

A Biased Correction of Exponential Growth Bias: Evidence and Theoretical Implications

Karna Basu*

September 2017

Abstract

Exponential growth bias—the tendency to under-estimate compound interest—has been documented in several papers. Using survey data from India, I find that the bias exists only for high interest rates and long time horizons. When interest rates are low or time horizons short, the bias is reversed and individuals over-predict compound interest yields. I argue that this two-directional bias reflects an excessively linearized correction of one’s initial under-prediction. I build a model to show how this results in sub-optimal saving and borrowing choices, time-inconsistent behavior, aversions to long-term savings and short-term debt, and simultaneous saving and borrowing.

*Hunter College and The Graduate Center, City University of New York (kbasu@hunter.cuny.edu). The fieldwork in this paper was conducted as part of a larger study funded by the International Initiative for Impact Evaluation and administered superbly by the Centre for Microfinance, IFMR. Additional funding was received from PSC-CUNY and the Hunter College President’s Fund for Faculty Advancement. I wish to thank several people who have assisted with this research: my collaborator on a related project (Shailendra Singh Bisht), field project heads (Anup Roy, Burgen Belltawn), field research assistants (Ujjawal, Tithee Mukhopadhyay, Nayantara Sarma), and data research assistant (Evan Bollens-Lund). I am also grateful to Mann Deshi Bank—in particular, Chetna Sinha (chair), Rekha Kulkarni, Vanita Shinde, and Sushma Shengde—for their partnership and assistance in the field. For additional comments, I thank Saurabh Bhargava, Tito Cordella, George Loewenstein, Kalle Moene, Jonathan Morduch, Kjetil Storesletten, Daniel Sia, Thom Thurston, and seminar participants at Carnegie Mellon University, Fordham University, The University of Oslo, and conference participants at the International Economic Association World Congress and the GC-CUNY Development Economics Colloquium.

1 Introduction

There is a large and growing literature on biases, or deviations from the classical model of decision-making, and how these might be of particular concern to the study of poverty and development. There are a number of reasons for this—biases may interact with developing economy institutions in particular ways (Ashraf, Karlan, & Yin, 2006; Basu, 2011, 2014; Fischer & Ghatak, 2010), biases could themselves be functions of poverty or under-education (Mullainathan & Shafir, 2013), and the consequences of sub-optimal choices could be dire for those who are already poor. Of the many ways in which preferences or choices may deviate from the classical model, *exponential growth bias* is well-documented but otherwise relatively under-examined.

Exponential growth bias refers to the tendency to under-estimate the values of exponential functions. This has an obvious application in financial decision-making. If individuals under-estimate compound interest yields, saving appears less attractive than it really is and borrowing more so. So, if the bias influences financial choices, individuals may fail to maximize utility. There is compelling evidence both that exponential growth bias exists and that it correlates with (and in some cases, causes) inferior economic outcomes.

This paper has two objectives—first, to contribute to the body of evidence on the nature and patterns of exponential growth bias; and second, to derive some theoretical predictions. On both, I find some unusual and nuanced results.

The data were collected as part of a study on long-term savings in a semi-rural district of Maharashtra, a state in western India. Respondents were asked to predict compound interest yields for varying interest rates (2% and 10%, compounded annually) and time horizons (1, 3, 5, 10, and 20 years). In part, the surveys were designed to examine whether exponential growth bias survives at low interest rates or over short horizons. I find that it does not. For the sub-sample of respondents who correctly answer some simple questions (i.e. those who understand the basic idea of interest accumulation), compound interest yields are *over-estimated* over short horizons (1, 3, and 5 years at either interest rate) or at low interest rates (2% over any horizon). It is only when interest rates are high (10%), horizons long (20 years) that yields are under-estimated as predicted under exponential growth bias. To the best of my knowledge, this is the first study to document such a two-directional bias in individual predictions of compound interest.

I suggest a plausible psychological explanation for this phenomenon. To some extent, individuals are conscious that their instinctive guess of compound interest yields leads to under-estimation. As some existing literature (discussed below) suggests, this under-estimation is driven by a tendency to linearize exponential functions, or to anchor compound interest on

simple interest. It is possible that individuals' attempts to correct their instinctive bias is also excessively linear—they recognize that compound interest diverges from simple interest, but don't account for the fact that this divergence happens late and then rapidly. A linear correction of exponential growth bias, then, will result in excessive correction over short horizons and inadequate correction over long horizons.

I build a simple model to show how this two-directional bias affects saving and borrowing decisions and corresponding outcomes. Some of these results, in modified form, also apply to standard exponential growth bias.

Consider an individual who must split an endowment across, current, near future, and distant future consumption using short-term and long-term savings. First, the fact that short-term savings appears excessively attractive, and long-term savings excessively unattractive, does not mean there will be short-term over-saving and long-term under-saving. Savings choices will in general be sub-optimal, but the direction of the error depends on whether income or substitution effects dominate. For example, if income effects dominate, the individual will accumulate sub-optimally high assets in the long-run (her belief that long-run saving is unattractive leads her to save too much). Second, this model can generate a liquidity premium without any uncertainty, since the individual would rather save repeatedly in short-term accounts than in a longer-term account offering the same interest rate. Third, if the individual saves for the distant future through a sequence of short-term savings accounts, she will revise her savings plan downwards in the near future. This resembles the behavior of an individual with time-inconsistent preferences, but is actually driven by the fact that in the near future she observes that her assets are smaller than she had predicted.

Next, consider an individual who receives her endowment in the distant future, and consumption in the present and the near future must be funded through debt. First, whether the individual borrows too much in the present (when the debt is to be repaid much later) and too little in the near future (when the debt is to be repaid soon) again depends on income and substitution effects. Second, the model generates a taste for long-term loans relative to short-term loans. Third, unlike under savings, there is no time-inconsistent behavior under borrowing, unless in the near future the individual observes debt accumulated so far and re-evaluates anticipated repayments in the following period.

In general, if both saving and borrowing are allowed, the individual would like to engage in simultaneous short-term savings and long-term borrowing, as this combination creates the illusion of money being created. The extent to which this happens must depend on the limits to borrowing and the gap between borrowing and saving interest rates. The model leaves a number of questions unanswered, especially about how interest rates are determined in equilibrium and how individuals learn from their mistakes. But these stripped-down

exercises demonstrate how a biased correction of exponential growth bias can affect behavior in some subtle ways and provide confounding explanations for some stylized facts.

This paper relates to the literature that studies the prevalence, causes, and consequences of exponential growth bias. Wagenaar & Sagaria (1975) provide early evidence of the bias in the context of general exponential functions. Lusardi (2008) and Lusardi and Mitchell (2011) find a low overall understanding of compound interest in the United States, and show that demographics, gender, and education matter. They find that the bias is correlated with poor retirement planning and low levels of stock market participation. McKenzie & Liersch (2011) study college students and Fortune 500 employees in the United States, and again find under-estimation of compound interest. They argue that this happens due to a tendency to linearize the exponential function. Binswanger & Carman (2010) show that the bias is greater when respondents guess future consumption given current savings (prospective bias) than when they guess current savings given future consumption (retrospective bias).

It could be argued that exponential growth bias, though it exists, is not particularly relevant to financial decisions, at least for those who have basic mathematical skills and access to a calculator. A number of papers show that this is not the case. In multiple papers, Stango & Zinman (2007, 2009) show how exponential growth bias is correlated with savings and borrowing behavior. Levy and Tasoff (2015) theoretically examine life cycle implications of exponential growth bias. Eisenstein & Hoch (2007) show that, in their sample where 90% of respondents under-estimate compound interest yields, "rule of 72" training substantially improves accuracy.¹ Goda, Manchester, & Sojourner (2012) show that providing interest accumulation information raises savings. Song (2011) finds that, in rural China, financial education training raises savings contributions by 40%. In light of the results in the current paper, interventions that emphasize the curvature of exponential functions, and how this depends on interest rates, may be particularly effective.

2 Evidence

2.1 Data

The data were collected from individuals selected to participate in a field experiment on the adoption of a pension savings product. The target population consists of low-income semi-rural households in Satara District, which lies in the western Indian state of Maharashtra. We partnered with Mann Deshi Bank, a local cooperative bank that offers a range of savings and

¹The rule of 72 provides a quick way to estimate doubling time under compound interest—divide 72 by the annual interest rate. For example, the rule predicts that at an annual interest rate of 10%, the principle will double in approximately 7.2 years (the correct answer is 7.27 years).

loan services. 3300 clients with active savings accounts in the bank were randomly selected (stratified by bank branch, gender, and type of savings account). These individuals are, therefore, partially banked, i.e. they have some familiarity with savings (and possibly loan) contracts and are appropriately placed to respond to the questions posed in our survey. The questions referred to in this paper were asked in the endline survey conducted in 2014. After attrition, 2938 of the initial respondents were interviewed. Earlier survey rounds collected data on respondents' wealth, education, other socioeconomic characteristics, and responses to core financial literacy questions.

Respondents were asked several hypothetical questions about compound interest yields. These were interspersed through the endline survey. The basic questions were the following: How much will a Rs. 100 deposit (approximately US \$1.50) yield after 1, 2, 3, 5, 10, and 20 years, at interest rates of 2% and 10% (compounded annually)? In addition, there were the following variations on the above: a Rs. 1000 deposit, a Rs. 1000 loan that is repaid in lump-sum, and a monthly recurring deposit of Rs. 100.

About half the sample was offered weak incentives—the top five respondents in each branch would receive a gift of a silver coin. While those offered incentives were spread across all branches, this was not randomly assigned (the announcement was made halfway through the execution of the endline survey). Surveyors were instructed to allow respondents to use pen and paper if they wished, but not other computing tools.

2.2 Results

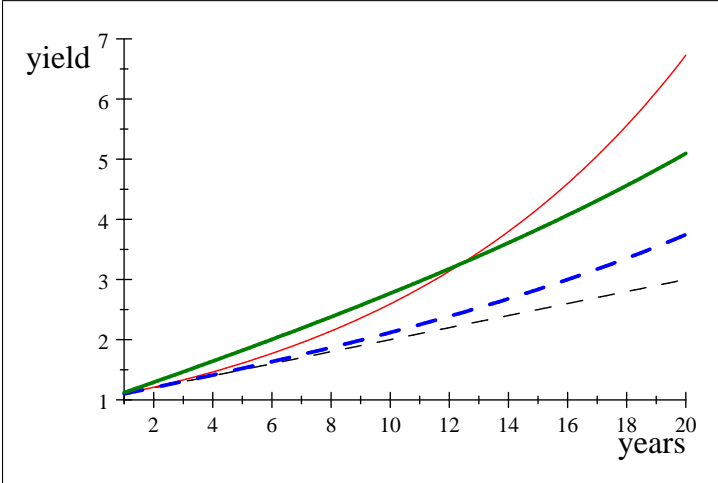


Figure 1: Compound interest (thin curve), simple interest (thin dashed curve), exponential growth bias (thick dashed curve), biased correction of exponential growth bias (thick curve) [$i = .1$]

I first describe hypothetical examples of both exponential growth bias and biased correction of exponential growth bias. Figure 1 depicts yields over time plotted for an annual interest rate of 10%. The thick curves describe possible biases, with dashes representing exponential growth bias (under-estimation throughout) and solid representing biased correction of exponential growth bias (over-correction followed by under-correction).

Next, figures 2-9 plot actual responses to the following questions:

- Rs. 100 deposit, 2% interest
- Rs. 100 deposit, 10% interest
- Rs. 100 monthly recurring deposit, 2% interest
- Rs. 100 monthly recurring deposit, 10% interest
- Rs. 1000 loan, 2% interest
- Rs. 1000 loan, 10% interest
- Rs. 1000 deposit, 2% interest
- Rs. 1000 deposit, 10% interest

For each question, three items are plotted: "all respondents" provides mean responses for the entire sample; "understood all questions" provides mean responses for those who correctly guessed one-year yields for all questions except those related to the monthly recurring deposits; and "correct value" plots the correct answer.

There are several ways to generate sub-populations to analyze responses, but my primary emphasis is on the "understood all questions" group. This separates those who have a minimal understanding of interest accumulation from those who do not. This leaves us with 294 responses, which constitutes 10% of the sample. It is striking that so many respondents fail to correctly answer one-year questions. One possible explanation is that they are attuned to think in terms of monthly recurring rather than one-time deposits (as is the case with Mann Deshi Bank accounts) and therefore assume that our questions relate to recurring deposits. This is indeed a significant concern, but not germane to the current paper.

Some straightforward observations can be made. At an interest rate of 2%, mean predictions are consistently higher than the correct answer, regardless of the time horizon. For low interest rates, the "all respondents" group over-predicts by less than the entire sample, which suggests that many in the sample may indeed be interpreting the questions in terms of recurring deposits. For the non-recurring deposits and the loan, we see over-prediction

for short horizons followed by under-prediction for long horizons. This two-directional bias is most apparent for the larger principle value of Rs. 1000. In most cases, p-values are small enough that we can reject the hypothesis that the mean responses are the same as the correct answer (see Tables 1 and 2). In general, we do not observe much variation in the bias across individual characteristics such as education, wealth, general mathematical skills, and treatment assignment in the separate experiment, but this remains a subject of continued investigation.

The results suggest that, in this case, the hypothesis of exponential growth bias can be rejected in favor of biased correction of exponential growth bias.²

3 Theoretical Predictions

3.1 Setup

Consider an individual who lives for 3 periods: 0, 1, 2. Her objective is to maximize discounted utility, $\sum_{t=0}^2 u_t(c)$, subject to a budget constraint. I assume $u(c)$ is concave, differentiable, and that $u'(0) = \infty$ (to eliminate corner solutions).

Intuitively, the spacing between periods should be interpreted in the following way: a gap of one period refers to the short-term and a gap of two periods refers to the long-term. Then, we can say that a given interest rate $r > 0$ is interpreted by the consumer as some $r_s > r$ if applied in the short-term and some $0 < r_l < r$ if applied in the long term. This is an admittedly simple setup (and would not apply for very low or very high interest rates), but captures the essential feature of the bias that is necessary for our analysis. For other approaches to modeling exponential growth bias, see Wagenaar & Sagaria (1975), Eisenstein & Hoch (2007), Stango & Zinman (2009), and Levy & Tasoff (2015).

3.2 Saving

3.2.1 Period 0 Decisions

Suppose the consumer receives an endowment y in period 0. In period 0, she must allocate it across current consumption, short-term future consumption, and long-term future consumption. Her maximization problem is:

²There are indeed other hypotheses consistent with this evidence. To take one, perhaps individuals have only invested in learning about certain key interest rates and time horizons—the ones that are most relevant to their everyday choices. Nevertheless, the following model does not depend on the psychological underpinnings of these mistakes; it simply takes these mistakes as given and examines the implications.

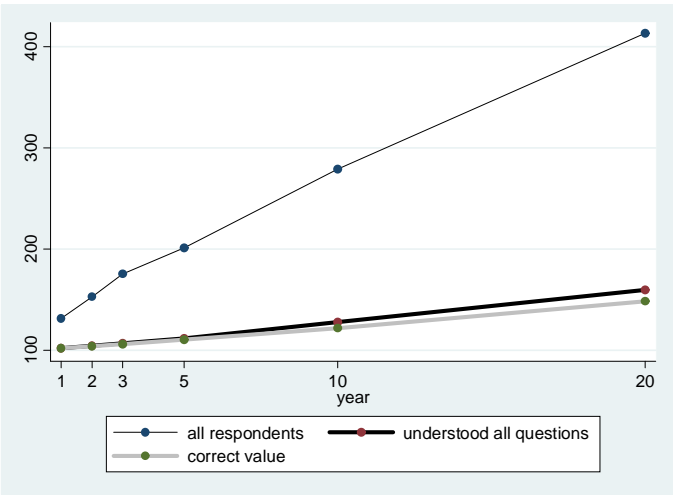


Figure 2: Rs. 100 deposit, 2% interest

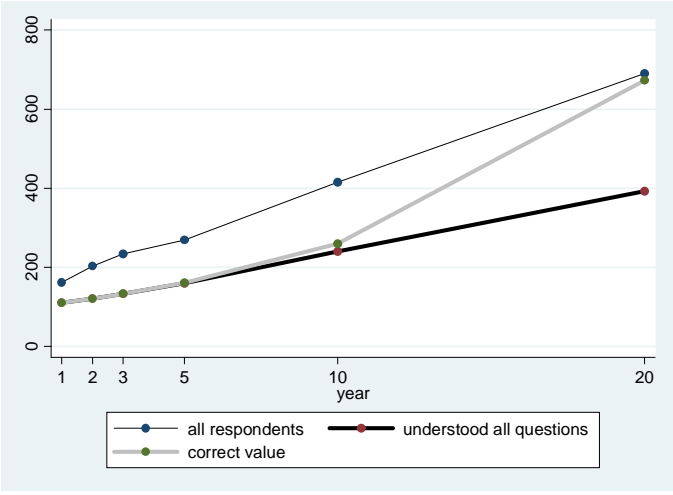


Figure 3: Rs. 100 deposit, 10% interest

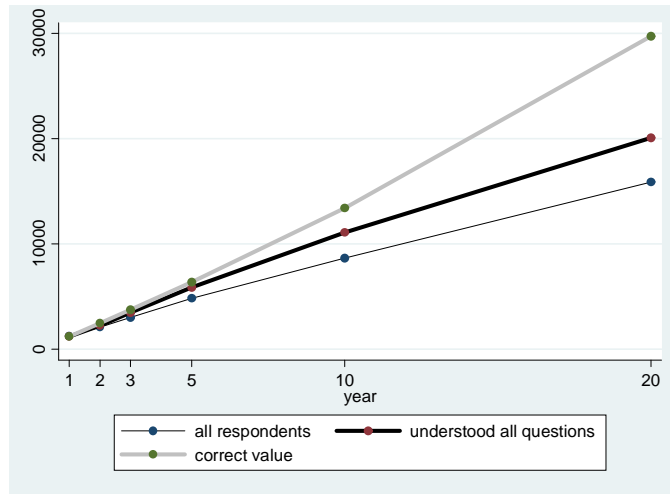


Figure 4: Rs. 100 monthly recurring deposit, 2% interest

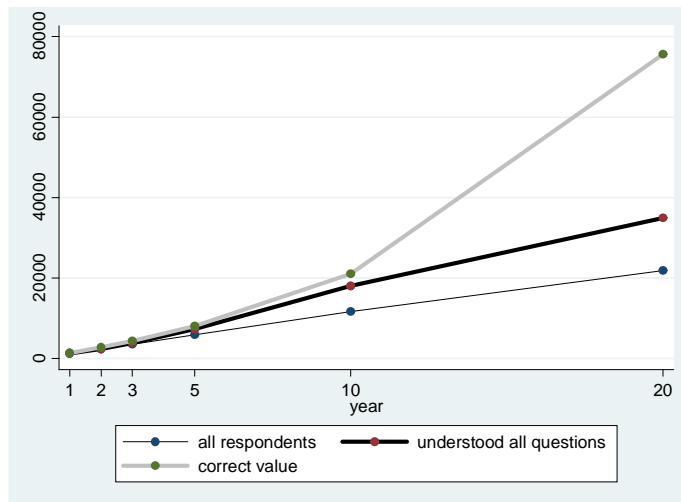


Figure 5: Rs. 100 monthly recurring deposit, 10% interest

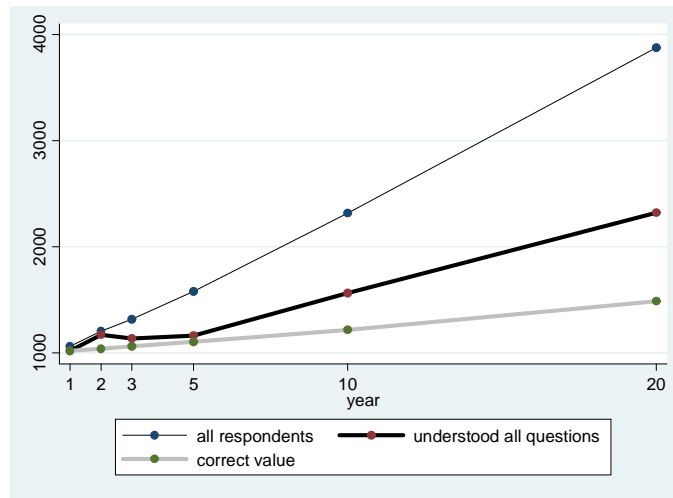


Figure 6: Rs. 1000 loan, 2% interest

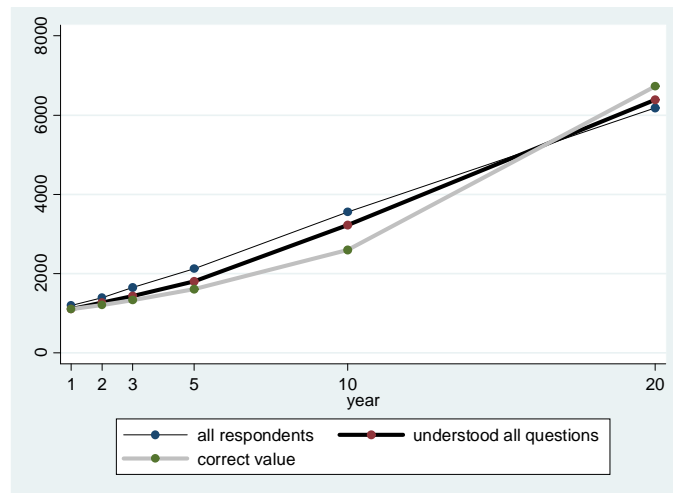


Figure 7: Rs. 1000 loan, 10% interest

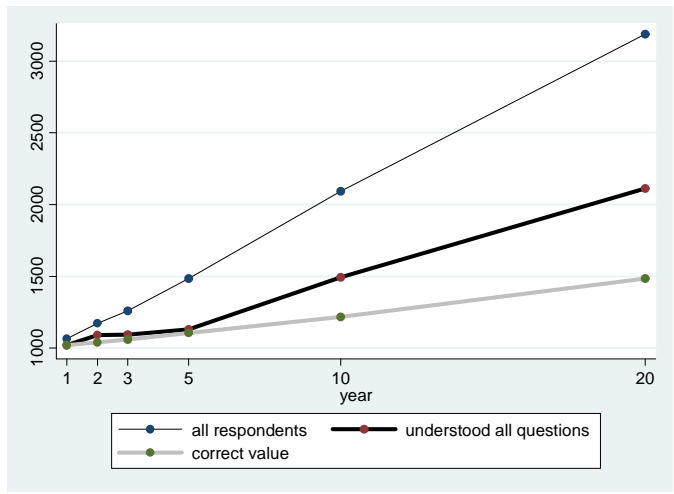


Figure 8: Rs. 1000 deposit, 2% interest

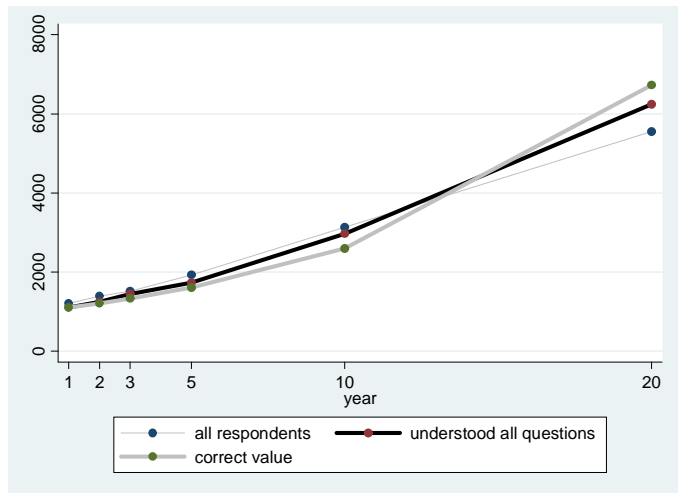


Figure 9: Rs. 1000 deposit, 10% interest

$$\begin{aligned} & \max_{c_0, c_1, c_2} u(c_0) + \delta u(c_1) + \delta^2 u(c_2) \\ \text{s.t.} \quad & c_0 + \frac{c_1}{1+r_s} + \frac{c_2}{(1+r_l)^2} \leq y \end{aligned}$$

This gives us the following first-order conditions:

$$u'(c_0) = \delta(1+r_s)u'(c_1) = \delta^2(1+r_l)^2u'(c_2)$$

Let the correct choices (i.e. those made in the absence of a bias) be denoted (c_0^*, c_1^*, c_2^*) . Comparing marginal utilities, we get:

$$\begin{aligned} \frac{u'(c_0)}{u'(c_1)} &= \delta(1+r_s) > \delta(1+r) = \frac{u'(c_0^*)}{u'(c_1^*)} \\ \frac{u'(c_0)}{u'(c_2)} &= \delta^2(1+r_l)^2 < \delta^2(1+r)^2 = \frac{u'(c_0^*)}{u'(c_2^*)} \\ \frac{u'(c_1)}{u'(c_2)} &= \frac{\delta(1+r_l)^2}{(1+r_s)} < \delta(1+r) = \frac{u'(c_1^*)}{u'(c_2^*)} \end{aligned}$$

Intuitively, period 1 is perceived as cheaper than it really is, and period 2 is perceived as more expensive than it really is. So the consumer plans to consume more in 1, and less in period 2, than she should. How consumption ends up looking depends on income and substitution effects. If substitution effects dominate, over-estimation means too little is consumed in 0 and too much saved for 1 (the opposite is true for under-estimation). If income effects dominate, over-estimation means too much is consumed in 0 and too little is saved for 1. This ambiguity can be seen in the following example.

Consider the relationship between periods 0 and 2, where the consumer exhibits standard exponential growth bias. Given an initial allocation of the endowment to these two periods (i.e. after subtracting the amount saved for period 1), the consumer's problem is described in Figure 10. The unbiased choice is marked **A**. As the result of the bias, the budget line is perceived as being flatter than it really is. As a result, the consumer may raise or lower c_0 , while intending to consume less in period 2 than under the unbiased choice. But because she misperceives the opportunity cost (price) of period 2 consumption, resulting period 2 consumption may be higher or lower than is optimal. Furthermore due to misperceptions about period 1, there will be a further parallel shift of the budget line in Figure 10. Applying a similar argument to the choices between periods 0 and 1, we see here that the consumption path will be suboptimal, but without a clear prediction of under-saving versus over-saving. A similar point could be made for standard exponential growth bias as well (so, the empirical

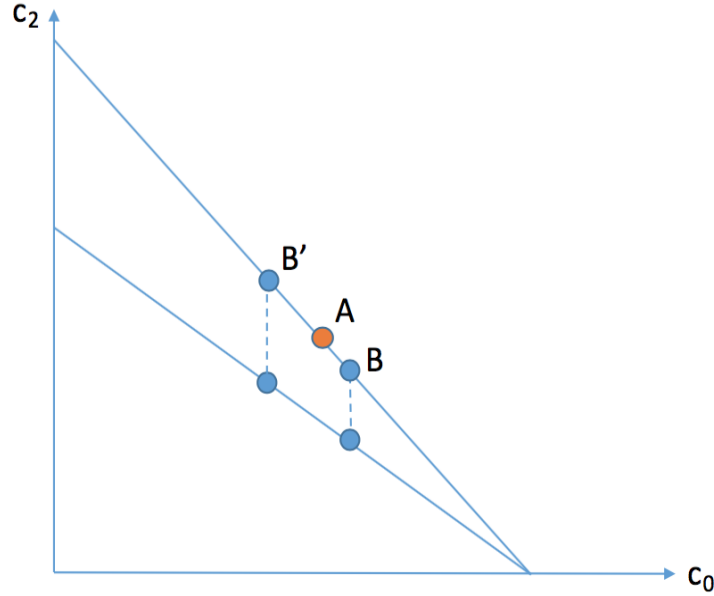


Figure 10: The consumer's choice between periods 0 and 2.

evidence of lower future savings under exponential growth bias is not a necessary theoretical prediction).

Testable Predictions It is possible to generate testable predictions using CRRA utility, where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. If the intertemporal elasticity of substitution is low ($\sigma > 1$), income effects dominate. Then, a larger fraction of the endowment will be allocated to periods 0 and 2 (relative to 1) for two reasons. First, Period 1 is misperceived as cheap. Second, the marginal utility of any allocation x to periods 0 and 2 is higher (this can be easily verified using the envelope theorem).

Next, for any amount x allocated to periods 0 and 2, too little will be consumed in 0 relative to the optimal. Therefore, there will be excessive long-term savings if $\sigma > 1$. If the intertemporal elasticity of substitution is high ($\sigma < 1$), a similar argument shows that there will be too little long-term savings.

3.2.2 Multi-Period Savings

Liquidity Premium We continue with the savings problem, but allow savings decisions to be made in any period. Now, in period 0, the individual need not save directly for period 2; she could instead save for period 1 and allow her next-period self to decide how to allocate consumption to period 2. Under time-consistent preferences, we can again solve the optimization problem from period 0's perspective. Any consumption for period 2 is best

delivered through short-term savings that matures in period 1 and can then be re-saved. In fact, for the agent to be willing to save in a long-term plan that directly matures in period 2, the interest rate on long-term savings would have to be some $\tilde{r} > r$ that satisfies:

$$(1 + \tilde{r}_l) > (1 + r_s)$$

This provides an alternative explanation for a liquidity premium in the absence of any uncertainty. For the same reason, if the individual had time-inconsistent preferences, this would make commitment savings less attractive than otherwise.

Time-Inconsistency Next, we can solve the optimization problem in which only short-term savings is engaged in. Ignoring the slack constraints, the maximization problem is:

$$\begin{aligned} & \max_{c_0, c_1, c_2} u(c_0) + \delta u(c_1) + \delta^2 u(c_2) \\ \text{s.t.} \quad & c_0 + \frac{c_1}{1 + r_s} + \frac{c_2}{(1 + r_s)^2} \leq y \end{aligned}$$

This problem could be analyzed similarly to the previous one, with the difference being that in this case any future consumption appears cheaper than it really is.

But there is an additional element that emerges. When choices can be made in any period, despite time-consistent preferences, we observe time-inconsistent behavior. In period 1, the consumer finds that she has less than she had thought she would. If period 0 had planned to leave y_1 to be split across 1 and 2, what period 1 observes is actually $\bar{y}_1 < y_1$. So, to equalize price-adjusted marginal utilities across periods 1 and 2, she must save less than originally planned. This behavior could be interpreted as evidence of time-inconsistent preferences even though that is not the case here. Note that under traditional exponential growth bias, where even short-term yields were under-predicted, we would observe a reverse time-inconsistency where savings plans were revised upwards.

3.3 Borrowing

3.3.1 Period 0 Decisions

Suppose the individual anticipates an endowment of y in period 2 and must borrow to fund consumption in periods 0 and 1. First, to parallel the first case studied under savings, suppose all borrowing decisions must be made directly against period 2. The maximization

problem is:

$$\begin{aligned} & \max_{c_0, c_1, c_2} u(c_0) + \delta u(c_1) + \delta^2 u(c_2) \\ \text{s.t.} \quad & c_0(1+r_l)^2 + c_1(1+r_s) + c_2 \leq y \end{aligned}$$

The first-order conditions are:

$$\frac{u'(c_0)}{(1+r_l)^2} = \frac{\delta u'(c_1)}{(1+r_s)} = \delta^2 u'(c_2)$$

Rearranging terms, we get the following relationships between marginal utilities:

$$\begin{aligned} \frac{u'(c_0)}{u'(c_1)} &= \frac{\delta(1+r_l)^2}{(1+r_s)} < \delta(1+r) = \frac{u'(c_0^*)}{u'(c_1^*)} \\ \frac{u'(c_0)}{u'(c_2)} &= \delta^2(1+r_l)^2 < \delta^2(1+r)^2 = \frac{u'(c_0^*)}{u'(c_2^*)} \\ \frac{u'(c_1)}{u'(c_2)} &= \delta(1+r_s) > \delta(1+r) = \frac{u'(c_1^*)}{u'(c_2^*)} \end{aligned}$$

When borrowing, the bias enters differently into the problem. Period 0 appears cheap relative to periods 1 and 2, and period 1 appears expensive relative to period 2. So, period 1 is misperceived as relatively expensive, compared to savings where period 2 is misperceived as relatively expensive.

Again, the overall predictions are ambiguous. Income effects could go in either direction (depending on relative over- and under-prediction). Combined with substitution effects, there could be over- or under-consumption in any period. Observe that this is fundamentally different from standard exponential growth bias: in that case, since income and substitution effects go in the same direction, there is a prediction of over-borrowing relative to optimal.

Long-Term Loans The problem does not change if the consumer is allowed to borrow against periods in which no income is earned. In other words, period 0 may borrow against period 1, and period 1 may borrow against period 2 to repay 0's loan and fund her own consumption. The solution stays the same as before since the individual prefers long-term loans to short-term ones. In fact, even if short-term loans get cheaper, the individual may prefer long-term loans to short-term ones. To make the individual prefer short-term loans in period 0, the interest on short-term savings would have to be $\tilde{r} < r$, so that:

$$(1 + \tilde{r}_s) < (1 + r_l)$$

This generates an aversion to short-term loans, again in a model without uncertainty. (This suggests that choices in home mortgages, to pick an example, are excessively biased towards 30-year loans relative to shorter ones.)

Finally, unlike under savings, the individual will not exhibit time-inconsistent behavior under borrowing, unless in period 1 she observes debt accumulated so far and re-evaluates how much will be paid in period 2.

3.4 Saving and Borrowing

So far, we have analyzed saving and borrowing decisions in isolation. If the individual can engage in both saving and borrowing, the bias discussed in this paper predicts that the consumer will engage in endless simultaneous saving (short-term) and borrowing (long-term) in the belief that she is creating money out of nothing. This is a problematic conclusion as it should be impossible to sustain the illusion that one is making money when in fact one is not. This leaves room for an extension of the model that incorporates learning. For present purposes, we can reasonably assume the consumer will not engage in endless saving and borrowing for two plausible reasons: limits to borrowing, and gaps between borrowing and saving interest rates. Nevertheless, the model suggests that the individual will be more inclined to simultaneously hold short-term savings and long-term debt than if she were not biased.

4 Conclusion

This paper provides evidence that miscalculation of compound interest may follow more complex patterns than pure under-estimation. In particular, when horizons are short or interest rates low, individuals over-predict compound interest yields. A simple model with savings and borrowing decisions shows how the bias can be predicted to affect behavior.

This suggests a number of areas for further investigation, some of which is ongoing work. It would be useful to better understand the psychological roots of the two-directional bias. How does it change as individuals get more educated or gain mathematical literacy? To what extent does the bias correlate with observed economic outcomes in the data?

Theoretically, the model could be extended to one with credit markets. How are interest rates determined in equilibrium? How does learning occur and how does this affect choices? A better understanding of this bias could lead to improved intervention designs and corresponding improvements in welfare.

References

- [1] Ashraf, Nava, Dean S. Karlan, and Wesley Yin. 2006. "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines." *Quarterly Journal of Economics*, 121(2): 635-672.
- [2] Basu, Karna. 2011. "Hyperbolic Discounting and the Sustainability of Rotating Savings Arrangements." *American Economic Journal: Microeconomics*, 3(4): 143-71.
- [3] Basu, Karna. 2014. "Commitment Savings in Informal Banking Markets". *Journal of Development Economics*. 107: 97-111.
- [4] Eisenstein, Eric, and Stephen Hoch. 2007. "Intuitive compounding: framing, temporal perspective, and expertise." Working paper. <http://eric-eisenstein.com/papers/Eisenstein%26Hoch-Compounding.pdf>
- [5] Ghatak, Maitreesh and Greg Fischer. 2010. "Repayment Frequency in Microfinance Contracts with Present-Biased Borrowers." Working paper, <http://sticerd.lse.ac.uk/dps/eopp/eopp21.pdf>
- [6] Goda, Gopi Shah, Colleen Flaherty Manchester, and Aaron Sojourner. 2012. "What Will My Account Really Be Worth? An Experiment on Exponential Growth Bias and Retirement Saving". Working paper. <http://www.nber.org/papers/w17927.pdf>
- [7] Levy, Matthew R. and Joshua Tasoff. 2015. "Exponential Growth Bias and Lifecycle Consumption". Working paper.
- [8] Mullainathan, Sendhil and Eldar Shafir. 2013. Scarcity: Why Having Too Little Means So Much. Time Books, Henry Holt & Company LLC, New York, NY.
- [9] Song, Changcheng. 2012. "Financial Illiteracy and Pension Contributions: A Field Experiment on Compound Interest in China." Working paper. http://faculty.chicagobooth.edu/workshops/finance/past/pdf/song_pension_12-01-12.pdf
- [10] Stango, Victor, and Jonathan Zinman. Fuzzy math and red ink: When the opportunity cost of consumption is not what it seems. Working Paper, 2007.
- [11] Stango, Victor, and Zinman, Jonathan. 2009. "Exponential Growth Bias and Household Finance," *Journal of Finance*, 64(6): 2807-2849.

- [12] Wagenaar, William A. and Sabato D. Sagaria. 1975. "Misperception of exponential growth". *Perception & Psychophysics*, 18 (6): 416-22.