Renegotiating with One's Self

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Abstract

This paper describes a hyperbolic discounter who is fully aware of her time-inconsistency but might make mistakes over equilibrium selection. The *semi-sophisticate* fails to anticipate future renegotiation of subgame perfect equilibria, while the *full-sophisticate* follows a renegotiation-proof equilibrium. I examine behavior when an individual must decide whether to purchase a good with immediate costs and deferred benefits. When faced with a deadline, a semi-sophisticate will behave like a sophisticated hyperbolic discounter, but without a deadline (when she faces multiple equilibria), she will appear naive. A welfare-maximizing government will offer the good only intermittently, for semi-sophisticates and possibly full-sophisticates. Monopoly might generate higher consumer surplus than competition, since a monopolist can commit to a pattern of sale and non-sale prices, thus effectively creating deadlines. If the individual is a Bayesian updater with inaccurate initial beliefs about her type, she will ultimately switch to a renegotiation-proof equilibrium with particular properties.

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1 Introduction

Hyperbolic discounters are typically classified by their beliefs about their preferences. A naif mistakenly believes her preferences are time-consistent. A sophisticate is fully aware of her time-inconsistency and engages strategically with her future selves. In this framework, it would appear that mistakes, or failures to implement one's plan, must emerge from some degree of naivete. I argue that this is not necessarily true and that there is a psychologically plausible way to model the behavior of individuals who understand their preferences but make mistakes. In this paper, the mistake comes from the sophisticate's failure to recognize that a subgame perfect Nash equilibrium might be subject to renegotiation. I classify sophisticates as *semi-sophisticates* or *full-sophisticates* based on how they select equilibria. This generates several insights into the role of deadlines, markets for durable goods and commitment products, and the nature of learning.

Since a hyperbolic discounter has time-inconsistent preferences, her notion of the optimal course of actions might vary over time. A natural way to analyze the sophisticated hyperbolic discounter's problem is to decompose the individual into distinct time-indexed selves. Each self places some weight on instant gratification–in addition to discounting subsequent periods by an exponential discount factor $\delta < 1$, she discounts the entire stream of future utilities by an additional factor $\beta < 1$.[1] The sophisticate recognizes that her future self, in any period t, will discount $t+1$ more severely than is ideal from the current perspective. She must therefore solve for a subgame perfect Nash equilibrium (SPNE) of a dynamic game with future players who don't share her preferences. This approach allows us to predict behavior in cases with a unique equilibrium. However, when there are multiple equilibria, the fact that a hyperbolic discounter is sophisticated does not inform us about the actions she will choose.

This concern is particularly relevant when the individual must select the timing of tasks with immediate costs and deferred benefits. Such games capture the essence of several important and realistic scenarios–typical costly tasks like homework or roof repair, the purchase of durable goods, or the takeup of commitment devices. Since sophisticated hyperbolic discounters know their financial decisions are subject to temptation, they might wish to voluntarily sign contracts that restrict choices or alter payoffs associated with future actions. However, the adoption of these contracts is frequently costly and difficult, both psychologically and, especially in develop-

[1] This describes *quasi-hyperbolic* discounting. For convenience, I drop the "quasi".

ing economies where financial institutions are dispersed, physically. If adoption involves even a simple transaction cost, the agent might have an incentive to postpone it to the following period. So, it doesn't follow from the fact that a product increases the individual's lifetime utility, that she will indeed decide to purchase it when offered. In the case of commitment devices, then, the question becomes: What can be done when I can't commit to commit?

Though postponed adoption would result in benefits being further deferred, it would also push the pain out of the present when the individual is most sensitive to it. If she is sophisticated, she understands that she cannot simply plan on completing the task in the next period, since she will be subject to the same impulse again. As O'Donoghue and Rabin (1999) show, under a finite horizon her actions will be determined by a unique SPNE. The outcome is less clear under an infinite horizon when there are multiple equilibria. In any period, she might wish to renegotiate with herself if the previously selected equilibrium does not maximize her continuation utility. This renegotiation problem must be taken seriously because there is no one the individual needs to communicate with. She can simple re-choose an equilibrium and communicate this to her future selves. Sophistication about preferences does not automatically provide a solution to the renegotiation problem.

I address the question of equilibrium selection by proposing a distinction between two types of sophisticated hyperbolic discounters. Full-sophisticates always seek a renegotiation-proof equilibrium.[2] Semi-sophisticates are sophisticated in the sense that they are aware of their future time-inconsistency. Yet, when faced with multiple equilibria, they fail to anticipate the possibility that their future selves might renegotiate to a different equilibrium. In this paper, an infinitely-lived,[3] sophisticated hyperbolic discounter has the option of purchasing a good which provides delayed benefits and involves immediate costs. I assume these costs and benefits are such that the agent is willing to buy the good immediately, but would ideally like to postpone the purchase to the next period. In this setting, the full-sophisticate will select an equilibrium from a restricted renegotiation-proof set, and will adhere to it. The semi-sophisticate, in each period, switches out of the equilibrium played by her past self if there is another that gives her strictly higher continuation utility. I examine the implications of this behavior under different market structures.

[2]I adapt renegotiation-proofness as defined in Farrell and Maskin (1989) and Kocherlakota (1996) to this setting.

[3]The infinite horizon can also be interpreted as an inter-generational setting or one with probabilistic survival.

First, I show that this results in strong deadline effects. Semi-sophisticated agents appear sophisticated under finite horizons and naive under infinite horizons. Unlike either full-sophisticates or naifs, a semi-sophisticate does much better under a deadline, even a distant one, relative to when there is no deadline at all. A deadline locks her in a unique SPNE. If it is lifted, she can always construct a SPNE under which the good will be purchased the following day. This allows her to perpetually renegotiate and never buy the good.

Second, with a changing population, a benevolent government that offers the good at marginal cost will fail to maximize consumers' welfare. To induce semi-sophisticates to buy the good, there must be stretches when it is not offered. As a result, agents born during those stretches will be forced to wait until the next date of availability. I also show that the government might prefer to offer the good in a similarly staggered manner to full-sophisticates. This can lead to more frequent purchases than with some renegotiation-proof equilibria under continuous access.

Next, I study markets with profit-maximizing firms. The model provides a rationalization of sale pricing of durable goods that does not rely on price discrimination. Firms list their stream of prices, based on which individuals make purchasing decisions. For semi-sophisticates, a monopolist will vary prices over time. By keeping prices high in certain periods, the firm effectively generates deadlines. This allows semi-sophisticates to play a unique equilibrium and buy the good during sale periods. If the population also consists of exponential discounters, they can continue to be attracted during high price periods.

Under perfect competition, there are multiple equilibria. There is an equilibrium where firms coordinate on sale and non-sale periods, but also one in which the good is always offered at marginal cost (and therefore never purchased). If the population contains any exponential discounters, competitive firms can no longer maintain the equilibrium with sales.

Finally, I ask how semi-sophisticates can learn from repeated observations of their mistakes. I build a simple model with Bayesian updating that allows sophisticates to learn from past actions whether they can expect future periods to follow an equilibrium selected today. Regardless of initial beliefs, a renegotiation-proof equilibrium is ultimately sustainable. This happens when the agent's beliefs about her ability to stick to her current equilibrium are sufficiently weakened. Only some renegotiation-proof equilibria are robust in the sense that they are supported by this learning process.

1.1 A Note on the Preferences

Several papers use hyperbolic discounting as a way to model problems of temptation and self-control.[4] Properties of hyperbolic discounting and resulting equilibria have been investigated in several papers, including Phelps and Pollack (1968), Laibson (1997), Bernheim, Ray, and Yeltekin (1999), Harris and Laibson (2001), O'Donoghue and Rabin (2001b), and Krusell and Smith (2003). O'Donoghue and Rabin (1999) solve, among other things, for SPNE in games with immediate costs and deferred benefits.

It is well understood that degrees of sophistication affect actions and welfare. Behavior by naifs, sophisticates, and those in between have been studied in different environments by O'Donoghue and Rabin (1999), Della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), and Heidheus and Koszegi (2010). O'Donoghue and Rabin (2001a) argue that people might also be *partially naive*; i.e. they underestimate the degree of future time-inconsistency. They show that even slight naivete can significantly increase procrastination. Ali (2011) builds a model in which individuals learn about their self-control problems, which leads them to behave as if sophisticated.

In this context, semi-sophistication can be viewed as identifying a different kind of mistake–it captures the idea of a disconnect between being aware of one's preferences and acting correctly on them. Unlike a semi-sophisticate, a full-sophisticate understands that subgame perfection is not enough. If she were asked to play a SPNE that gives her a lower discounted utility than another, she has an incentive to renegotiate the choice of equilibrium. Knowing this, the individual must restrict her choices to those equilibria that are renegotiation-proof. The concept of renegotiation-proofness in repeated games has been developed, most notably, by Bernheim & Ray (1989), Farrell and Maskin (1989), and Abreu, Pearce, and Stacchetti (1993). Kocherlakota (1996) extends this to games with time-inconsistency, terming it *reconsideration-proofness.*

I suggest that semi-sophistication is interesting not only because of its predictions, but also because it is a new and reasonable description of preferences. This captures the idea that people are not under the illusion that their future selves are exponential discounters, but are nevertheless frustrated about the choices they make and continue to make. The model allows people to be fully aware of their hyperbolic tendencies even as they make mistakes along another dimension.

[4] For some other approaches to related problems, see Gul and Pesendorfer (2001), Fudenberg and Levine (2006), and Banerjee and Mullainathan (2010).

1.2 Applications and Evidence

There is growing empirical evidence of time-inconsistent preferences. See Laibson, Repetto, and Tobacman (2001), Ashraf, Karlan, and Yin (2006), and Gugerty (2007) for examples. One goal of this paper is to contribute to our understanding of how these preferences might translate into behavior in a class of games. Section 3 argues that we must take deadlines into account in an attempt to infer preferences, since the same individual can appear sophisticated or naive depending on the set of equilibria she faces. Even if individuals are full-sophisticates, the multiplicity of renegotiation-proof equilibria creates difficulties in backing out preferences from behavior. Also, the analysis with competitive firms shows how markets could fail to deliver certain products even when individuals value them.

The design of commitment contracts under hyperbolic discounting has been widely studied.[5] Carrillo and Dewatripont (2008) examine the role of hard-to-break promises as a form of commitment. Basu (2011) shows how roscas can serve as commitment devices by improving savings patterns relative to autarky. Ghatak and Fischer (2010) and Basu (2012) study commitment in simple lending and borrowing contracts. Ashraf, Karlan, and Yin (2006) find that time-inconsistency (elicited through hypothetical questions) is a significant predictor of take-up of basic illiquid savings accounts.

Those who perceive the need for a commitment product must be sophisticated hyperbolic discounters.[6] Of those, the ones who are semi-sophisticated and face high transaction costs will not join. Here, a limited-time offer will create a unique equilibrium for semi-sophisticates, and generate higher takeup than an open offer would. This paper should also help answer one of the questions posed by Ashraf, Karlan, and Yin (2006): "A natural question arises concerning why, if commitment products appear to be demanded by consumers, the market does not already provide them" (p. 638). While their focus is on the design of the product, I suggest here that patterns of access to the also matter.

This problem can be particularly acute in developing economies.[7] Apart from the fact that transaction costs are high, individuals lack access to tools that help them commit to committing. Examples of such tools include pre-allocation of income to retirement funds, or infrequent benefits-enrollment opportunities that help generate a unique equilibrium. If staggered offers are

[5] See Gharad, Karlan, and Nelson (2010) and Bond and Sigurdsson (2011) for a discussion.
[6] These are products that naifs do not perceive a need for.
[7] Mullainathan (2004) lays out the problem of time-inconsistency in the context of development.

infeasible, attempts to reduce the first-period cost (such as shortening the distance the person needs to travel to sign up) can prove more beneficial than other models would suggest.

This model also provides a framework for distinguishing between goods that have a natural deadline, and those that don't. Take, for example, a roof that needs to be fixed. Upon completion, this will yield a stream of benefits. The constraint is that the roof must be fixed before winter sets in and makes outdoor work impossible. The model predicts that a naif will fix the roof on the last possible day, an exponential discounter will do it early, and a semi-sophisticate (and fully-sophisticate) will do it sometime in between. Now consider the case where the agent resides in a climate with perpetual summer. Under these circumstances, the semi-sophisticate, like the naif, will never fix the roof.

Choi, Laibson, Madrian, Metrick (2001) find that employees are much more likely to join and remain in 401(k) savings plans if these plans are offered as the default option. If we are to assume that, in either case, most people would prefer to be enrolled in 401(k) than not, but that there is a small cost to deviating from the default option, then the empirical results are consistent with my model. When the 401(k) option is not a default, sophisticated hyperbolic discounters fail to adopt it. In contexts such as this, semi-sophistication might provide an alternative interpretation for status-quo bias. Also, Cialdini (2001) writes about how the illusion of scarcity can make people more willing to purchase a good: "A great deal of evidence shows that items and opportunities become more desirable to us as they become less available." This could be taken as further indirect evidence of semi-sophistication.

2 Model Setup

2.1 Preferences and the Game

Time is discrete: $t \in \{1, 2, 3, ...\}$. Infinitely-lived individuals are born in every period. An individual's instantaneous utility in period t is denoted u_t, and her discounted utility starting in period t is:

$$V_t \equiv u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau \tag{1}$$

Here, $\delta \in (0, 1)$ is the exponential discount factor and $\beta \in (0, 1)$ is the hyperbolic discount factor. If β were equal to 1, the individual would have time-consistent preferences. With $\beta < 1$, she is a hyperbolic discounter–in any period t, the discounting between t and $t+1$ is heavier than the discounting between $t+1$ and $t+2$.

The individual must choose when, if ever, to purchase a good (or, equivalently, perform a task). At most one such purchase can be made over a lifetime. In period 0, before the game begins, a stream of prices $P \equiv (p_1, p_2, p_3, ...)$ is revealed (let P_t denote the stream of prices starting in period t). If the individual purchases in some period t', she pays an instantaneous price $p_{t'}$. I assume that the price of the good is not transferrable across periods–this could be because of financial market failures or because the price includes a transaction cost. The good yields a benefit of b in the period after which it is purchased. This benefit could be viewed as a one-period consumption benefit, or the discounted lifetime benefit from a durable good, or the future benefits of a commitment product.

Instantaneous utility is a strictly increasing function of consumption: $u_t = u(c_t)$. In the absence of a purchase, the individual's consumption is 0, and we set $u(0) = 0$. Suppose the individual were to purchase the good in period t' at a price $p_{t'}$. We can then describe his discounted utility in any period $t \leq t'$ by:

$$U_t\left(t', p_{t'}\right) \equiv \begin{cases} u\left(-p_{t'}\right) + \beta\delta u\left(b\right), \text{ if } t' = t \\ \beta\delta\left(u\left(-p_{t'}\right) + \delta u\left(b\right)\right), \text{ if } t' > t \end{cases} \tag{2}$$

The individual's objective in period t is to maximize V_t. She takes current and future prices as given and makes an plan, which is given by a strategy $R_t \equiv \left(\rho_t, \rho_{t+1}, \rho_{t+2}, ...\right)$, where each $\rho_i \in [0,1]$. A strategy can be interpreted in the following manner: if, by period t', the good has not been purchased, then it will be purchased with probability $\rho_{t'}$ in t'. This mixed-strategy notation also accommodates pure strategies (which would involve restricting the action space to 0 or 1). In this framework, discounted utility from a strategy R_t under prices P_t is given by:

$$V_t\left(R_t, P_t\right) \equiv \sum_{\tau=t}^{\infty}\left(\left(\prod_{\gamma=t}^{\tau-1}\left(1-\rho_\gamma\right)\right)\rho_\tau U_t\left(\tau, p_\tau\right)\right) \tag{3}$$

Figure 1 illustrates the game. We can separate the individual into a sequence of time-indexed players. Each node describes the individual's choices in a particular period. The moment she purchases the good, the game is over. What the individual will actually do depends on her preferences and her awareness of her preferences, as explained in the next subsection.

Finally, I make three assumptions to restrict attention to the parameter regions of interest.

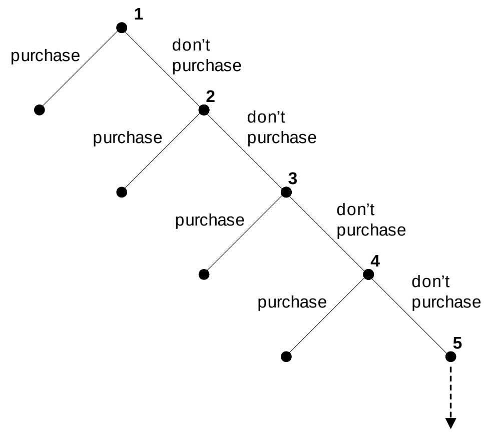

Figure 1: The game

Suppose the marginal cost of production for the good is given by a constant $c > 0$. Then:

1. $U_t(t, c) > 0$. When the product is offered at marginal cost, it is better to purchase today than never at all.

2. $U_t(t+1, c) > U_t(t, c)$. The hyperbolic discounter strictly prefers to purchase tomorrow than today. This captures the essential problem of time-inconsistency: if a purchase is deferred from the present, the costs diminish by $\beta\delta$ while the benefits diminish by δ. This assumption ensures that the diminished pain from deferring costs is sufficient to make up for the postponement of the benefits.

3. For any $t' > t$, $U_t(t', c) \neq U_t(t, c)$. This is a "no-indifference" assumption, used to simplify the analysis without loss of generality.

2.2 Strategies and Equilibrium

A hyperbolic discounter can be *naive* or *sophisticated*. The naif, in any period, believes that the preferences of her future selves are characterized by $\beta = 1$. A sophisticate knows that her future selves discount the future exactly as she does, by placing relatively greater weight on instantaneous utility.[8] To predict behavior, consider O'Donoghue and Rabin's (1999) concept of a *perception-perfect strategy (PPS)*. In any period, the PPS describes the individual's plan, or the best of the strategies she considers feasible.

[8] A hyperbolic discounter is sophisticated if, at any period t, her preferences are common knowledge across all t'-selves, where $t' \geq t$.

A strategy R is a PPS for a naif iff it has the following properties:

$$\begin{aligned}
\rho_t &= 1, \text{ if } [U_t(t,p_t) > 0 \text{ and for all } t' > t,\ U_t(t,p_t) > U_t(t',p_{t'})] \\
\rho_t &= 0, \text{ if } [U_t(t,p_t) < 0 \text{ or for some } t' > t,\ U_t(t,p_t) < U_t(t',p_{t'})] \\
\rho_t &\in [0,1], \text{ otherwise}
\end{aligned}$$

In any period, the naif determines her ideal purchase period, and sets her strategy to purchase then. She will purchase the good in the current period only if she cannot do better by purchasing it in the future. The naif forms her plan just as an exponential discounter does–if, from today's vantage point, she ideally wants to purchase the good in period t', she assumes that when t' arrives she will be similarly inclined to purchase it. Of course, unlike an exponential discounter, she fails to recognize that her t'-self will also be time-inconsistent and might reject this plan.

For sophisticates, it is natural to frame the problem as a game with infinite agents, where each agent decides whether to purchase the product, given that it has not been purchased so far. Here, a PPS is equivalent to a subgame perfect Nash equilibrium (SPNE). An agent will choose to purchase the good in any period as long as it is better than waiting for the next time it is purchased under a given equilibrium. In any period, she understands that future periods will choose actions that are optimal from their own perspective, not hers.

A strategy ρ is a PPS for a sophisticate iff it has the following properties (i.e. is a SPNE):

$$\begin{aligned}
\rho_t &= 1, \text{ if } \left[U_t(t,p_t) > 0 \text{ and } U_t(t,p_t) > \sum_{\tau=t+1}^{\infty}\left(\left(\prod_{\gamma=t+1}^{\tau-1}(1-\rho_\gamma)\right)\rho_\tau U_t(\tau,p_\tau)\right)\right] \\
\rho_t &= 0, \text{ if } \left[U_t(t,p_t) < 0 \text{ or } U_t(t,p_t) < \sum_{\tau=t+1}^{\infty}\left(\left(\prod_{\gamma=t+1}^{\tau-1}(1-\rho_\gamma)\right)\rho_\tau U_t(\tau,p_\tau)\right)\right] \\
\rho_t &\in [0,1], \text{ otherwise}
\end{aligned}$$

The fact that a sophisticate plays a SPNE does not by itself predict behavior when there is a multiplicity of equilibria, especially when there is disagreement across selves about the best equilibrium. Suppose period t chooses her *optimal* SPNE, or one that maximizes V_t. Now suppose that in period $t' > t$, the equilibrium in play is no longer optimal. She would like to renegotiate to a difference SPNE. Moreover, since there is no other player she needs to instantaneously coordinate with, renegotiation here simply involves *reconsideration* (to use

Kocherlakota's (1996) term). Having discarded the original equilibrium and replaced it with another, she can just broadcast the new strategy to her future selves.

A hyperbolic discounter who is cognizant of this problem must agree with her future selves to choose from a restricted set of equilibria, such that there is no incentive to replace an equilibrium in that set with another from that set. To define this set, I modify the definitions of renegotiation-proofness developed for repeated games by Farrell and Maskin (1989), and for hyperbolic discounters by Kocherlakota (1996).

Definition 1 *A SPNE R is weakly renegotiation-proof iff the following holds for any two distinct nodes* $\hat{t}$ *and* t'*: if* $P_{\hat{t}} = P_{t'}$*, then* $V_{\hat{t}}(R_{\hat{t}}, P_{\hat{t}}) = V_{t'}(R_{t'}, P_{t'})$*.*

Definition 2 *A SPNE R is strongly renegotiation-proof iff it is weakly renegotiation-proof and the following holds for any two (not necessarily distinct) nodes* $\hat{t}$ *and* t'*: if* $P_{\hat{t}} = P_{t'}$*, then* $V_{\hat{t}}(R_{\hat{t}}, P_{\hat{t}}) = V_{t'}\left(\tilde{R}_{t'}, P_{t'}\right)$ *for all weakly renegotiation-proof equilibria* $\tilde{R}$*.*

Definition 3 *A sophisticated hyperbolic discounter is fully-sophisticated if she plays a strongly renegotiation-proof equilibrium when one exists.*

A full-sophisticate restricts herself to a set of equilibria in which each has the following property: at any node, given a stream of prices, there is no other equilibrium in the set that would provide a higher discounted utility. Since the agent is always aware that certain equilibria would tempt future selves to renegotiate, she eliminates those from the set that she chooses from.

Definition 4 *A hyperbolic discounter is semi-sophisticated if, in any period* t*, she selects the SPNE that maximizes* V_t*.*

Semi-sophisticates too are aware of their future preferences, so they always play a SPNE. However, in any period, a semi-sophisticate chooses the equilibrium that maximizes her continuation utility, while failing to anticipate that future selves may switch to a different equilibrium. In a sense, she fails to recognize that in future periods, her effective strategy sets also include a choice over available equilibria. This describes a mistake that is fundamentally different from naivete. In particular, we can see that full-sophisticates and semi-sophisticates will behave identically in games with a unique equilibrium, but their outcomes can diverge under multiple equilibria.

3 Benchmark: Pricing at Marginal Cost

Assume that the good is always offered at the marginal cost, c, so there is no room for strategic pricing. This lends itself to a number of interpretations. The good in question could be considered a task: a job application, a trip to Ikea, or the first phone call to a romantic interest. It could also be the adoption of a bank account or commitment savings product where the price is the fixed cost of enrollment, such as opening fees or travel time. All such tasks share the property of immediate costs and delayed benefits. By eliminating the role of markets, we are able to make some interesting predictions about outcomes for different types of agents. This section analyzes how deadlines affect outcomes.

When there is a deadline, the task can be completed at cost c in any period up to a deadline T. The results here intuitively match those in O'Donoghue and Rabin (1999). An exponential discounter completes the task in period 1 (by Assumption 1). A naif completes it in period T. Both full-sophisticates and semi-sophisticates play the unique SPNE and complete the task by some date in between.

When we lift the deadline, outcomes diverge dramatically for full-sophisticates and semi-sophisticates. A full-sophisticate plays a strongly renegotiation-proof SPNE and performs the task with some probability in each period. However, a semi-sophisticate finds that, in each period, she can switch to a new SPNE that allows her to postpone the task. As a result, she never completes it. A semi-sophisticated agent appears sophisticated when there is a deadline, but naive when there isn't. The distance of the deadline has an effect on the timing of completion, but can be considered small relative to the impact of having a deadline at all.

A key driver of intuition below is the following. From Assumptions 1 and 2, a one-period delay is desirable:

$$U_t\left(t+1,c\right)=\beta\delta\left[u\left(-c\right)+\delta u\left(b\right)\right]>u\left(-c\right)+\beta\delta u\left(b\right)=U_t\left(t,c\right)>0 \tag{4}$$

But if completion is pushed back further, it becomes relatively less attractive compared to immediate completion. In other words, $U_t\left(t+\alpha,c\right)=\beta\delta^{\alpha}\left[u\left(-c\right)+\delta u\left(b\right)\right]$ is strictly dropping in α. If the individual knows that the next completion date is sufficiently far off, she will choose to complete it today.

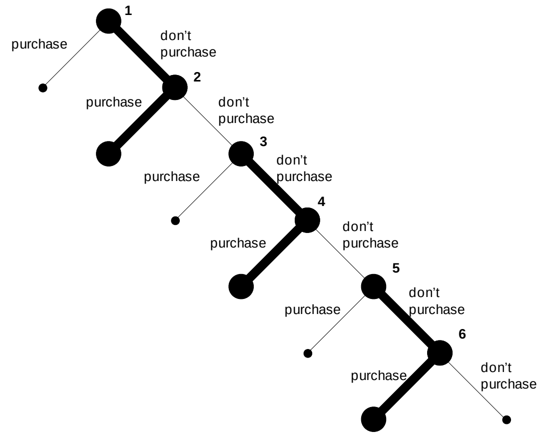

Figure 2: A possible SPNE when the task cannot be completed after period 6.

3.1 Deadline

First, consider the case with a deadline. The behavior of exponential discounters and naifs is straightforward. The exponential discounter plays a strategy $\rho_t = 1$ for all $t \leq T$. She completes the task in the first period of her life. The naif would always like to postpone by one period. In any period $t < T$, her PPS involves $\rho_t = 0$ and $\rho_{t+1} = 1$. On the last possible day, locked in by the deadline, she performs the task. She procrastinates until T because, in any period before that, she believes that her next period self will find it optimal to complete the task.

We can also determine the unique SPNE for a sophisticate. The intuition for this is easily gained using backward induction. We know that the agent in period T will certainly complete the task if it has not yet been completed. Consider period $T-1$. By Assumption 2, she would rather wait until T. Now consider period $T-2$. If she does not complete it immediately, she must wait two periods, which yields a strictly lower discounted utility than waiting one period. At this point, she might prefer to complete it immediately rather than wait. A repetition of this argument generates a SPNE in which the task is performed every other period. A more hyperbolic individual would have a SPNE will less frequent completion. Figure 2 provides a possible equilibrium (the thick lines are the equilibrium strategies). In this case, $T = 6$, and the unique SPNE results in the task being completed by period 2.

The following proposition establishes that there is a unique integer α^* that describes the frequency with which a task is completed.[9]

Proposition 1 *If the good is offered with a deadline T and is priced at marginal cost ($p_t = c$),*

[9] All proofs are in the appendix.

then there is a unique SPNE for the sophisticated hyperbolic discounter. This SPNE satisfies, for some integer $\alpha^* > 1$*,*

$$\rho_t = \begin{cases} 1, \text{ if } t = T - k\alpha^*, \text{ where } k \text{ is any non-negative integer} \\ 0, \text{ otherwise} \end{cases}$$

α^* is the smallest integer that satisfies:

$$u(-c) + \beta\delta u(b) \geq \beta\delta^{\alpha}(u(-c) + \delta u(b)) \tag{5}$$

It is interesting to note here that, while α^* is inversely related to β, it does not follow that a less hyperbolic person will complete a task sooner.

3.2 No Deadline

Suppose the task can be completed in any period at price c. There is a unique PPS for the exponential discounter: for all t, $\rho_t = 1$. She will again complete it in the first period of her life. Since the naif consistently mis-predicts her future preferences, and given Assumption 2, in any period t she plays a SPNE with $\rho_t = 0$ and $\rho_{t+1} = 1$. She ends up never completing the task.

The sophisticate must choose from a large set of equilibria. Recall that, under a deadline, the SPNE involved completing the task every α^* periods. Even without a deadline, any pure-strategy SPNE will have the same property. However, there are now α^* such SPNE: one in which the agent completes the task every α^* periods starting in period 1, another where she completes it every α^* periods starting in period 2, and so on.

We can also solve for mixed-strategy SPNE. For an agent to play a mixed strategy in any period, she must be indifferent between completing the task today and leaving it for the future. To have the agent be similarly indifferent in each period, there must be some fixed probability with which she completes the task in every period starting with $t = 2$. This probability, ρ^*, is uniquely defined for the following reason: if the agent plays probability $\rho' > \rho^*$ in any period, her previous self loses the incentive to play a mixed strategy and prefers to play $\rho = 0$. If the agent plays $\rho'' < \rho^*$ in any period, her previous self now has a strict incentive to play $\rho = 1$.

To derive ρ^*, note that the following indifference condition must be satisfied at any t:

$$U_t(t,c) = \sum_{\tau=t+1}^{\infty} \left(\left(\prod_{\gamma=t+1}^{\tau-1} (1-\rho_\gamma) \right) \rho_\tau U_t(\tau, p_\tau) \right) \tag{6}$$

Combining indifference conditions for t and $t+1$, we get:

$$\begin{aligned} & U_t(t,c) = \rho_{t+1} U_t(t+1,c) + \delta \left(1-\rho_{t+1}\right) U_{t+1}(t+1,c) \\ \Rightarrow \quad & \rho_{t+1} = \rho^* \equiv \frac{(1-\delta) U_t(t,c)}{U_t(t+1,c) - \delta U_t(t,c)} = \frac{(1-\delta)\left[u(-c) + \beta\delta u(b)\right]}{-(1-\beta)\delta u(-c)} \end{aligned} \tag{7}$$

Note that ρ^* is strictly increasing in β.

So, we end up with a set of fully mixed-strategy SPNE that all look like $(\tilde{\rho}, \rho^*, \rho^*, \rho^*, ...)$, where $\tilde{\rho} \in [0,1]$. Period 1 is free to mix in any way she likes since this does not affect the choices of her future selves. There can also be partly-mixed-strategy SPNE. Among these are equilibria that follow pure strategies until some time $\hat{t}$ and then switch to fully mixed-strategies forever.

Given the multiplicity of SPNE, there will be a stark difference between the outcomes of semi-sophisticates and full-sophisticates. In any period, the semi-sophisticate's optimal SPNE involves completion in the following period (one of the pure strategy SPNE always achieves this). She repeatedly renegotiates her equilibrium, and never completes the task. The full-sophisticate, on the other hand, must seek a strongly renegotiation-proof equilibrium. It turns out that any weakly renegotiation-proof equilibrium is also strongly renegotiation-proof. Each has the property that the individual's continuation utility in any period t is exactly $U_t(t,c)$. To see why, note that any equilibrium in which two different nodes had different discounted utilities, there would be an incentive to renegotiate at one of those nodes. These results are summarized in the following proposition.

Proposition 2 *Suppose the good is always offered at marginal cost. (a) The semi-sophisticate will, in any period t, select a SPNE with $\rho_t = 0$ and $\rho_{t+1} = 1$. She will never purchase the good. (b) The full-sophisticate will choose and stick to an equilibrium with the following property: for all t, $V_t = U_t(t,c)$. These equilibria look like the following: $(\tilde{\rho}, \rho^*, \rho^*, \rho^*, ...)$ for all $\tilde{\rho} \in [0,1]$, and $(1, \hat{\rho}, \rho^*, \rho^*, \rho^*, ...)$ for all $\hat{\rho} \in [0, \rho^*]$, with ρ^* defined by Equation 7 in each case.*

3.3 Implications

The analysis above suggests a number of empirically relevant points. We have seen that, even though semi-sophisticates and full-sophisticates would display identical choices under a deadline, they behave very differently when the deadline is lifted. The semi-sophisticate appears sophisticated under finite horizons but naive under infinite horizons. So, depending on the rules of the game, the same preferences can lead to disparate behaviors that are typically interpreted as evidence of two different types of preferences. This provides one framework to think about the psychological salience of deadlines.

The full-sophisticate, unlike the semi-sophisticate, will always complete the task, regardless of deadline. In the absence of a deadline, the likelihood of purchase rises as she gets less hyperbolic.

There is also a difficulty associated with backing out preferences in finite horizon settings. Since individuals are locked into unique SPNE, it is entirely possible that a more hyperbolic individual will complete the task before a less hyperbolic individual will.

4 Government Provision

Suppose the population is fixed at N, but that any individual alive in t survives to $t+1$ with probability δ. This means that there are $(1-\delta)N$ newborns in every period. Consider a well-meaning government that wishes to maximize the probability that an individual completes the task in her lifetime. The government must announce, in period 0, in which periods the task will be offered. When it is offered, it will be at the lowest possible price, c.

Clearly, welfare as described here will be highest if each individual were to complete the task in the first period of her life. This would be achievable if the population was static–the government could offer the task only in period 1. But with a changing population, this would require that the government offer it everyday, and as a result semi-sophisticates and naifs would never complete it.

4.1 Semi-Sophisticates

When the population consists of semi-sophisticates, the government must ensure that there is some window of $(\alpha^* - 1)$ periods when the task is unavailable. One such scheme involves only making the task available every α^* periods. This locks semi-sophisticates into a unique equilibrium under which they complete the task in certain periods. As a result, some individuals will not survive to complete it.

Proposition 3 *Suppose the government sells the good at marginal cost and must decide in which periods to offer it, and its goal is to maximize the probability that a given individual will purchase the product in his lifetime. This probability will be maximized by an offer every α^* periods.*

If the government has a larger set of tools at its disposal, it can achieve greater takeup. In cases where the offer can be conditioned on birthdate, the government can immediately maximize total welfare. Secondly, in cases where it is at all feasible for the government to reduce prices below c, it would make sense to lower them enough (so that hyperbolic discounters prefer to purchase the good immediately), while making up the revenue difference through taxes.

4.2 Other Types

Government provision to other types of consumers depends on the nature of the task being performed. Suppose the task involves takeup of a commitment device–a fixed deposit account, a rosca, or a layaway. Such products are of limited value to exponential discounters (because they don't need commitment) and naifs (because they believe they don't need commitment). The pattern of offers in such cases can focus on maximizing takeup among sophisticates. The main lesson here is that commitment device offers without deadlines can miss out on semi-sophisticates altogether.

Suppose instead the task is of value to all types. For naifs, the government strictly prefers to offer every α^* periods as described in Proposition 3. While distant deadlines would be equally effective with semi-sophisticates, they would leave naifs strictly worse off–a naif will not complete the task until she actually hits against an immediate deadline lasting at least $(\alpha^* - 1)$ periods. For exponential discounters, on the other hand, daily offers are ideal. The government's ultimate decision must depend on the fractions of each type in the population.

Finally, it is instructive to think about optimal offers to full-sophisticates. Recall that full-sophisticates choose from the following set of equilibria: $(\tilde{\rho}, \rho^*, \rho^*, \rho^*, ...)$ for all $\tilde{\rho} \in [0, 1]$, and $(1, \hat{\rho}, \rho^*, \rho^*, \rho^*, ...)$ for all $\hat{\rho} \in [0, 1]$. If the individuals pick the equilibrium that involves Y in period 1, the government could offer the product everyday and maximize takeup. It is arguable that, when a hyperbolic discounter finds herself indifferent across equilibria, she picks the one that maximizes her "period 0" welfare (i.e. the discounted utility she would have had with $\beta = 1$).

However, there is no clear reason to assume this. In fact, in Section 6.1 I show that equilibria like $(\tilde{\rho}, \rho^*, \rho^*, \rho^*, ...)$ with $\tilde{\rho} < \rho^*$ have a particular advantage–they eliminate the semi-sophisticate's desire to postpone adoption of the equilibrium into the following period. Suppose individuals play the SPNE with identical probabilities in each period: $(\rho^*, \rho^*, \rho^*, \rho^*, ...)$. For such agents, we could ask whether the government has a clear preference for unlimited provision or staggered provision.

Under a renegotiation-proof equilibrium, the probability that an individual completes the task is:

$$Q_{\rho^*} \equiv \frac{\rho^*}{1 - (1 - \rho^*)\,\delta} \tag{8}$$

Under a unique equilibrium emerging from staggered provision, the probability with which an individual completes the task depends on her distance between her birthdate and the next offer period. On average, this is:

$$Q_{\alpha^*} \equiv \left(\frac{1}{\alpha^*}\right)\left(\frac{1 - \delta^{\alpha^*}}{1 - \delta}\right) \tag{9}$$

The proposition below shows that if α^* satisfies Equation 5 with equality, then $\rho^* < \frac{1}{\alpha^*}$, from which it follows mechanically that $Q_{\rho^*} < Q_{\alpha^*}$. The intuition for this is the following. Suppose, under the mixed strategy, the expected number of days an individual had to wait for the good to be purchased was exactly α^*. Then, her discounted utility from waiting would be greater than $\beta\delta^{\alpha^*}[u(-c) + \delta u(b)]$, because the probability of buying it early is overweighed (due to discounting) relative to the probability of buying it late. But then this means that the RHS of Equation 7 is greater than the LHS–waiting is strictly better than purchasing now. So this mixing strategy cannot be a SPNE. Therefore, the only acceptable value of ρ^* must be one that leads to an expected wait that is longer than α^*.

Proposition 4 *Suppose, when the task is offered everyday, the individual plays $(\rho^*, \rho^*, \rho^*, ...)$. If α^* satisfies Equation 5 with equality, then the average probability of completion when the task is offered every α^* periods is higher than when it is offered everyday.*

The reason this does not always hold is that time is discrete, and therefore α^* might be larger than the non-integer value that satisfies Equation 5 with equality. However, it does suggest that even for full-sophisticates, there is no clear advantage to offering the task everyday. If the renegotiation-proof equilibrium has a low purchase probability in the first period, the government is even more likely to improve completion rates by offering the task every α^* periods rather than everyday.

5 Markets

5.1 Monopoly

Continue to assume that the population has a fixed size N, with an individual's probability of survival δ. Consider an infinitely-lived monopolist with a discount factor of δ_f.[10] This firm must post a menu of prices in period 0.[11] As we have seen before, if it offers the good at a constant price above marginal cost in each period, then semi-sophisticates will never buy the good. To encourage agents to buy the product, the firm will have to occasionally raise prices.

This leads to a new rationalization of sales. Among existing theories, Sobel (1984) proposes a model where each firm has monopoly power over a group of impatient buyers who value the product highly, and all firms compete for access to a set of patient buyers with lower valuations of the product. Firms keep prices high as long as they can make higher profits from their core clients than they can by selling to the patient agents. Once enough patient clients develop, prices are lowered.

In this paper, price variation can be justified not as a form of price discrimination, but as a way to generate deadlines. These deadlines come in the form of high prices, and help to lock semi-sophisticates into a unique equilibrium. While the actual pattern of prices offered by a firm depend on the tradeoff between frequency of sales and price at which sales are made (higher price at time of purchase means that these sale periods must be more staggered), we can see that there is always a credible pricing rule that produces strictly higher profits than flat pricing.

Definition 5 *A stream of prices is credible if, at any time t, given the future stream of prices, the firm has no incentive to change the current price.*

Suppose the firm decides to have a sale every α periods, with high prices the remaining

[10] δ_f could reflect the monopolist's preferences or the net present value of future earnings: $\delta_f = \frac{1}{1+r}$.

[11] The "price" could also be viewed as a decision of where to locate or how many branches to open–the further the branch is from the consumer, the higher the transaction costs incurred by her.

time. The highest price it can charge in the sale period is given by the consumer's indifference condition. Define $p(\alpha)$ as satisfying:

$$U_t(t, p(\alpha)) = U_t(t + \alpha, p(\alpha)) \geq 0 \tag{10}$$

This yields a unique solution for $\alpha \geq \alpha^*$. Any such $p(\alpha)$ every α periods, with higher prices in between, generates a unique SPNE.

Proposition 5 *With a population of semi-sophisticated buyers, there is always a credible pricing strategy with sales that is more profitable for the firm than any strategy with constant prices.*

The firm's profits are:

$$\pi(\alpha) = [p(\alpha) - c]\left[N + N(1 - \delta^\alpha)\frac{\delta_f^\alpha}{1 - \delta_f^\alpha}\right] \tag{11}$$

The term in the first square brackets represents the profit per unit sold. The term in the second square brackets represents the discounted sum of units sold: N in the first period and $N(1 - \delta^\alpha)$ during every sale period thereafter. If a firm adopts such a pricing scheme, the profit-maximizing choice of α depends on several factors, and in particular two that did not figure in earlier sections: the curvature of the utility function (downward pressure on α if more concave) and the magnitude of δ_f (upward pressure on α if high).

The firm's pricing scheme will also respond to the makeup of types in the population. Naifs encourage frequent sales. With full-sophisticates, firm behavior will depend on the choice of equilibrium, which might involve immediate purchase. Finally, exponential discounters provide firms with a reason to think carefully about prices in non-sale periods as well. Suppose the sale price is p'. In the period before the sale, the exponential discounter is willing to buy the good at any price $p'' > p'$ that satisfies: $u(-p'') + \delta u(b) \geq \delta[u(-p') + \delta u(b)]$. Since such a price will not alter the SPNE played by the semi-sophisticate, the firm can sell at higher prices to exponential discounters without losing the semi-sophisticates in the next period.

5.2 Competition

Consider m infinitely-lived competitive firms, each able to produce the product at a constant marginal cost, c. In period 0, each firm i posts a menu of prices $P^i = (p_1^i, p_2^i, p_3^i, ...)$. The firms have a discount rate of δ_f. At any time t, the firm's discounted value of profits is given by:

$$\Pi_t^i = \pi_t^i + \delta_f \pi_{t+1}^i + \delta_f^2 \pi_{t+2}^i + ... \tag{12}$$

The timing is as described before. After the firms publicize their respective menus of prices, each individual plays according to her optimal equilibrium in each period. The relevant set of prices for the consumer is simply the lowest posted price for each period t. Let us denote this menu of prices $\bar{P} = (\bar{p}_1, \bar{p}_2, \bar{p}_3, ...)$. A competitive equilibrium is defined as a set of prices with the following property: there is no period t in which any firm is able to strictly increase Π_t^i by deviating from its proposed price. Note that any firm that posts $p_t^i > p_t$ will sell 0 units in period t. Suppose in some period t, that q firms each set some price p_t. Then, each firm's expected profit from that period is given by the total expected profits divided by p.

First, it is clear that firms cannot make profits in competitive equilibrium. If there is any period t in which a firm makes a profit by selling with positive probability at $p_t > c$, every other firm has an incentive to sell at some $p_t - \varepsilon$. Two zero-profit outcomes can emerge in this setting–one in which prices are always so low that semi-sophisticates never buy the good, and another in which semi-sophisticates buy the good as frequently as under government provision.

Proposition 6 *(a) There is always a competitive equilibrium in which semi-sophisticates never buy the good. (b) There is also always a competitive equilibrium in which semi-sophisticates buy the good every α^* periods at price c.*

This proposition shows us that competition can lead to both the worst and the best possible outcomes. In one equilibrium, no individual ever buys the good. In another equilibrium, total welfare is equal to the highest total welfare under government provision. While this leads to an ambiguous prediction about the relative merits of monopoly and competition, it is nevertheless noteworthy that competitive equilibrium can lead to strictly lower total welfare than under a monopoly.

This comparison becomes more favorable to monopoly if the population also contains a small fraction of exponential discounters. As long as this fraction is sufficiently small, the monopolist will still find it in its interest to sell to semi-sophisticates by using deadlines. However, regardless of the fraction of exponential discounters, there will no longer be a competitive equilibrium in which semi-sophisticates buy. To see this, suppose μ fraction of the population are exponential discounters. In competitive equilibrium, if there is some period in which a semi-sophisticate purchases the good with a positive probability, then there must be a period t in which the lowest market price is $p_t > c$. Then, in period t, a firm can strictly raise its profits by lowering its price by a sufficiently small amount, thereby capturing all newborn exponential discounters. So prices must collapse to c in each period.

6 Additional Considerations

6.1 Learning

Semi-sophistication describes a fundamental mistake being made by the hyperbolic discounter. Though she is aware of her preferences and thinks strategically about her actions, she fails to recognize that the equilibrium chosen today might be replaced with another equilibrium tomorrow. Given that people are capable of learning from their mistakes, it is worth examining how learning might occur. It turns out that there is a natural interpretation of "learning from one's mistakes" in this model, and that this also gives us reason to narrow the set of renegotiation-proof equilibria to a smaller set of reasonable ones.

I describe an individual who thinks strategically about equilibrium selection given uncertainty about her type: she could be a *renegotiator*–the type that is always struck with the urge to renegotiate her previously selected equilibrium unless it is strongly renegotiation-proof; or she could be a *follower*–the type that sticks to the previously selected equilibrium unless she is struck with an instantaneous urge to renegotiate (which happens with probability $s \in [0, 1)$ only if the previously selected equilibrium is not strongly renegotiation-proof).

Consider the individual born in period 1. She believes with probability p_1 that she is a renegotiator, and with probability $(1 - p_1)$ that she is a follower. She chooses an equilibrium subject to these beliefs. Suppose p_1 is sufficiently small that she prefers to select her optimal equilibrium (which is not renegotiation-proof). In period 2, she finds that the previously selected equilibrium requires purchase today, and also observes that she no longer wants to play this equilibrium. This means that she is either (a) a renegotiator or (b) a follower who has been struck by the renegotiation urge. Applying Bayes' Rule, she updates her probabilities to get $p_2 = \frac{p_1}{p_1+(1-p_1)s} > p_1$. If she were to repeatedly renegotiate, p_t would approach 1 and she would ultimately learn that she was a renegotiator. The proposition shows that, under some conditions, she will ultimately switch over to a renegotiation-proof equilibrium.

Proposition 7 *Suppose the individual believes with probability $p_1 \in (0, 1]$ that she is a renegotiator, and with probability $(1 - p_1)$ that she is a follower. In each period, she observes her reaction to the previously selected equilibrium and updates her beliefs using Bayes' Rule. Based on her updated beliefs, she chooses between following the previously selected equilibrium and renegotiating to a new equilibrium. Then, the following is true: For a renegotiation-proof equilibrium of the form $(\rho_1, \rho^*, \rho^*, \rho^*, ...)$, where ρ^* is defined by Equation 7 and ρ_1 is sufficiently small, there will be some period t^* such that: in any $t < t^*$, she strictly prefers to select her optimal equilibrium; in t^*, she weakly prefers to select the renegotiation-proof equilibrium; and for $t > t^*$, she weakly prefers to stick to the renegotiation-proof equilibrium.*

The intuition for this result is straightforward. By the cutoff period t^*, the individual is sufficiently certain that she is a renegotiator that if she selects her optimal equilibrium today, tomorrow she will likely switch out of it and into a renegotiation-proof equilibrium. If she knows she is likely to switch to a renegotiation-proof equilibrium tomorrow, under what conditions would she rather switch today? The renegotiation-proof equilibrium in question must have a starting probability of purchase $\rho_1 < \rho^*$. Then, the additional deferment of the equilibrium play is undesirable. If this wasn't the case, the individual would prefer to choose her optimal equilibrium for the small chance it would be complied with, since the alternative of a deferred renegotiation-proof equilibrium was anyway more attractive than starting it immediately. This highlights the relative merits of some renegotiation-proof equilibria over others–the strategy should not start out with such high purchase probabilities that the individual would rather wait for the next period to begin.

This throws up a new set of implications for government provision and firm pricing. Especially if learning about one's type affects behavior along other dimensions beyond the scope of the model, it might make sense to allow the individual to learn from her mistakes. Locking her into a unique equilibrium eliminates the prospect of learning. A monopolist too would have to now consider the option of waiting for the consumer to learn so that she purchases even under flat pricing. In particular, if the population consists of a large proportion of exponential discounters, the firm might prefer to enforce this learning process so that it can maintain flat pricing. However, it should be noted that t^* could be very large, leaving the individual to renegotiate for many periods before she switches to a renegotiation-proof equilibrium.

6.2 Finite Horizon

The distinction between full-sophistication and semi-sophistication is only meaningful when there are multiple Pareto-unranked equilibria, which occur in infinite-horizon games such as the one described in this paper. An infinite horizon does not mean individuals must be infinitely-lived–the same insights apply to inter-generational models or, as in Sections 4-5, cases where individuals survive probabilistically into the next period.

In hyperbolic discounting games with a well-defined final period, backward induction typically rules out multiple equilibria. However, if there is some indifference built into the payoff structure of the game, there is once again a substantive difference between the outcomes of full-sophisticates and semi-sophisticates. Consider the investment game in Figure 3. A hyperbolic discounter with a linear utility function must decide whether to make an initial investment (of size 1) in period 1. If she does not invest, the game ends. In period 2, she decides whether to top up the initial investment, which would cost her x. The combined investment decisions yield results in period 3. If top-up investment occurred, there is a return of y, which can be consumed immediately or left untouched until period 4, by when it grows with interest to $\frac{y}{1+r}$. If top-up investment did not occur, there is instead a return of z, and period 3 must again decide when to consume it. Now, suppose $1+r$ is exactly equal to $\beta\delta$. This means that, no matter what period 2 decides, period 3 is indifferent across her choices.

It is easy to verify that there is a parameter region where the following are satisfied:

1. Period 1's optimal outcome involves investing in both periods and consuming late: $\left[-1-\beta\delta x+\beta\delta^3\left(\frac{y}{\beta\delta}\right)\geq 0\right]$.
2. If period 2 were to not invest, period 1 would rather not invest either: $\left[-1+\beta\delta^3\left(\frac{z}{\beta\delta}\right)<0\right]$
3. Period 2's optimal outcome involves not investing and consuming late: $\left[\beta\delta^2\left(\frac{z}{\beta\delta}\right)>-1+\beta\delta^2\left(\frac{y}{\beta\delta}\right)\right]$.
4. Full investment can be supported by a SPNE (period 3 consumes early if there was no investment, late otherwise): $\left[-1+\beta\delta^2\left(\frac{y}{\beta\delta}\right)\geq\beta\delta z\right]$.

Here, period 2 prefers that period 3 consume late. Since period 3 is indifferent, she can condition her action on period 2's action, thereby creating a credible punishment.

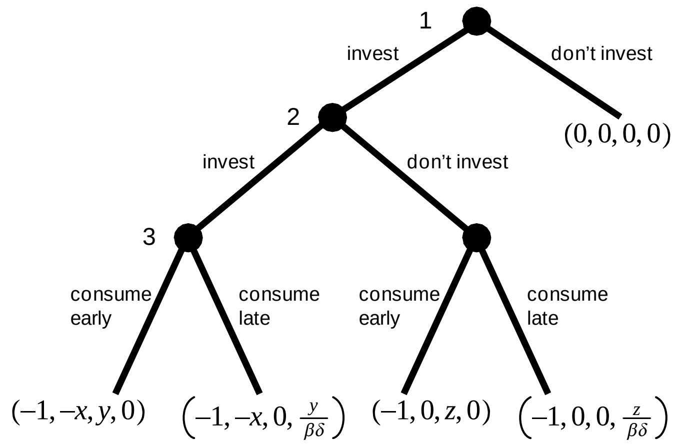

Figure 3: Finite horizon investment game with multiple equilibria: the payoffs refer to instantaneous consumption in periods 1, 2, 3, and 4 respectively.

In this game, the semi-sophisticate will start out playing the equilibrium with investment. However, in period 2, she will renegotiate to an equilibrium that does not punish her for not investing. As a result, period 2 will not invest and period 3 will consume late. A full-sophisticate, on the other hand, would recognize immediately that her preferred equilibrium was not renegotiation-proof, and would select the equilibrium without any investment. While this example is admittedly contrived, it demonstrates the role of indifference in a finite-horizon game. One could in fact interpret period 1's choices as being over the purchase of an "indifference technology" for period 3–as a semi-sophisticate, she would mistakenly believe that such a technology could serve as a commitment device and would therefore pay for it it.

7 Conclusion

In this paper, I have proposed a new way to classify sophisticated hyperbolic discounters based on their beliefs about equilibrium selection in the future. I argue that semi-sophistication is a reasonable way to think about individuals who are aware of their preferences but still make mistakes. By investigating the behavior of semi-sophisticates when they must purchase products that have immediate costs and delayed benefits, I have shown how deadlines can matter. This also provides an alternate theory of sale pricing of durable goods. It is possible for competitive equilibria to lead to lower welfare than markets with a monopolist. Finally, the paper demonstrates how learning can occur, resulting in a switch from semi-sophistication to full-sophistication. The model contains some stark predictions about the takeup of commitment devices under different settings, and it would be instructive to test these in future empirical studies.

8 Appendix

Proof of Proposition 1. The SPNE can be constructed by backward induction. Given the deadline, $\rho_t = 0$ for $t > T$. It follows from Assumption 1 that $\rho_t = 1$.

By Assumption 2, $U_{T-1}(T, c) > U_{T-1}(T-1, c)$. Consider any $\alpha > 1$. $U_{T-\alpha}(T, c)$ is strictly decreasing in α and $\lim_{\alpha\to\infty} U_t(T, c) = 0$. Then, given Assumption 3, there must exist α^* such that $[U_{T-\alpha^*}(T-\alpha^*, c) > U_{T-\alpha^*}(T, c)$ and for all $\alpha < \alpha^*$, $U_{T-\alpha}(T-\alpha, c) < U_{T-\alpha}(T, c)$.

If $\alpha^* \geq T$, then $\rho_t = 0$ for all $t < T$. If $\alpha^* < T$, then $\rho_t = 1$ for $t = T - \alpha^*$ and $\rho_t = 0$ for $t \in \{T - \alpha^* + 1, T - \alpha^* + 2, ..., T - 1\}$. Repeating the above steps using backward induction, the unique SPNE is characterized. ■

Proof of Proposition 2. (a) In any period t', the hyperbolic discounter's optimal outcome involves purchase in $t' + 1$. There is always a SPNE that achieves this. Consider, for any non-negative integer k and $t \geq t'$, $\rho_t = \begin{cases} 1, \text{ if } t = t' + 1 + k\alpha^* \\ 0, \text{ otherwise} \end{cases}$.

(b) Since prices are constant, any weakly renegotiation-proof SPNE must have a constant continuation utility at every node. Such a SPNE cannot satisfy $V_t > U_t(t, c)$ for all t. (If it did, there would have to be some t' at which the product is purchased with positive probability. But then, $V_{t'} = U_{t'}(t', c)$.) Such a SPNE cannot satisfy $V_t < U_t(t, c)$ for all t. (If it did, the agent could do strictly better by deviating and purchasing the good at any node.) Therefore, any weakly renegotiation-proof SPNE satisfies $V_t = U_t(t, c)$, and is also strongly renegotiation-proof.

We can now construct all such equilibria. From $t = 2$ onwards, the agent cannot strictly prefer to purchase the good (if there were any t' where this was false, $V_{t-1} > U_t(t, c)$). From $t = 1$ onwards, the agent cannot strictly prefer not to purchase the good (if there were any t' where this was false, $V_t > U_t(t, c)$). This means that Condition 6 must be satisfied at any t, and for any $t \geq 3$, $\rho = \rho^*$ as defined in Equation 7. By Assumptions 1 and 2, $\rho^* \in (0, 1)$. Then, the only SPNE that satisfy $V_t = U_t(t, c)$ for all t, are: $(\tilde{\rho}, \rho^*, \rho^*, \rho^*, ...)$ for all $\tilde{\rho} \in [0, 1]$, and $(1, \hat{\rho}, \rho^*, \rho^*, \rho^*, ...)$ for all $\hat{\rho} \in [0, \rho^*]$. ■

Proof of Proposition 3. Suppose, in some period t', there is an optimal SPNE in which the good is purchased with positive probability. Then, period t' must also have an optimal equilibrium R^* involving $\rho_{t'} = 1$. Consider any period $t \in [t' - \alpha^*, t' - 1]$. In each of these periods, immediate purchase is strictly dominated by a SPNE that satisfies $\rho_t = 0$ for $t \in [t' - \alpha^*, t' - 1]$ and is identical to R^* thereafter. So, if the agent has a weak incentive to purchase in any period, there will be no purchases for the previous α^* periods. Therefore, no pattern of offers can do better than inducing a purchase every α^* periods.

If the good is offered in periods $\{1, 1 + \alpha^*, 1 + 2\alpha^*, ...\}$ and not offered in any other period, all agents are automatically locked into a unique SPNE in which they purchase at the first opportunity. ■

Proof of Proposition 4. Since α^* satisfies Equation 5 with equality,

$$u(-c) + \beta\delta u(b) = \frac{\rho^* \beta\delta [u(-c) + \delta u(b)]}{[1 - (1 - \rho)\delta]} = \beta\delta^{\alpha^*} [u(-c) + \delta u(b)]$$

$$\Rightarrow \quad \rho^* = \frac{\delta^{\alpha^* - 1} - \delta^{\alpha^*}}{1 - \delta^{\alpha^*}} \tag{13}$$

Since $\delta < 1$, ρ^* is increasing in δ. Applying L'Hopital's rule, we get $\lim_{\delta=1} \rho^* = \frac{1}{\alpha^*}$. Therefore, $\rho^* < \frac{1}{\alpha^*}$ for $\delta < 1$. Simplifying Equations 8 and 9 gives $Q_{\rho^*} = \frac{u(-c)+\beta\delta u(b)}{\beta\delta(u(-c)+\delta b)}$ and $Q_{\alpha^*} = \left(\frac{1}{\alpha^*}\right) \frac{(1-\beta)c}{(1-\delta)\beta(u(-c)+\delta b)}$. Now,

$$\rho^* < \frac{1}{\alpha^*}$$

$$\Rightarrow \quad (\rho^*) \frac{(1 - \beta) c}{(1 - \delta) \beta (u(-c) + \delta b)} < Q_{\alpha^*}$$

$$\Rightarrow \quad Q_{\rho^*} < Q_{\alpha^*}$$

■

Proof of Proposition 5. First, it is easy to see that a firm will have zero profits with constant prices. If the firm sets a price p in each period, it must be the case that $p \geq c$ and $u(-p) + \beta\delta u(b) \geq 0$. By Assumption 2, it will continue to be true that $U_t(t, p) > U_t(t + 1, p)$. So, for any fixed price strategy, the optimal SPNE for the buyer will involve purchasing the good in the next period. Therefore, the good will never be bought. Regardless of the price the firm sets, it will always have zero profits.

Regardless of δ_f, there is at least one pricing strategy with sales that yields positive profits and is credible. Consider the following pricing rule for any integer $\alpha \geq 2$: the good is offered at price $p(\alpha)$ every α periods and at $\bar{p}$ in all other periods. Let $\bar{p} > \beta\delta b$, so an agent would never purchase at that price. By assumption 3, $p(\alpha) > c$. By construction of $p(\alpha)$, the agent will always buy the good when it is offered at the low price.

These prices are credible. The firm has no incentive to lower prices in high price periods since agents will not buy anyway. It has no incentives to lower prices in sale periods since all agents are anyway buying at $p(\alpha)$. Finally, it has no incentive to raise prices in sale periods since this will break Equation 10 and all consumers will choose to wait until the next sale period.

■

Proof of Proposition 6. (a) Consider a pricing strategy $p_t^i = c$ for every firm for every period t. This is a competitive equilibrium, since no firm can raise profits by changing its price in any period. As proved before, semi-sophisticates will never purchase the good under such a pricing strategy.

(b) Consider a pricing strategy $p_t^i = c$ for every firm in $t \in (1, 1+\alpha, 1+2\alpha, ...)$ and p_t^i satisfies $u\left(-p_t^i\right) + \beta\delta u(b) < 0$ in all other periods. This is a competitive equilibrium. No firm can make a sale by lowering prices in a high price period. No firm can make positive profits by changing prices in a low price period. ■

Proof of Proposition 7. Consider any renegotiation-proof equilibrium strategy. Call it $\bar{R}$. Define $\bar{V}$ as the discounted utility of starting a renegotiation-proof equilibrium today, with prices constant at c. Define $\bar{W}$ as the discounted utility of starting a renegotiation-proof equilibrium tomorrow, with prices constant at c. Suppose the individual has been renegotiating. Then, by Bayes' Rule, $p_t = \frac{p_{t-1}}{p_{t-1}+(1-p_1)s}$ has been rising. p_t limits to 1 as t approaches ∞.

Find the lowest t^* such that the individual with probabilities p_{t^*} weakly prefers to switch to the renegotiation-proof equilibrium. At this t^*, the following must be satisfied:

$$\bar{V} \geq (1-p_{t^*})(1-s)\,U_t(t+1,c) + [1-(1-p_{t^*})(1-s)]\,\bar{W} \tag{14}$$

This can be satisfied if p_{t^*} is high enough and $\bar{W} < \bar{V}$. The second part will be true if the probability of purchase in the first period in which a renegotiation-proof equilibrium is played is strictly less than ρ^* (if $\rho_1 > \rho^*$, then by construction of ρ^*, you strictly prefer to wait than start the equilibrium today). So we can restrict $\bar{R}$ to be of the form $(\rho_1, \rho^*, \rho^*, \rho^*, ...)$ with $\rho_1 < \rho^*$. Consider any $t^* > t$. We know that no more updating of beliefs has happened, so she continues to prefer to play the renegotiation-proof equilibrium.

Consider t^*-1. By Bayes' Rule, we know that $p_{t^*-1} < p_{t^*}$. By construction of t^*, we know that at $t^* - 1$, the individual would rather reselect to her optimal equilibrium and see what happens:

$$\bar{V} < (1 - p_{t^*-1})(1 - s) U_t(t+1, c) + [1 - (1 - p_{t^*-1})(1 - s)] \bar{W} \equiv X_{t^*-1} \tag{15}$$

Consider $t^* - 2$. Again, $p_{t^*-2} < p_{t^*-1}$. We need to check if she will also prefer to renegotiate to her optimal equilibrium rather than switch to a renegotiation-proof equilibrium. If she were to switch to a renegotiation-proof equilibrium, she would get $\bar{V}$, as above. If she were to choose her optimal equilibrium, she would expect to get:

$$X_{t^*-2} \equiv (1 - p_{t^*-2})(1 - s) U_t(t+1, c) + [1 - (1 - p_{t^*-2})(1 - s)] \delta X_{t^*-1} \tag{16}$$

If $\delta X_{t^*-1} \geq \bar{W}$, then in $t^* - 2$ she also prefers to select her optimal equilibrium. Note that $\bar{V} = \beta\delta \frac{\rho^*(u(-c)+\delta u(b))}{1-(1-\rho^*)\delta}$ and $\bar{W} = \beta\delta\rho_1(u(-c) + \delta u(b)) + \beta\delta^2(1 - \rho_1)\frac{\rho^*(u(-c)+\delta u(b))}{1-(1-\rho^*)\delta}$. From Condition 15, we know that $X_{t^*-1} > \bar{V}$. Then, $\delta X_{t^*-1} > \delta\bar{V}$. As ρ_1 drops to 0, $\bar{W}$ drops and approaches $\delta\bar{V}$. Therefore, if ρ_1 is small enough, $t^* - 2$ strictly prefers to not switch to a renegotiation-proof equilibrium. Applying the argument recursively, if ρ_1 is small enough, this will be true for all $t \in \{1, 2, ..., t^* - 1\}$. ■

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