A behavioral model of simultaneous borrowing and saving

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Abstract

Why do individuals borrow and save at the same time? This paper proposes a new explanation in settings where savings are not secure. A sophisticated hyperbolic discounter who foresees an investment opportunity would like to ensure that her future self has both the liquid cash and the incentives necessary to invest. I derive conditions under which she will rationally choose to save while borrowing to fund the investment. The combination of non-secure savings and a loan serves as a commitment device that generates self-inflicted punishments for non-investment. I argue that this model is particularly applicable to, but not limited to, microfinance and informal banking.

JEL classifications: D91, G21, O16.

1 Introduction

There are a number of economic explanations for simultaneous borrowing and saving, a widespread practice that on the surface appears irrational. Individuals may borrow while maintaining liquid savings for their option value. They could combine hidden savings and visible loans to disguise their wealth in the face of social pressures (Baland et al., 2011). Perhaps people frame their choices through mental accounts, so saved money is treated differently from borrowed money (Gross and Souleles, 2000 and Ianole, 2012). If consumers have time-inconsistent preferences, they may hold illiquid assets while smoothing short-term shocks with credit card debt (Laibson et al., 2003). Here, the cost of engaging simultaneously in low-returns saving and expensive borrowing is in part the price of commitment–a way to ensure that one's future selves do not consume assets too quickly.

The arguments above are individually compelling, though each is best suited to particular institutional contexts. This paper proposes a new rationalization, also using time-inconsistent preferences but with a unique mechanism. The model is motivated by the contractual environments and experiences of microfinance institutions and other informal banking groups. First, one of the principle innovations of modern microfinance has been the introduction of savings technologies (Armendariz and Morduch, 2010), and many clients borrow while saving more than is required (Karlan, 2005; Baland et al., 2011; Dehejia et al., 2012; and Atkinson et al., 2013). Second, savings are illiquid until the end of a loan cycle, and in some cases deposits are loaned out, so they may be insecure or perceived as insecure (Karlan, 2007 and Collins et al., 2010). Third, loans are frequently used for small-business investment (Banerjee et al., 2015). And fourth, MFI participation has been connected, empirically and theoretically, to time-inconsistency and the concomitant need for commitment or self-discipline (Banerjee and Duflo, 2010; Fischer and Ghatak, 2010; Morduch, 2010; Bauer et al., 2012; and Basu, 2014).

Studies of commitment frequently focus on a consumer's need to accumulate a target level of assets for investment or durable goods (see Ashraf et al., 2003 and Bryan et al., 2010 for surveys). I introduce an added question that is relevant if returns to investment are themselves delayed. How does a hyperbolic discounter ensure that her future self has both the accumulated cash *and* incentives necessary for investment?

Consider an individual who lives for three periods. All banking decisions are made in period 0, the start of a microfinance contract 'cycle'. In period 0 she anticipates an investment opportunity in period 1, which would yield returns in period 2.[1] Ideally, she would like to borrow some l^* and have her future self invest. But she knows that because she is present-biased, in period 1 she may lack the incentive to invest. Now, suppose in period 0 she borrows some additional amount ϕ while also saving ϕ in the bank (the savings mature in period 2, but without certainty). Period 1 is left with the same quantity of liquid cash, but she now has a stronger incentive to invest, since not doing so exposes period 2 to possible adversity if savings don't mature. I derive sufficient conditions under which the agent will choose to simultaneously save and borrow with the bank.

This stylized model aims to provide a partial but plausible explanation for a prevailing practice, and to also make a more general comment about commitment. An effective commitment device is one that triggers penalties associated with undesirable actions but is otherwise cheap. In the current setting, commitment is achieved in a novel way–because the utility function is assumed concave, the small possibility of disappeared savings can be painful under non-investment (because of generally low consumption levels) but of little consequence otherwise. The presence of a few unreliable borrowers can thus provide valuable commitment to others.

2 The model

2.1 Assumptions

There are three periods: 0, 1, and 2. In period 0, the agent observes her endowment, w, and makes formal banking decisions. She chooses some loan, $l \geq 0$, to be repaid in period 2. She also chooses some savings, $0 \leq s_0 \leq w + l$, that matures in period 2. The

[1] In particular, I focus on forms of investment that can plausibly be construed as safe, such as inventory or working capital.

remainder, $w + l - s_0$, is passed on to period 1.

In period 1, the agent consumes $c_1 \leq w + l - s_0$. Funds permitting, she may choose to invest p, a fixed investment amount (if she is indifferent, she invests). The remainder, denoted s_1, is passed on to period 2.

In period 2, the agent receives s_0 with probability $(1 - \varepsilon)$ (with some probability ε, the bank savings disappear). She receives s_1 with certainty. And if the investment was made, she receives some return b. She repays the loan l and consumes the remainder, c_2 (which may be negative).[2] For simplicity, interest rates are assumed to be 0.

The timing is summarized in Figure 1.

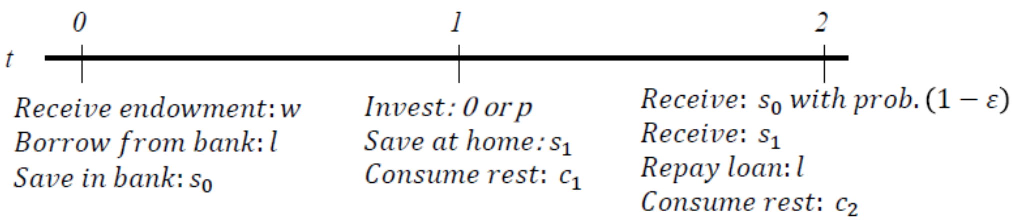

Figure 1: Timeline

I assume that $b > p$ (so that investment is attractive) and $b > w - p$ (so that investment returns are large relative to initial wealth).

The agent is a sophisticated quasi-hyperbolic discounter with $\beta \in (0, 1]$ (the hyperbolic discount factor) and $\delta = 1$ (the exponential discount factor). In any period in which consumption occurs, $t \in \{1, 2\}$, her instantaneous utility function $u(c_t)$ (sometimes denoted u_t), defined over $\mathbb{R}$, is strictly concave and differentiable. Discounted expected utilities are denoted $U_0 = \beta(u_1 + Eu_2)$ and $U_1 = u_1 + \beta Eu_2$, from the perspectives of periods 0 and 1, respectively. This implies that period 1's optimal plan differs from period 0's optimal plan–in particular, period 1 cares more about instantaneous consumption than period 0 would like.

[2] I explain later how the same argument can be made if consumption is bounded below at 0 and there is a penalty for loan default.

2.2 The period-0 perspective

From period 0's perspective, the optimal outcome is straightforward. First, observe that if she had complete control over period 1's choices, she would never save in the bank since the technology is strictly inferior to simply passing down savings directly.

If no investment were to be made, her utility would be maximized from an equal division of consumption, so $c_1 = c_2 = \frac{w}{2}$.

But, since $b > p$ and borrowing is interest-free, it is clear that she prefers to invest. Since $b > w - p$, her optimal outcome involves borrowing. She solves:

$$\max_{l \geq 0} u\left(w + l - p\right) + u\left(b - l\right) \tag{1}$$

The solution satisfies the first-order condition, $u_1' = u_2'$, resulting in equal consumption across periods. Let the solution be denoted l^*.

2.3 The period-1 perspective

Suppose the agent borrowed some l and saved some s_0 in period 0. In period 1, she must decide whether to invest. Due to her time-inconsistent preferences, she views investment less favorably than she did in period 0.

If she does not invest, she solves the following:

$$\max_{s_1 \geq 0} u_1\left(w + l - s_0 - s_1\right) + \beta\left[\left(1 - \varepsilon\right) u\left(s_0 + s_1 - l\right) + \left(\varepsilon\right) u\left(s_1 - l\right)\right] \tag{2}$$

Let the solution be denoted $s_1^N\left(s_0, l\right)$ and the resulting discounted utilities $U_0^N\left(s_0, l\right)$ and $U_1^N\left(s_0, l\right)$, where the superscript N denotes 'no investment'. If there is an interior solution, it satisfies the first-order condition:

$$u_1' = \beta\left[\left(1 - \varepsilon\right) u'\left(s_0 + s_1 - l\right) + \left(\varepsilon\right) u'\left(s_1 - l\right)\right] \tag{3}$$

If there is a corner solution at $s_1 = 0$, then from period 1's perspective, the marginal utility of current consumption must be higher than the discounted expected marginal utility of future consumption.

Alternatively, the agent may choose to invest. In that case, she solves:

$$\max_{s_1 \geq 0} u_1 \left(w + l - s_0 - s_1 - p\right) + \beta \left[\left(1 - \varepsilon\right) u \left(s_0 + s_1 + b - l\right) + \left(\varepsilon\right) u \left(s_1 + b - l\right)\right] \quad (4)$$

Let the solution be denoted $s_1^I (s_0, l)$ and the resulting discounted utilities $U_0^I (s_0, l)$ and $U_1^I (s_0, l)$, where the superscript I denotes 'investment'. Again, at an interior solution, discounted marginal utilities are equalized from period 1's perspective, while at a corner solution $s_1 = 0$ and the marginal utility of current consumption is higher than the discounted expected marginal utility of future consumption.

Of particular interest is the question of whether the agent will invest if in period 0 she had borrowed the optimal amount l^*. If she does invest, she would certainly be at a corner solution with $s_1 = 0$: since l^* was selected so that after investment $u_1' = u_2'$, in period 1 the agent would ideally like a larger loan so as to satisfy $u_1' = \beta u_2'$. Despite l^* not being optimal for her, she will invest if the future returns to investment are sufficiently attractive. She weakly prefers to invest if:

$$U_1^I (0, l^*) \geq U_1^N (0, l^*) \quad (5)$$

$$\iff u \left(w + l^* - p\right) + \beta u \left(b - l^*\right) \geq u \left(w + l^* - s_1^N (0, l^*)\right) + \beta u \left(s_1^N (0, l^*) - l^*\right) \quad (6)$$

If the agent is sufficiently time-inconsistent (i.e. if β is small enough), the promise of future returns becomes less salient and the agent prefers to raise immediate consumption by forgoing investment.

Lemma 1 *Suppose $l = l^*$ and $s_0 = 0$. There is some $\bar{\beta} \in (0, 1)$ such that the agent weakly prefers to invest if and only if $\beta \geq \bar{\beta}$.*

All proofs are in the appendix.

2.4 Inducing investment

Assume $\beta < \bar{\beta}$, so that period 0 is unable to attain the optimal outcome. Then, she must compare discounted utilities from the following options.

First, she could abandon investment. The utility from this is bounded above by an outcome in which periods 1 and 2 consume identical amounts: $2u\left(\frac{w}{2}\right)$.

Second, she could induce investment by taking a larger loan. As shown in Proposition 1, there is some loan size $\tilde{l} > l^*$ at which period 1 would be willing to invest. A larger loan lowers period 1's marginal utility and raises period 2's marginal utility, so period 1 is willing to save more. Ultimately, if the loan is sufficiently large, investment requires a sufficiently small additional sacrifice of immediate consumption that period 1 will opt to engage in it.

Finally, period 0 could engage in simultaneous saving and borrowing. To convince period 1 to voluntarily invest, period 0 needs to ensure that her future self has both the liquidity and incentives to do so. A loan creates both liquidity and incentives. Savings intensifies the incentives, since if period 1 does not invest, there is a possibility that period 2 will be left at a very low level of consumption (if the savings don't mature). If the consequences of non-maturing savings are sufficiently severe, period 1 will invest. Next, I show by construction that simultaneous saving and borrowing may be period 0's preferred option.

2.4.1 Simultaneous saving and borrowing: existence

Consider some $\phi > 0$. Assume period 0 takes a loan of $l = l^* + \phi$ and saves $s_0 = \phi$. If ε is sufficiently small (but not 0), period 0's discounted utility from this strategy, conditional on investment in period 1, is strictly better than the other options. To see this, notice that if savings mature, the outcome is identical to the optimal. So, as ε approaches 0, the likelihood of savings defaulting vanishes and we are left with the optimal outcome.

Now, we must check if period 0 will indeed opt to invest. Recall that if period 0 had simply borrowed l^*, period 1 would prefer to save $s_1^N(0, l^*)$ than to invest. Now, if she chooses not to invest, she will save more than $s_1^N(0, l^*)$ to partially protect period 2 from the consequences of default (the possibility of default raises period 2's expected marginal utility). If the utility costs of defaulted period 0 savings are sufficiently large (i.e. under default, period 2's marginal utility is sufficiently high), period 1 finds that she would have to save so much to insure period 2 that investment becomes her preferred option.

In essence, period 0 takes advantage of the concavity of her utility function to create conditions where, if her future self fails to invest, the consequences are severe, but if she does invest, they are not (defaulted savings hurt period 2 only if her consumption is already low).

Figures 2 (representing $u(c_1)$) and 3 ($u(c_2)$) depict this argument visually. (A_1, A_2) represent period 0's optimal outcome, while (B_1, B_2) show period 1's choices under a loan of l^*. Under simultaneous saving and borrowing, if period 1 continues to not invest, period 2's expected utility would drop sharply, to B'. This gives her the incentive to invest, resulting in the outcome (A_1, A_2'), which is close to optimal.

Proposition 1 formalizes the argument above.

Proposition 1 *There exist parameter regions and utility functions under which the agent strictly prefers $s_0 > 0$ and $l > 0$ in period 0.*

2.4.2 Simultaneous saving and borrowing: optimal choices

Conditional on simultaneous saving and borrowing being preferred, period 0's actual allocation depends on a straightforward utility maximization problem. Consider some $\tilde{l}$ and $\phi\left(\tilde{l}\right)$ so that $l = \tilde{l} + \phi$ and $s_0 = \phi$ (so ϕ represents the extent of simultaneous saving and borrowing). $\phi\left(\tilde{l}\right)$ is the lowest level of ϕ at which period 1 is willing to invest, and is dropping in $\tilde{l}$. Period 0 solves:

$$\max_{\tilde{l}} u\left(w + \tilde{l} - p\right) + (1 - \varepsilon) u\left(b - \tilde{l}\right) + (\varepsilon) u\left(b - \tilde{l} - \phi\left(\tilde{l}\right)\right) \quad (7)$$

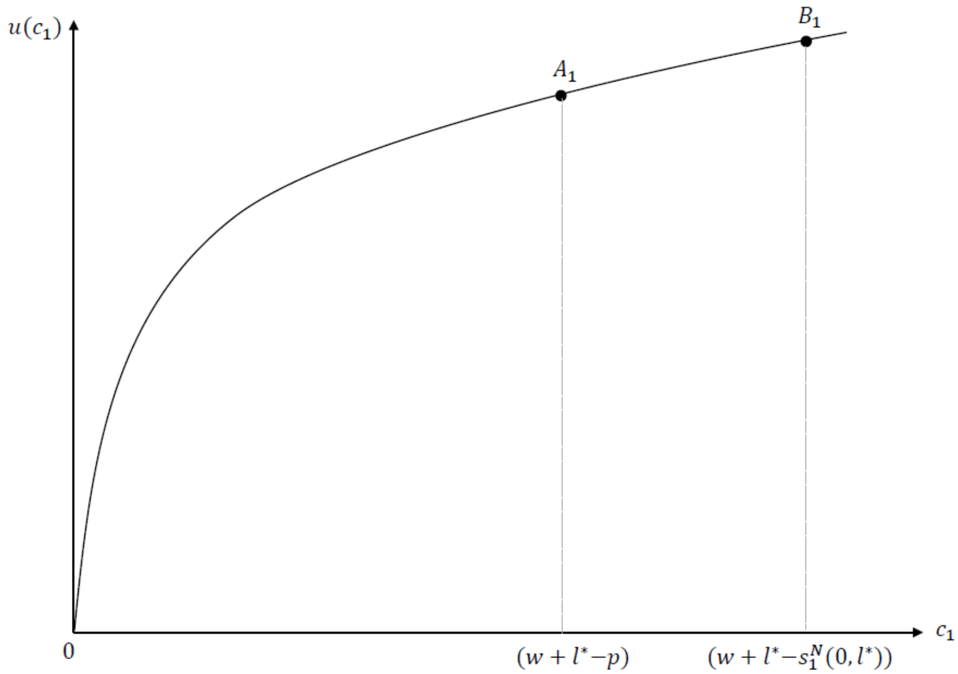

Figure 2: Period 1's instantaneous utility

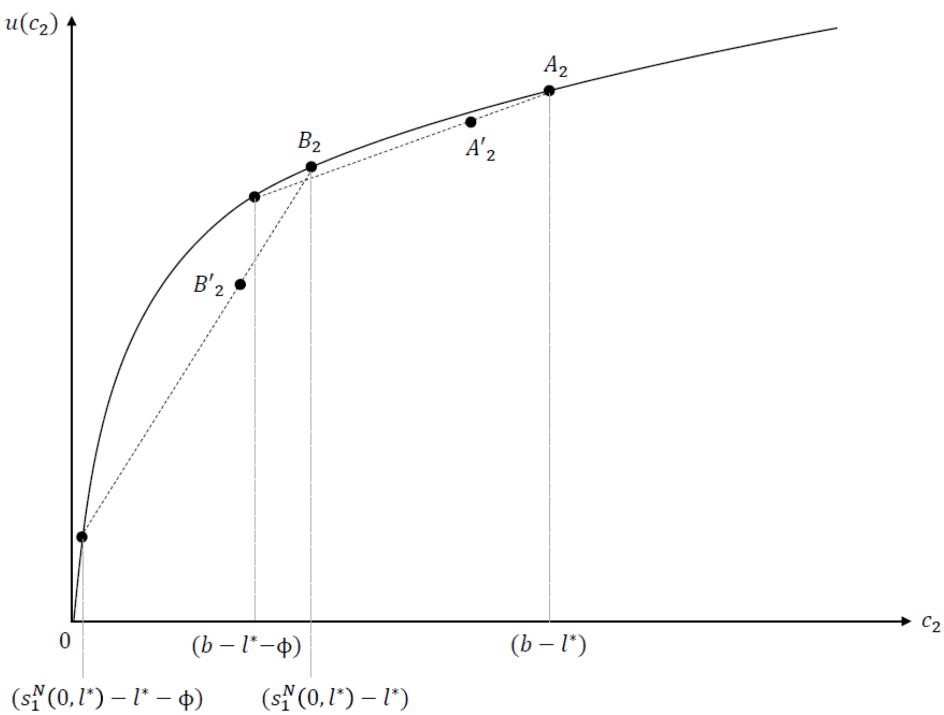

Figure 3: Period 2's instantaneous utility.

The derivative of the maximand with respect to $\tilde{l}$ is:

$$u'\left(w+\tilde{l}-p\right)-(1-\varepsilon)\,u'\left(b-\tilde{l}\right)-(\varepsilon)\,u'\left(b-\tilde{l}-\phi\left(\tilde{l}\right)\right)\left(1+\phi'\left(\tilde{l}\right)\right) \tag{8}$$

The derivative may be positive or negative at $\tilde{l}=l^*$, so the solution to the maximization problem could yield an outcome in which period 1 gets to consume either more or less than the first-best. If it is costly to induce investment (i.e. $\phi'(l)$ is highly negative), the derivative above is positive at $\tilde{l}=l^*$–period 0 should allow period 1 to consume more. If it is not costly to induce investment, period 0 should further reduce period 1's consumption, thereby providing additional insurance to period 2. The actual direction in which $\tilde{l}$ deviates from l^* therefore depends on the trade-off between the direct utility effect (smaller loan) and the incentivization of investment (larger loan).

3 Conclusion

This paper provides a new rationale for the phenomenon of simultaneous borrowing and saving. When agents are sophisticated hyperbolic discounters, access to a non-secure source of saving can be useful–by creating the threat of a large punishment in the event of non-investment, the agent can induce her future selves to invest. Actual utility loss in equilibrium is limited if the probability of default is low.

If, instead, the individual only had access to secure savings (at a lower interest rate than borrowing), then she would never choose to borrow and save simultaneously. To see this, consider any loan-savings combination that induces investment. Period 0 could reduce both loan and savings in such a way that total wealth rises (less money is burned), and the benefits accrue to period 1. Period 1's incentive to invest would remain intact.

The model makes some simplifying assumptions but these are not central to the argument. Similar results could be derived with positive interest rates or risky investments. If a penalty for default were introduced into the model, simultaneous saving and borrowing may become more attractive to period 0, since she could now take advantage of a utility discontinuity from failing to repay her loan.

There are some natural directions for further work. The practical relevance of the model's basic mechanism could be investigated in empirical studies of microfinance. The model could also be extended to other, related settings, with possibly interesting implications–other forms of uncertain savings such as stocks, savings accounts whose maturity date is uncertain, multi-period deposit decisions, and other informal banking institutions such as ASCAs, ROSCAs, and VSLAs.[3]

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References

[1] Armendariz, B. and Morduch, J.. 2010. The Economics of Microfinance. The MIT Press, Cambridge.

[2] Ashraf, N., Gons, N., Karlan, D., and Yin, W. 2003. "A Review of Commitment Savings Products in Developing Countries". Working paper #45, Economics and Research Department, Asian Development Bank, Manila.

[3] Atkinson, J, de Janvry, A., Mcintosh, C., and Sadoulet, E.. 2013. "Prompting Microfinance Borrowers to Save: A Field Experiment from Guatemala". *Economic Development and Cultural Change.* 62: 21-64

[4] Baland, J., Guirkinger C., and Mali, C.. 2011. "Pretending to be poor: borrowing to escape forced solidarity in credit cooperatives in Cameroon". *Economic Development and Cultural Change*. 60: 1-16.

[5] Banerjee, A. V. and Duflo, E.. 2010. "Giving Credit Where It Is Due". *Journal of Economic Perspectives.* 24: 61–80.

[3] For descriptions of their structures, see Collins et al. (2010), Salas (2014), and Basu (2011), respectively.

[6] Banerjee, A. V., Duflo, E., Glennerster, R., and Kinnan, C.. 2015. "The Miracle of Microfinance? Evidence from a Randomized Evaluation" *American Economic Journal: Applied Economics.* 7: 22-53.

[7] Basu, K. 2011. "Hyperbolic Discounting and the Sustainability of Rotational Savings Arrangements". *American Economic Journal: Microeconomics.* 3: 143–71.

[8] Basu, K. 2014. "Commitment Savings in Informal Banking Markets". *Journal of Development Economics.* 107: 97-111.

[9] Bauer, M., Chytilova, J., and Morduch, J. 2012. "Behavioral Foundations of Microcredit: Experimental and Survey Evidence from Rural India", *American Economic Review.* 102: 1118–1139.

[10] Bryan, G., Karlan, D., and Nelson, S. 2010. "Commitment Devices". *Annual Review of Economics.* 2: 671-98.

[11] Collins, D., Morduch, J., Rutherford, S., and Ruthven, O.. 2010. Portfolios of the Poor: How the World's Poor Live on $2 a Day. Princeton University Press, Princeton.

[12] Dehejia, R., Montgomery, H., and Morduch, J.. 2012. "Do Interest Rates Matter? Credit Demand in the Dhaka Slums". *Journal of Development Economics.* 97: 437-449.

[13] Fischer, G. and Ghatak, M. 2010. "Repayment Frequency in Microfinance Contracts with Present-Biased Borrowers". EOPP Working paper #21, STICERD, London School of Economics and Political Science, London.

[14] Gross, D. and Souleles, N.. 2000. "Consumer Response to Changes in Credit Supply: Evidence from Credit Card Data". Working paper, Wharton Financial Institutions Center, University of Pennsylvania, Philadelphia.

[15] Ianole, R. 2012. "The Puzzle of Simultaneous Savings and Debts". *Challenges of the Knowledge Society.* 2: 1379-1384.

[16] Karlan, D. 2005. "Using Experimental Economics to Measure Social Capital and Predict Financial Decisions". *American Economic Review.* 95: 1688-1699.

[17] Karlan, D. 2007. "Social Connections and Group Banking". *The Economic Journal.* 117: F52-F84.

[18] Laibson, D., Repetto, A., and Tobacman, J.. 2003. "A Debt Puzzle". in Knowledge, Information, and Expectations in Modern Economics: In Honor of Edmund S. Phelps by Aghion, P., Frydman, R., Stiglitz, J. E., and Woodford, M. (eds.). Princeton University Press, Princeton.

[19] Morduch, J. 2010. "Borrowing to Save". *Journal of Globalization and Development.* 1: 1-11.

[20] Salas, L. M. 2014. "Public vs. Private Mental Accounts: Experimental Evidence from Savings Groups in Colombia". Working paper, http://luzsalas.ws.gc.cuny.edu/files/2014/05/JMPaper-v.5.pdf (last accessed February 18, 2016)

Appendix: proofs

Proof of Lemma 1. $\frac{dU_1^I}{d\beta} = u(b - l^*)$. $\frac{dU_1^N}{d\beta} = -[u'(w + l^* - s_1^*) - \beta u'(s_1^* - l^*)] \frac{ds_1^*}{d\beta} + u(s_1^* - l^*) = u(s_1^* - l^*)$. So, $\frac{dU_1^I}{d\beta} > \frac{dU_1^N}{d\beta}$.

U_1^I and $U_1^N$are continuous in β. By construction of l^*, at $\beta = 1$, $U_1^I > U_1^N$. Also, since $\lim_{\beta \to 0} s_1^* = 0$, $\lim_{\beta \to 0} U_1^N > \lim_{\beta \to 0} U_1^I$.

Therefore, there exists some $\bar{\beta} \in (0, 1)$ at which $U_1^I = U_1^N$, with $U_1^I > U_1^N$ for $\beta > \bar{\beta}$ and $U_1^I < U_1^N$ for $\beta < \bar{\beta}$. ■

Proof of Proposition 1. Suppose $\beta < \bar{\beta}$. Then period 0's optimal outcome is not attainable.

No investment: If period 0 chooses not to induce investment, her discounted utility can never be higher than $2u\left(\frac{w}{2}\right)$. So, $U_0^N \leq 2u\left(\frac{w}{2}\right) < U_0^I(0, l^*)$.

Alternatively, period 0 could choose to induce investment.

Investment with pure loan: Suppose the agent does not invest in period 1. There must be some $\hat{l} \geq 0$, such that for $l < \hat{l}$, $s_1^N = 0$ and $u'\left(w + l - s_1^N\right) > \beta u'\left(s_1^N - l\right)$, and for $l \geq \hat{l}$, $u'\left(w + l - s_1^N\right) = \beta u'\left(s_1^N - l\right)$. Alternatively, suppose the agent does invest in period 1. There must be some $\bar{l} > l^*$ such that for $l < \bar{l}$, $s_1^I = 0$ and $u'\left(w + l - s_1^I - p\right) > \beta u'\left(s_1^I + b - l\right)$, and for $l \geq \bar{l}$, $u'\left(w + l - s_1^I - p\right) = \beta u'\left(s_1^I + b - l\right)$. It can easily be seen that $\hat{l} < \bar{l}$.

Therefore, at any $l \geq 0$, $\frac{dU_1^I}{dl} = \left[u'\left(w + l - s_1^I - p\right) - \beta u'\left(s_1^I + b - l\right)\right]\left(1 - \frac{ds_1^I}{dl}\right) \geq \frac{dU_1^N}{dl} = \left[u'\left(w + l - s_1^N\right) - \beta u'\left(s_1^N\right)\right]\left(1 - \frac{ds_1^N}{dl}\right)$. Combining this with the continuity of U_1^I and U_1^N, and the fact that $U_1^I(0, l^*) < U_1^N(0, l^*)$ (by assumption) and $\lim_{l \to \infty} U_1^I(0, l) > \lim_{l \to \infty} U_1^N(0, l)$, we know there must be some $\tilde{l}$ such that the agent in period 1 weakly prefers to invest if and only if $l \geq \tilde{l}$.

Investment with pure savings: Since the agent will not invest at a loan of l^*, we know that a reduction to $l = 0$ with any positive savings further reduces period 1's incentives to invest. So, investment cannot be induced with pure savings.

Simultaneous saving and borrowing: We can now derive sufficient conditions under which the agent will strictly prefer to simultaneously borrow and save in period 0. Consider some $\phi > p - s_1^N(0, l^*)$. Let $l^{**} = l^* + \phi$ and $s_0^{**} = \phi$. Since $U_0^I(s_0^{**}, l^{**})$ is continuous in ε and $\lim_{\varepsilon \to 0} U_0^I(s_0^{**}, l^{**}) = U_0^I(0, l^*)$, there exists some $\bar{\varepsilon} > 0$ such that, at $\varepsilon = \bar{\varepsilon}$, $U_0^I(s_0^{**}, l^{**}) > \max\left\{2u\left(\frac{w}{2}\right), U_0^I\left(0, \tilde{l}\right)\right\}$. Therefore, conditional on period 1 choosing to invest, this combination of savings and loan strictly dominates pure loans and no saving (from the period 0 perspective).

Now, we find conditions under which period 1 prefers to invest given the savings and loan combination above. If she does not invest, from the maximization problem in 2, the solution is: $s_1 = 0$ or

$$u'(w + l^{**} - s_0^{**} - s_1) = \beta\left[(1-\varepsilon)u'(s_0^{**} + s_1 - l^{**}) + (\varepsilon)u'(s_1 - l^{**})\right] \quad (9)$$

$$\Longleftrightarrow \quad u'(w + l^* - s_1) = \beta\left[(1-\varepsilon)u'(s_1 - l^*) + (\varepsilon)u'(s_1 - l^* - \phi)\right] \quad (10)$$

Up to this point, the entire analysis above depends on the shape of $u(c)$ only for $c \geq s_1^N(0, l^*) - l^*$ (all other conditions derived above rely on higher values of c). Hold $u(c)$ fixed for all $c \geq s_1^N(0, l^*) - l^*$. Consider $p - l^* - \phi < s_1^N(0, l^*) - l^*$ (by construction of ϕ). There must be some value $\bar{u}'$ such that, if $u'(p - l^* - \phi) \geq \bar{u}'$, $s_1^N(s_0^{**}, l^{**}) \geq p$. Assume $u'(p - l^* - \phi) \geq \bar{u}'$. Clearly, since not investing results in a saved amount that weakly exceeds the amount needed to invest, we have $U_1^I(s_0^{**}, l^{**}) > U_1^N(s_0^{**}, l^{**})$, so period 1 will invest. ■