfy Bayes' rule (where applicable).*

We further refine the set of equilibria by requiring that the (off-equilibrium-path) beliefs satisfy the Cho and Kreps (1987) intuitive criterion. For brevity, from now on we will refer to a Perfect Bayesian Equilibrium that satisfies the intuitive criterion just as an *intuitive equilibrium.*

Finally, define a *symmetric-information* benchmark as a variant of our environment where signals are publicly observable. We will compare equilibrium outcomes in our model to those in the benchmark.

3 Equilibrium Characterization

In this section, we establish the uniqueness of the intuitive-equilibrium outcome. We demonstrate that there is always an intuitive equilibrium that features credit-history building and analyze costs and benefits associated with such credit-history building.

For notational convenience, we will refer to a lender who observes a signal realization P (a signal realization N) as a P-lender (an N-lender). We will refer to a borrower for whom the pair of signal realizations for the two lender classes are P and N as a PN-borrower. Similarly, the PP-borrowers (NN-borrowers) are those for whom both classes of lenders observe a P (an N) signal realization. Notice that whether a borrower is PP, PN, or NN

the offer set O_1, only the borrower's choice from that set.

is initially unknown to both the borrower and lenders. Whether borrowers or lenders may be able to infer this information will depend on the strategies these agents choose.

3.1 Preliminary Results

To rule out "uninteresting" equilibria where all borrowers obtain the same small risk-free loans, we make a simple assumption that guarantees that the borrowers want larger loans:

Assumption 1. *The equilibrium loan size with uninformative signals exceeds φe_ℓ.*

This assumption ensures that (some) equilibrium loans are subject to default risk, and thus, that the signals contain relevant information. The first preliminary result is that this information is not ignored in equilibrium.

Lemma 1. *Equilibrium outcomes do not involve PP-, PN-, and NN-borrowers all having the same aggregate loan/ price terms at the end of period I.*

The result is quite intuitive. Assumption 1 rules out pooling on a safe loan. Pooling on a risky loan cannot happen in equilibrium either. Competition implies that such a loan would be priced at the pooled actuarially-fair interest rate. But P-lenders could then make positive expected profits by offering interest rates between the one that is actuarially-fair given the P signal and the pooling rate, while N-lenders would strictly prefer not to make a loan at the pooled rate.

Lemma 1 implies that offers made by lenders reflect their information. Because borrowers observe offers made to them, it immediately follows that in any equilibrium, borrowers infer the signals that lenders receive about them (at least) by the end of period I.

The fact that borrowers infer the signals in turn leads to the next key result that the equilibrium outcomes reflect not just the information of a given lender, but the *aggregated* information. In particular, even though P-lenders cannot distinguish PP- and PN-borrowers at the beginning of stage 1, in equilibrium the information of the other class of lenders must be incorporated, and PP- and PN-borrowers must end up with different loan/price terms at the end of stage 2.

Lemma 2. *Equilibrium outcomes do not involve PP- and PN-borrowers having the same aggregate loan/price terms at the end of period I.*

Proof of Lemma 2. The proof follows from the usual cream-skimming argument, familiar, e.g., from Rothschild and Stiglitz (1976). Consider a candidate equilibrium in which PP- and PN-borrowers are pooled. Under Assumption 1, these two types of borrowers perceive their own probabilities of repaying a given loan differently, a lender may construct a profitable deviation—i.e., an offer which earns strictly positive profits. Typically, such a deviation features a slightly smaller loan at a better interest rate that appeals to PP-borrowers, but not to PN-borrowers. This deviation destroys the candidate equilibrium. □

Lemma 2 guarantees that information is aggregated in equilibrium. That is, the equilibrium allocation reflects the information of *both* classes of lenders. One way to aggregate information of different lenders is to have (at least some) lenders make offers in stage 1, and (at least some) borrowers accept these offers. Then lenders whose offers do not get accepted, learn information of the lenders of the other class by observing the contract that the borrower accepted in stage 1. In particular, a PP-borrower has incentives to take on a loan in stage 1 from a lender of class j in order to signal to the other class of lenders the positive information received by class-j lenders. A borrower's taking on an early loan with the purpose of facilitating information aggregation constitutes *credit-history building.*

To facilitate our discussion, we will restrict attention to parameter values that generate equilibrium loan sizes in the set $\{\varphi e_L, \varphi e_M, \varphi e_H\}$. We will refer to the loans of these sizes as small, medium, and large, respectively. Intuitively, since for (total) loan sizes in each of the intervals $(0, \varphi e_L]$, $(\varphi e_L, \varphi e_M]$, $(\varphi e_M, \varphi e_H]$ the default probability is the same, the corresponding equilibrium loan prices will be constant as well. So if a borrower is sufficiently "hungry" (sufficiently willing to borrow), she will not choose an interior loan size, but will prefer to be at the corner.[10]

Note that credit-history building entails costs for PP-borrowers. One such cost is *cross subsidization.* When PN-borrowers accept the same loan as PP-borrowers in the first stage, the early loans are costly to PP-borrowers—they cross-subsidize PN-borrowers. The second cost, which we refer to as *excessive borrowing*, occurs when the resulting loans of PP-borrowers are larger than what they would have taken on under symmetric information. To understand excessive borrowing more clearly, suppose model parameters are such that if all lenders observed both signals in stage 1 then PP-borrowers would only take on a medium loan. We define excessive borrowing in the model when lenders only observe their

[10]This requires a mild restriction on preferences and/or endowments of the borrower to ensure that interior debt levels are not preferred to these "corner" values.

own signal as occurring when PP-borrowers end period I with a large loan. Thus, excessive borrowing occurs when PP-borrowers take on "too much" credit due to the informational frictions.[11]

Equilibria with excessive borrowing highlight the important distinction between credit-history building and improving one's credit score. Since the purpose of a credit score is to proxy a borrower's probability of repayment and information aggregation leads to larger and hence riskier loans, the credit-history building that emerges in these equilibria would result in *lower* credit scores. Taking on an early loan communicates positive information to other lenders, lowering the posterior probability of default on a given loan size. But, since information aggregation induces borrowers to take on a *larger* loan, the resulting probability of default is increased (relative to those borrowers who do not take on early loans).

Importantly, note that if PN-borrowers accept an early loan in equilibrium, they do so to take advantage of cross subsidization, not to aggregate information. They do not gain from letting an N-lender know that there is a P-lender, as borrowing from an N-lender who knows the other signal is P is the same as borrowing from a P-lender who knows the other signal is N. Only PP-borrowers have incentive to facilitate the information aggregation.

In spite of the costs associated with information aggregation, we next argue that there exist intuitive equilibria that feature credit-history building. As common to models with adverse selection, we next make a parametric restriction to ensure existence of equilibria in pure strategies.[12]

Assumption 2. *The parameters are such that, under symmetric information, PP-borrowers*

- *either prefer a large loan to a medium loan (both at actuarially-fair prices):*

$$\begin{aligned} & u(q_h^{PP}\varphi e_h) + \beta E_{PP} u\left((1-\varphi)e\right) \\ & \geq u(q_m^{PP}\varphi e_m) + \beta E_{PP} u\left(\max\left\{e-\varphi e_m, (1-\varphi)e\right\}\right), \end{aligned} \tag{8}$$

- *or, if they prefer a medium loan, they would not dilute the smallest loan $\underline{x}$ to a large*

[11] We define excessive borrowing in terms of the face value of the loan X.

[12] Our characterization of equilibrium outcomes of (possibly) mixed-strategy equilibria when Assumption 2 is violated is ongoing.

loan instead of a medium one:

$$\begin{aligned} &u(q_m^{PP}\varphi e_m) + \beta E_{PP} u\left(\max\left\{e - \varphi e_m, (1-\varphi)e\right\}\right) \\ &\geq u(q_m^{PP}\underline{x} + q_h^{PP}(\varphi e_h - \underline{x})) + \beta E_{PP} u\left((1-\varphi)e\right). \end{aligned} \tag{9}$$

Note that as $\underline{x}$ approaches 0, the set of parameters that violate Assumption 2 becomes vanishingly small.

We are now ready to state one of the main results of the paper:

Claim 1. *There is always an intuitive equilibrium with credit-history building.*

That is, there is always an intuitive equilibrium in which P- and N-lenders make different offers in the first stage and PP-borrowers accept an offer in the first stage in order to facilitate information aggregation.[13] Since PN-borrowers receive the same offer in the first stage (but only from one class of lenders), the early loans are costly to PP-borrowers—they either have to cross-subsidize PN-borrowers or pay interest rates high enough to discourage PN-borrowers from accepting this loan. Thus, the only reason PP-borrowers take on the early loan is to aggregate the information across lenders.

To highlight the process of information aggregation in the equilibrium with credit-history building, it is helpful to sketch out the equilibrium actions. For clarity of exposition, consider the equilibrium where only P-lenders make an offer in the first stage. Observing the offers in stage 1, the borrower infers the signals of both classes of lenders. Lenders only see their own signal, and do not directly observe the signal of the other class of lenders. But they can try to infer it from the actions of the borrower in the first stage. If they see a borrower accept a loan from the other lender class in the first stage (and that loan is the one prescribed by the equilibrium), they infer that the other lender class received the P signal. Note that the lender whose offer was accepted in stage 1, and all lenders from the same class, do not learn anything new from the borrower's actions in the stage 1. The ones who learn are the ones whose offer was not accepted.[14]

[13] In this equilibrium, N-lenders do not make an offer in the first stage. For some parameter values, there is an outcome-equivalent equilibrium in which N-lenders do make offer in the first stage which is distinct from that made by P-lenders.

[14] The reason we have multiple lenders in each class is to avoid monopoly power of an informed lender in the second stage, which would discourage the borrower from costly credit-history building in the first stage.

For some parameter values there may be other ways to achieve separation that do not involve credit-history building. That is, for some parameter values, one can do as well with information aggregation in stage 2 as with information aggregation in stage 1. But for many parameter values one cannot.

3.2 Features of Equilibrium Outcomes

We now provide an overview of possible equilibrium outcomes. The equilibrium strategies we consider are those where only P-lenders make offers in the first stage. In such equilibria, it is natural to classify equilibrium outcomes according to whether PP- and PN-borrowers both accept an offer made by a P-lender or whether only a PP-borrower accepts such an offer. If PP- and PN-borrowers accept the same loan in stage 1, then P-lenders must necessarily price their loans at a pooled price implying that such an equilibrium features cross subsidization from PP- to PN-borrowers. In contrast, if PP- and PN-borrowers do not accept the same loan in stage 1, then PP-borrowers are fully separated from PN- and NN-borrowers and hence in such equilibria there is no cross subsidization from PP- to PN-borrowers.

We may therefore classify equilibria according to whether they feature cross subsidization from PP- to PN-borrowers or not. Consider first equilibria with such cross subsidization so that both PP- and PN-borrowers accept the same loan in stage 1. Two types of cross-subsidizing equilibria emerge in our environment. The first equilibrium outcome features complete separation of PP-, PN- and NN- borrowers on loan *sizes*. That is, in such an equilibrium, PP-, PN- and NN-borrowers all accept different-size (and different-price) loans in stage 2. Recall that such separation is feasible because a P-lender whose stage-1 offer is rejected observes a credit history associated with the borrower accepting another P-lender's offer, and understands that such a borrower is a PP-borrower. Similarly, an N-lender who observes the same credit history understands that the borrower is a PN-borrower, while an N-lender who observes no credit history understands that the borrower is an NN-borrower. Of course, in stage 2, these differentially-informed lenders all offer loans at competitive, actuarially-fair prices. Hence, the only cross subsidization that occurs in such an equilibrium occurs from the pooling of the loan in stage 1.

The second type of equilibrium outcome with cross subsidization features pooling of PP- and PN-borrowers on loan sizes but separation on (stage-2) prices. In such an equilibrium,

PP- and PN-borrowers receive and accept the same loan size but at different prices in stage 2. In this equilibrium, NN-borrowers accept a weakly smaller loan size at lower prices (higher interest rates).

Consider next equilibria which do not feature cross subsidization from PP- to PN-borrowers. In such equilibria, only PP-borrowers accept a loan in stage 1. Similar to cross-subsidizing equilibria, equilibrium outcomes without cross subsidization may feature separation on loan sizes or partial pooling. In this case, the outcome may be *full separation* so that PP-, PN- and NN-borrowers all receive different-sized loans (again, with PP-borrowers the only borrowers who accept an offer in stage 1) or *partial pooling* on loan sizes—but at different prices—of PN- and NN-borrowers in stage 2 with separation of PP-borrowers.

As we saw in Claim 1, there is always an intuitive equilibrium that features credit-history building. Our next result states that if there are several intuitive equilibria (for the same set of parameter values), then they all must lead to the same equilibrium outcome. Combining this with the result of Claim 1, we may then conclude that focusing on equilibria that feature credit-history building is without loss of generality: if there is an intuitive equilibrium that does not feature credit-history building, there is always one that does, and it delivers the same equilibrium outcome.

Claim 2. *For any given set of parameter values there is a unique intuitive-equilibrium outcome.*

The intuitive criterion basically selects the equilibrium outcome most preferred by the "best" borrowers, i.e., the PP-borrowers. For example, if there is cross subsidization from PP- to PN-borrowers in stage 1, the intuitive criterion selects cross subsidization on the smallest visible loan, and rules out using larger loans as a way of building credit histories. While there are Perfect Bayesian Equilibria with larger than minimal visible loans in the first stage, these equilibria do not satisfy the intuitive criterion, which puts restrictions on the off-equilibrium-path beliefs. Specifically, since a smaller stage-1 loan would be preferred by PP-borrowers (since they have to pay a worse, cross-subsidizing price on the stage-1 loan), and not by PN-borrowers, the intuitive criterion rules out beliefs necessary to sustain the Perfect Bayesian Equilibrium with larger loans (which assigned an unfavorable posterior to borrowers with minimal visible loans).

As mentioned above, information aggregation (i.e., credit-history building) benefits PP-

borrowers, but also comes at a cost to these borrowers—either at the cost of cross subsidizing PN-borrowers or at the cost of excessive borrowing, or both. Yet, since the intuitive criterion always pick PP-borrowers' favorite allocation, the costly credit-history building is *preferred* by the best borrowers to the allocation without information aggregation. To be more precise, we define the *costs* of information aggregation relative to the symmetric-information benchmark—the outcome of our environment if both signals were publicly observable from the beginning (and thus all lending took place in a single stage in equilibrium). In contrast, when we say that PP-borrowers *prefer* to bear these costs, we are comparing the equilibrium outcome to the outcome of P-borrowers in an environment with a single class of lenders.

3.3 Comparative Statics of Equilibrium Outcomes and Credit-History-Building Costs

In this section, we now describe how equilibrium outcomes vary with parameter values by examining comparative statics with respect the signal precision ρ. To illustrate the costs of credit-history building that arise in our model, we fully describe equilibrium strategies for a set of high values of the signal precision. We then show which costs are present in equilibrium at various levels of the signal precision.

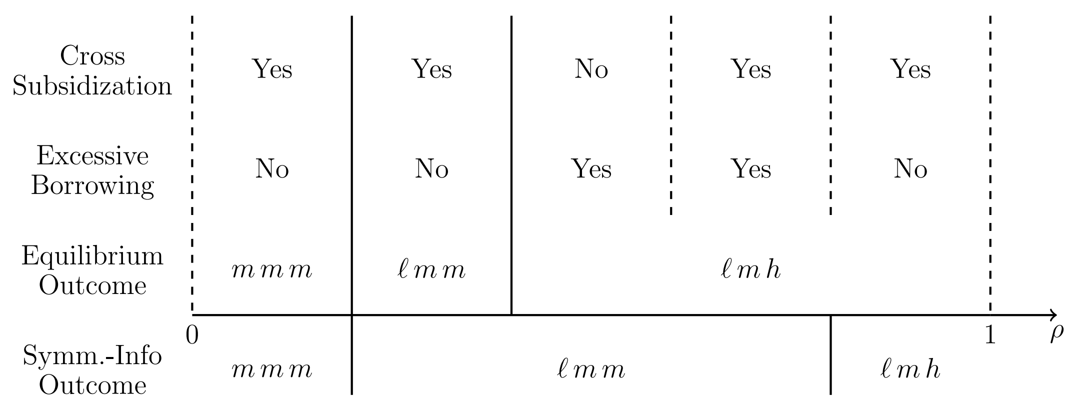

Figure 1: **Comparative statics with respect to the signal precision** ρ. Notation: ℓmh means φe_ℓ to NN-borrowers, φe_m to PN-borrowers, φe_h to PP-borrowers.

Figure 1 illustrates (from the bottom to the top) equilibrium outcomes under symmetric information, equilibrium outcomes in our game, and the costs of credit-history building.

For equilibrium outcomes, we only report the total loan sizes, using the following notation: xyz, with $x, y, z \in \{\ell, m, h\}$, meaning that NN-borrower's total loan is φe_x, PN's is φe_y, and PP's is φe_z. In the figure, we assume that the equilibrium outcome with uninformative signals would be mmm, i.e., a medium loan for all borrowers. Moreover, we assume that with arbitrarily informative signals (ρ close to one), under symmetric information there is full separation by loan size, i.e., we get the ℓmh outcome.[15]

For ρ high enough, the equilibrium depicted in the figure (columns 4 and 5) is as follows. In stage 1, N-lenders make no offers, and P-lenders offer the smallest visible loan $\underline{x}$ at price q^P defined as

$$q^P = \Pr(PP|P)q_h^{PP} + \Pr(PN|P)q_m^{PN}, \tag{10}$$

where

$$q_h^{PP} = \Pr(\text{repaying large loan}|PP)\bar{q} = \Pr(e = e_h|PP)\bar{q}, \tag{11}$$

$$q_m^{PN} = \Pr(\text{repaying medium loan}|PN)\bar{q} = \Pr(e \in \{e_m, e_h\}|PN)\bar{q}. \tag{12}$$

All borrowers with such an offer accept one. In stage 2, P-lenders whose offer was not accepted, and who see that the accepted offer was made by a lender from the other class, conclude that the borrower is PP. They offer a loan $\varphi(e_h - e_\ell)$ (i.e., top up to a large loan) at price q_h^{PP}. A PP-borrower accepts such an offer. P-lenders whose offer was accepted (or whose offer was not accepted, but the accepted offer came from a lender of the same class), or N-lenders who observed that an offer was accepted, offer $\varphi(e_m - e_\ell)$ at price q_m^{PN}. A PN-borrower accepts such an offer from one of those lenders. (Notice that P-lenders making such an offer correctly predict that only a PN-borrower would accept their offer.) Finally, N-lenders who see that no offer was accepted, conclude that this is an NN-borrower, and offer her a risk-free small loan φe_ℓ and $\bar{q}$. An NN-borrower accepts such an offer.

Notice that on the small loan in stage 1, PP-borrowers cross-subsidize PN-borrowers. In addition, the small stage-1 loan will be necessarily diluted in stage 2 to either a medium loan (for PN-borrowers) or a large loan (for PP-borrowers). Hence the price q^P of the stage-1 loan is a weighted average between the price of a large loan that only PP-borrowers accept, and a medium loan that only PN-borrowers accept.

[15] Specifically, the parameter values in the illustrative example are $e_\ell = 3$, $e_m = 8$, $e_h = 15$, $\varphi = 0.3$, $\alpha = 0.2$, $\pi(e_\ell, g) = 0$, $\pi(e_m, g) = 0.4$, $\pi(e_h, g) = 0.6$, and $\pi(e_\ell, b) = 0.7$, $\pi(e_m, b) = 0.3$, $\pi(e_h, b) = 0$, $u(c) = \ln c$, $\beta = \bar{q} = 1/1.05$.

The distinguishing feature of this equilibrium is that a PN-borrower is willing to accept the same stage-1 loan as a PP-borrower. For this to be the case, a PN-borrower must prefer the equilibrium loan outcomes to rejecting the stage-1 loan and receiving a medium loan at the PN-actuarially-fair price in stage 2. This incentive constraint simply reduces to a requirement that the stage-1 loan price q^P be better than the actuarially-fair price for PN-borrowers on a medium-sized loan, q_m^{PN}. It follows from (10) that $q^P > q_m^{PN}$ is equivalent to

$$q_h^{PP} > q_m^{PN}. \tag{13}$$

Note that (13) is the condition for cross subsidization if PP-borrowers and PN-borrowers end stage 2 with different-size loans in equilibrium. Of course, cross subsidization necessarily occurs in any equilibrium in which PP- and PN-borrowers end stage 2 with the same size loans.

Since we have described features of the equilibrium for high values of signal precision ρ, we next consider how equilibrium outcomes change as ρ falls from 1 to 0. We have chosen our parameter values so that for ρ sufficiently close to 1, the symmetric-information equilibrium outcome features full separation on loan sizes. As we have argued above, the equilibrium features cross subsidization and PP-borrowers ending stage 2 with a large loan. Since PP-borrowers take on a large loan under symmetric information, there is no excessive borrowing in this case, and since the probability of receiving mixed signals (PN) is close to zero, the cost of cross subsidization becomes arbitrarily small as ρ tends to 1. Hence, the costs of credit-history building are small when ρ is sufficiently large.

As ρ decreases, the size of the loan that a PP-borrower takes in the symmetric-information equilibrium falls from a large loan to a medium loan. The reason is that as ρ declines, a PP-borrower's perceived probability of receiving a high endowment in period II declines. That is, PP-borrowers become more pessimistic about their endowment process and choose to borrow less in period I (see the bottom row of Figure 1).

Consider what happens in our environment with private signals as ρ falls. Note that the switch from a large to medium loan by a PP-borrower does not happen at the same value of ρ in the asymmetric-information environment. Since PP-borrowers cross subsidize PN-borrowers on the stage-1 loan, their period-I consumption is lower than they would obtain in the symmetric-information benchmark for the same size loan. To increase consumption in period I, PP-borrowers end up with a large instead of a medium loan. This is the excessive-borrowing feature that we have discussed earlier. In this equilibrium, excessive borrowing occurs because of cross subsidization. This need not always be the case: excessive borrowing can emerge even in equilibria without cross subsidization, as in column 3 on Figure 1, the case that we discuss next.

As ρ decreases further, the likelihood a PP-borrower repays a large loan falls and with it the price, q_h^{PP}. On the other hand, q_m^{PN} remains unchanged. The leads to a violation of the cross-subsidization equation (13) for sufficiently low signal precision. That is, for low enough ρ, PN-borrowers would no longer receive a subsidy if they were to take a stage-1 loan and thus prefer to wait for an actuarially-fair-priced loan in stage 2.

Hence we switch to an equilibrium without cross subsidization, where only PP-borrowers accept an early loan—column 3 in the figure. To see why this equilibrium still features excessive borrowing despite the absence of cross subsidization, consider what would happen if PP-borrowers were to get a medium loan instead of a large one. Then, the equilibrium price of the stage-1 loan for them would be q_m^{PP}, which is larger than the price q_m^{PN} that PN-borrowers receive on their loan (in stage 2). So PN-borrowers would not reject the stage-1 loan in that case, making it impossible to not have cross subsidization in equilibrium. Thus, in the equilibrium in column 3, PP-borrowers over borrow to separate themselves from PN-borrowers and to avoid cross subsidization. Note that for high signal precision (column 4) excessive borrowing occurs because of cross subsidization while for lower signal precision (column 3) it occurs to prevent it.

As ρ decreases further, PP-borrowers endowment prospects become less and less favorable, which causes their price of a large loan to fall. Ultimately, these borrowers prefer to switch from a large to a medium-size loan (column 2) in equilibrium. Of course, as that happens, PN-lenders start accepting the early loan, and we again have cross subsidization.

Finally, as ρ gets sufficiently close to zero, the information content of the signals vanishes. As a consequence, borrowers with different signal combinations have sufficiently similar endowment prospects. In equilibrium (as well as in the symmetric-information benchmark), all borrowers obtain a medium loan. Note that cross subsidization only happens between PP- and PN-borrowers.

As Figure 1 illustrates, cross subsidization takes place for large enough and small enough values of the signal precision, while excessive borrowing occurs for intermediate values of signal precision. Moreover, the two costs can occur simultaneously, or one at a time.

3.4 More Dilution is Better

Our model generates a novel empirical prediction, as can be seen in the equilibrium depicted in columns 4 and 5 of Figure 1. Notice that in this equilibrium there is uncertainty for the stage-1 lender about how much his loan will be diluted in stage 2. If the borrower turns out to be a PP-borrower, the stage-1 loan will be diluted to a large loan, and if the borrower turns out to be a PN-borrower, the loan will be diluted to a medium loan. Although the lender earns zero profit ex-ante, in which of these two scenarios is he better off? In other words, in which of the two cases is the probability of being repaid higher? The answer immediately follows from equation (13) and the definitions of q_h^{PP} and q_m^{PN} (equations (11) and (12))—in this equilibrium, a PP-borrower is more likely to repay a large loan than a PN-borrower to repay a medium loan. That is, the incumbent lender is more likely to be repaid if he is diluted by more. We refer to this implication as the "more-dilution-is-better" result.

The result is contrary to the conventional wisdom that more dilution increases the probability of default. Indeed, for a borrower of a given risk/quality, a larger loan increases the probability of default (dilution effect). But here, less risky/better quality borrowers take out larger loans (selection effect). This selection effect dominates the dilution effect in the considered equilibrium. It is important to note that information aggregation is key for the "more-dilution-is-better result": a larger top-up loan conveys positive information of the diluting lender.

Can more dilution ever be worse in this model? The answer is No. To see this, consider what happens if (13) is violated. In that case PN-borrowers do not find it optimal to accept stage-1 loan, and hence only PP-borrowers will accept it. As a result, there is no heterogeneity in the size of the top-up loan. As Figure 1 illustrates, the "no-dilution-is-better" result is relevant for large enough values of the signal precision parameter ρ.

4 Conclusion

We have put forward a mechanism of credit-history building (by taking on loans) as a way of aggregating information across heterogeneously-informed lenders. We illustrate this mechanism in a parsimonious model, which allows us to analyze costs and benefits associated with the credit-history building and yields testable empirical implications. One particularly striking model implication concerns debt dilution. The standard mechanism, which is present in our model, implies that when a borrower of a given quality increases her overall loan size, she also increases her probability of default. On the other hand, the novel information-aggregation channel present in our model suggests that larger loans are chosen by higher quality (or less risky) borrowers. Hence, in our model, a lender prefers to see his borrower taking on a larger, rather than a smaller, additional loan from a competing lender. We plan on testing this empirically by using individual loan-level data.

Another testable implication is the very notion that taking out a loan makes a borrower more likely to be approved for other loans in the future. While we obviously do not expect this to be universally true (as taking on a loan may also signal an onset of a negative income or expense shock), we expect to find evidence of this channel when informational heterogeneity across lender is particularly salient. We are currently working on obtaining data that will enable us to test these model predictions.

References

Arellano, C. and A. Ramanarayanan (2012): "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120, 187–232.

Athreya, K. (2005): "Equilibrium Models of Personal Bankruptcy: A Survey," *Federal Reserve Bank of Richmond Economic Quarterly*, 91, 73–98.

Bizer, D. S. and P. M. DeMarzo (1992): "Sequential Banking," *Journal of Political Economy*, 100, 41–61.

Chatterjee, S., D. Corbae, K. P. Dempsey, and J.-V. Ríos-Rull (2016): "A Theory of Credit Scoring and Competitive Pricing of Default Risk," Mimeo.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75, 1525–1589.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–2699.

——— (forthcoming): "A Seniority Arrangement for Sovereign Debt," *American Economic Review*.

Cho, I.-K. and D. M. Kreps (1987): "Signaling Games and Stable Equilibria," *Quartely Journal of Economics*, 102, 179–221.

Hu, L., X. Huang, and A. Simonov (2017): "Credit Score Doctor," Unpublished manuscript.

Liberman, A., D. Paravisini, and V. Pathania (2017): "High-Cost Debt and Borrower Reputation: Evidence from the U.K." SSRN Working Paper.

Livshits, I. (2015): "Recent Developments in Consumer Credit and Default Literature," *Journal of Economic Surveys*, 29, 594–613.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer Bankruptcy: A Fresh Start," *American Economic Review*, 97, 402–418.

Parlour, C. A. and U. Rajan (2001): "Competition in Loan Contracts," *American Economic Review*, 91, 1311–1328.

Rothschild, M. and J. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 629–650.