Commitment Savings in Informal Banking Markets

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Abstract

I study the provision of commitment savings by informal banks to sophisticated hyperbolic discounters. Since a consumer is subject to temptation in the period that he signs a contract, banks might exploit his desire for instant gratification even as they help him to commit for the future. Without banking, savings decisions and welfare are not monotonic in the degree of time-inconsistency. Consequently, commitment savings will lower welfare for moderately time-inconsistent agents. If loan contracts are enforceable, pure commitment savings will disappear. This will further lower welfare if the lender is a profit-maximizing bank, but raise welfare if the lender is a welfare-maximizing NGO. Finally, I consider the coexistence of a bank and NGO. There will be zero takeup of NGO-provided commitment savings if there is competition from a moneylender. But the NGO’s offer will raise the agent’s reservation utility, thus reducing the surplus that can be extracted by the moneylender.

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1 Introduction

This paper addresses some questions related to informal banking under time-inconsistency. First, if an individual values commitment savings, under what conditions will such a product be offered by a bank? Second, when does voluntary adoption of commitment raise the individual’s welfare? And third, what are the implications for equilibrium contracts if a welfare-minded NGO enters a region served by a profit-minded monopolist?

Hyperbolic discounters, who in any period place an emphasis on instant gratification, can make inefficient financial decisions. Suppose an individual would like to save up for a nondivisible good or investment. His savings decision today depends on his future selves’ willingness to continue saving. If he fears that his future selves will not follow through, he might abandon saving altogether. In this context, it is well understood how access to commitment devices, or contracts that restrict future choice sets, can improve welfare. In particular, consider commitment savings, which I define as a contract that makes savings balances illiquid until a specified date. Illiquidity, by raising future selves’ incentives to save, gives the current self a reason to save as well.

The fact that markets will respond to a demand for commitment does not itself inform us about equilibrium contracts and individual welfare. I show that, depending on time preferences and the contracting environment, traditional commitment savings might not be offered or adopted, and that if adopted, it can lower welfare relative to autarky. In this paper, I follow O’Donoghue and Rabin (1999b) and subsequent papers in assuming that an agent’s welfare is what his lifetime utility would be if he were time-consistent (equivalently, it is his discounted utility from the perspective of a hypothetical "period 0", just before he actually starts making decisions).

The model isolates some key mechanisms through which predictions about contracts and welfare are made. Consider a sophisticated quasi-hyperbolic discounter who, in any period, discounts the sum of future utilities by a factor $\beta < 1$. His preferences are time-inconsistent since, in any period, he places greater value on immediate consumption than his past selves would like him to. Much of the intuition in this paper comes from the analysis of the strategic interaction across different incarnations of the same agent. In particular, the period 1 self makes decisions that must take into account the optimal response of the period 2 self. The fact that banking decisions are made by period 1, who is himself subject to temptation even as he tries to curb the temptation of his future selves, allows us to see how markets might fail to maximize welfare.

As a starting point, I show than, in the absence of banking, the agent’s savings patterns

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1This can be interpreted as, say, the preferences parents have over their children’s lives. This approach is also reasonable if we are interested in thinking about how people might vote on future changes in policy such as new banking structures and new forms of contract enforcement. Given that an agent’s intertemporal preferences vary over his lifetime, this welfare criterion does not legitimize myopia in a particular period while rejecting the same preferences later in life.
and welfare are not monotonic in the degree of time-inconsistency. Suppose he is saving for a nondivisible good in periods 1 and 2, to be consumed in period 3. If he became more time-inconsistent (i.e. $\beta$ dropped), the changes in his behavior would be driven by two considerations. First, in period 1 he would wish to transfer more of the savings burden to period 2. Second, in period 2, he would face a greater temptation to simply consume his accumulated assets rather than continue saving. At high values of $\beta$, the second consideration would not be binding and a drop in $\beta$ would result in slower, or more imbalanced, saving. At lower values of $\beta$, the second consideration enters into play. Even though the period 1 self would like to save less, he will find himself saving more than before in order to induce period 2 to continue saving. Therefore, as $\beta$ drops, the agent’s period 1 savings will fall, then rise, and ultimately, when period 2 becomes sufficiently uncooperative, drop to 0.

The characterization of autarky equilibrium establishes that a commitment contract is sometimes valuable. If period 2 does not have access to period 1’s deposits, he has an improved incentive to save. At the point of adopting a contract, the hyperbolic discounter has two objectives. He wants to improve the behavior of his future selves, but to also limit the sacrifices required of his current self. A profit-maximizing bank will seek to capitalize on both objectives.

In the paper, Section 6 introduces a monopolist bank. There are two possible cases: (a) only commitment savings contracts can be offered (if the bank cannot adequately enforce repayment on loans), or (b) both commitment savings and loan contracts are feasible. Under a commitment savings contract, welfare will rise if it enables the agent to save when he otherwise could not. However, if the agent adopts commitment saving when he was already saving in autarky, his welfare will fall. To see why this is the case, consider the autarky outcome when the agent is slightly hyperbolic. In period 1, he is saving less than the welfare maximizing amount (but not as little as he would like). Now, access to commitment allows him to save even less by giving period 2 a greater incentive to make up the balance. This serves to make savings patterns more imbalanced than in autarky.

If the bank is able to enforce lending contracts, commitment savings will no longer be offered. The bank will offer a loan instead. This will cause the agent’s welfare to drop relative to both autarky and commitment savings. Borrowing is not inherently bad since it allows the nondivisible to be purchased while creating commitment through a fixed repayment schedule. However, pulling nondivisible consumption into the present creates such a large surplus for the hyperbolic discounter that the bank can extract high future repayment in exchange for the instant gratification.

In Section 7, I carry out the same exercise for a welfare-maximizing NGO. The NGO, unlike the bank, will not charge fees for commitment savings. However, it will deny
access to those agents whose welfare would be hurt by commitment. If repayment is enforceable, it too will offer loans instead of commitment savings, but at better terms and with different loan sizes than the bank. In sharp contrast to the bank, the NGO achieves the first-best welfare through lending, since it can ensure that the nondivisible is purchased while preventing over-borrowing, enforce commitment through the repayment schedule, and return surplus to the agent.

Section 8 examines equilibrium contracts when an NGO and bank coexist. This is of interest to both practitioners and experimental researchers investigating nonprofit entry in areas dominated by a monopolist. When both entities offer the same product, the NGO must expand its offers to serve those who would otherwise turn to the bank. While this erodes some of the welfare gains that an NGO could achieve if it operated alone, it eliminates the monopoly rents that a bank could earn. It is also reasonable to consider the coexistence of a bank that can lend and an NGO that cannot, since NGOs often lack the information and enforcement power that local moneylenders possess. In this case, the NGO’s commitment savings product will not be adopted by any agent. This is because a moneylender can always design a loan contract that is preferable from period 1’s perspective. However, the NGO’s offer improves the individual’s outside option, which reduces the amount of surplus the bank can extract from him. Zero take-up of commitment savings, therefore, does not imply that it was ineffective.

Finally, Section 9 discusses the results in the context of empirical research in development economics. While a number of the results have relevance beyond informal banking, the motivating setting for this paper is a low-income region where people lack access to the more complicated financial instruments and contract enforcement technologies of industrialized nations. Several recent empirical papers have examined the provision and takeup of commitment savings in developing countries. This paper aims to provide a theoretical complement by generating predictions about the relationship between time preferences, adoption of banking services, and welfare. The results have implications for the design of commitment savings contracts and allow us to put some structure on empirical hypotheses. This is pertinent in light of concerns about market provision of commitment and ambiguous welfare effects of microfinance.

For a broader discussion, see Conning and Udry (2007).

2 Related Literature

Starting with Phelps and Pollack (1968) and subsequently popularized by Laibson (1997), several papers have studied the theoretical properties of hyperbolic discounting. Harris and Laibson (2001), Krusell and Smith (2003), and Bernheim, Ray and Yeltekin (2013) all develop techniques for solving consumption-savings problems. Two papers in particular share some of the intuition of Section 4, which analyzes how period 2 incentives affect period 1 behavior: In the context of addictive goods, O'Donoghue and Rabin (1999a) argue that sophisticated hyperbolic discounters are driven by two forces—a "pessimism effect" (If I am more likely to indulge later, I might as well indulge now) and an "incentive effect" (if I indulge now, I am more likely to indulge later, so I should restrain now). Diamond and Koszegi (2003) study how the option of early retirement affects savings decisions for hyperbolic discounters.

There is now a significant body of empirical work that points to consumer demand for commitment. Ashraf, Karlan, and Yin (2006) find, through a field experiment, that agents most interested in commitment savings display relatively greater time inconsistency and are aware of their preferences. Additional evidence on the advantages of commitment savings is provided by Benartzi and Thaler (2004), Brune, Gine, Goldberg and Yang (2013), and Dupas and Robinson (2013). There is also growing evidence that commitment embedded in other forms of informal banking plays a significant role. For example, roscas (rotational savings and credit associations) can serve as effective commitment devices.

Of more direct relevance to this paper is the idea that microfinance too can be viewed as a form of commitment. This is discussed in Banerjee and Duflo (2011) and Bauer, Chytilova and Morduch (2012). Basu (2008) argues that simultaneous saving and borrowing in microfinance can be rationalized as a form of commitment. Fischer and Ghatak (2010) show that a particular feature of microfinance contracts—frequent repayment—allows hyperbolic discounters to access larger incentive compatible loans than under infrequent repayment.

Finally, a number of papers study contracts between firms and time-inconsistent agents. Amador, Werning and Angeletos (2006) and Bond and Sigurdsson (2009) look at the tradeoffs between commitment and flexibility under uncertainty. DellaVigna and Malmendier (2004) explore the implications of differing levels of naivete for firm pricing of leisure goods and investment goods. Eliaz & Speigler (2006) show how screening contracts can be designed when the agent's degree of naivete is unobservable to the seller. And Heidheus and Koszegi (2010) model equilibrium credit contracts when competitive

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4 Hyperbolic discounting is one of a few different ways to model problems of temptation and self-control. Other approaches include Gul and Pesendorfer (2001), Fudenberg (2006), and Banerjee and Mullainathan (2010).
5 However, in the papers described, the "period 2" decision is discrete, unlike in this model.
lenders face consumers of varying naivete. (Unlike in their model, the agent described below is fully sophisticated, but subject to the desire for instant gratification at the time he takes a loan). In the context of this literature, this paper can be viewed as focusing specifically on commitment savings contracts, through both savings and credit, offered by profit-maximizing and welfare-maximizing banks to consumers who vary in their levels of time-inconsistency.

3 The Model: Assumptions and Definitions

An agent lives for 3 periods, $i \in \{1, 2, 3\}$. He has a non-stochastic income in each period, which is normalized to 1. His per-period utility function, $u(c)$, is strictly concave and twice differentiable, with $u'(0) = \infty$. The agent can consume a numeraire good or a nondivisible good. The nondivisible good has a price of $p$ and yields an instantaneous benefit of $b$, where $3 > p \geq 2$ and $b > p$.

The agent is a sophisticated quasi-hyperbolic discounter—he is aware that his future selves continue to be time-inconsistent. In any period $i$, his discounted utility is given by:

$$U_i(\beta; c_1, c_2, c_3) \equiv u(c_i) + \beta \sum_{j=i+1}^{3} u(c_j)$$

where $0 < \beta \leq 1$. I implicitly assume there is no other discounting. The agent’s welfare is evaluated from a hypothetical period 0:

$$U_0(\beta; c_1, c_2, c_3) \equiv \sum_{j=i+1}^{3} u(c_j)$$

In the absence of banking, the agent can allocate current wealth to consumption and savings. Let $s_1$ denote savings in period 1 and $s_2$ denote cumulative savings in period 2 ($s_2 = 1 + s_1 - c_2$). In this setting, the nondivisible can only be purchased in period 3, and if $s_2 \geq p - 1$.

Notice that, if a time-consistent ($\beta = 1$) agent were to save for the nondivisible, the savings burden would be spread evenly across periods 1 and 2, so that $s_1 = \frac{p-1}{2}$ and $s_2 = p - 1$. The following assumption ensures that the time-consistent agent prefers to save than to simply consume the numeraire good in each period:

$$U_1\left(1; 1 - \frac{p-1}{2}, 1 - \frac{p-1}{2}, b\right) > U_1(1; 1, 1, 1)$$  (1)

Depending on the nature of the banking market, the agent might have access to com-

\footnote{The results of the model would be qualitatively similar if the nondivisible good were also durable.}
mitment savings or loans. These banking services can be provided by a profit-maximizing bank/moneylender or a welfare-maximizing NGO, or both. I assume that the service providers have access to funds at an external interest rate of 0. This assumption allows us to isolate commitment motives for saving independent of interest-based considerations.

4 Autarky Equilibrium

I first characterize the agent’s equilibrium outcome in the absence of banking. This allows us to establish benchmarks against which banking outcomes can be assessed. Assumption 1 ensures that, in period 0, the agent would like his future selves to save for the nondivisible. How saving actually occurs depends on period 2’s willingness to add to period 1’s savings, and on period 1’s actions based on period 2’s anticipated response. The outcome will be determined by the Subgame Perfect Nash Equilibrium of a consumption-savings game played by the agent’s consecutive selves.

The results below are intuitively explained in Section 4.3.

4.1 Period 1’s Optimal Outcome

The parameter that distinguishes the exponential discounter from the hyperbolic discounter is \( \beta \): the exponential discounter has \( \beta = 1 \), while the hyperbolic discounter has \( \beta < 1 \). Consider the agent with \( \beta \) slightly below 1. In period 1, he wishes to save for the nondivisible, but with a relatively greater savings burden on period 2. Once \( \beta \) is small enough, period 1’s optimal outcome will involve no saving at all (in this region, he is reluctant to sacrifice any immediate consumption for the nondivisible).

Suppose period 1 could fully control period 2’s savings decisions. Since the nondivisible provides the only motive to save, he would choose either \( s_2 = s_1 = 0 \) or, if he wished to purchase the nondivisible, \( s_2 = p - 1 \) and \( s_1 = s_{1yes}(\beta) \), with \( s_{1yes}(\beta) \) as defined below:

\[
s_{1yes}(\beta) \equiv \arg \max_{1 \geq s_1 \geq p-2} U_1(\beta; 1 - s_1, 2 + s_1 - p, b) \tag{2}
\]

This would involve setting \( u'(1 - s_{1yes}) = \beta u'(1 + s_{1yes} - (p - 1)) \). The more time-inconsistent the agent gets, the greater the savings burden he would like to transfer to period 2.

Period 1 prefers to save for the nondivisible only if \( \beta \geq \beta_{min} \), where \( \beta_{min} \) is the solution to:

\[
U_1(\beta_{min}; 1 - s_{1yes}(\beta_{min}), 2 + s_{1yes}(\beta_{min}) - p, b) = U_1(\beta_{min}; 1, 1, 1) \tag{3}
\]

When \( \beta \) drops below \( \beta_{min} \), the perceived benefits of a future nondivisible are so low that they cannot justify the immediate sacrifice.\(^8\)

\(^8\)Assume the following tie-breaking rule: when indifferent between saving and not saving, the agent
This gives us period 1’s optimal savings decision at any $\beta$, denoted $s_{1}^{\text{opt}}(\beta)$:

$$s_{1}^{\text{opt}}(\beta) \equiv \begin{cases} s_{1}^{\text{yes}}(\beta), & \text{if } \beta \geq \beta_{\text{min}} \\ 0, & \text{if } \beta < \beta_{\text{min}} \end{cases}$$ (4)

The corresponding values of $s_2$ are automatically determined. When $s_1 = s_{1}^{\text{yes}}(\beta)$, $s_2 = p - 1$ and the nondivisible is consumed in period 3. Otherwise, no saving takes place.

The above claims are summarized in Lemma 1. All proofs are in the appendix.

**Lemma 1** (a) $\beta_{\text{min}} \in (0, 1)$ exists and is uniquely determined. (b) $s_{1}^{\text{opt}}(\beta)$ is the optimal savings pattern. (c) For $\beta \geq \beta_{\text{min}}$, $s_{1}^{\text{opt}}(\beta)$ is continuous and strictly increasing, with $s_{1}^{\text{opt}}(\beta_{\text{min}}) > p - 2$ and $s_{1}^{\text{opt}}(1) = \frac{p-1}{2}$.

### 4.2 Equilibrium

Under hyperbolic discounting, period 2 might be unwilling to implement period 1’s optimal plan. To predict actual behavior, we can decompose the agent into three independent time-indexed agents and use backward induction to solve for equilibrium. In periods 1 and 2, the agent decides how much money, $s_i$, to send to the next period. This decision is a function of the current wealth, $s_{i-1} + 1$.

Since, by assumption, $b > p$, the period 3 decision is straightforward. He will consume the nondivisible if it is affordable. His consumption decision, $c_{3}^{\text{aut}}(s_2)$, is given by:

$$c_{3}^{\text{aut}}(s_2) = \begin{cases} b + (s_2 - (p - 1)), & \text{if } s_2 \geq p - 1 \\ 1 + s_2, & \text{otherwise} \end{cases}$$ (5)

Period 2 observes $s_1$ and decides how much to send to period 3. If he saves less than $p - 1$, period 3 cannot experience the consumption jump from the nondivisible. Period 2’s savings decision is given by:

$$s_{2}^{\text{aut}} \equiv \arg \max_{0 \leq s_2 \leq 1 + s_1} u(1 + s_1 - s_2) + \beta u\left(c_{3}^{\text{aut}}(s_2)\right)$$

This determines $c_{2}^{\text{aut}}(s_1)$.

Anticipating period 2’s decision, period 1 saves the following:

$$s_{1}^{\text{aut}} \equiv \arg \max_{0 \leq s_1 \leq 1} u(1 - s_1) + \beta u\left(1 + s_1 - s_{2}^{\text{aut}}(s_1)\right) + \beta u\left(c_{3}^{\text{aut}}(s_{2}^{\text{aut}}(s_1))\right)$$ (6)

This specifies the equilibrium outcome.

To explicitly describe equilibrium savings decisions at any $\beta$, we need to focus on the interplay of periods 1 and 2’s maximization problems. In particular, period 1 must assess
period 2’s willingness to "top up" any savings that he receives. For any \( s_1 \geq p - 2 \), define:

\[
\begin{align*}
    s_2^{yes} & \equiv \arg\max_{s_2 \geq p - 1} u (1 + s_1 - s_2) + \beta u (b + (1 + s_2 - p)) \\
    s_2^{no} & \equiv \arg\max_{s_2} u (1 + s_1 - s_2) + \beta u (1 + s_2)
\end{align*}
\]

\( s_2^{yes} \) is period 2’s optimal savings decision conditional on the nondivisible being purchased in period 3, and \( s_2^{no} \) is period 2’s optimal savings decision conditional on the nondivisible not being purchased. Now we can define \( s_1^{min} \) as the lowest value of \( s_1 \) for which period 2 is willing to save for the nondivisible:

\[
s_1^{min} \equiv \min \left\{ \begin{array}{l}
    s_1 \geq p - 2 : U_2 (\beta; 1 - s_1; 1 + s_1 - s_2^{yes}; b + 1 + s_2^{yes} - p) \\
    \geq U_2 (\beta; 1 - s_1; 1 + s_1 - s_2^{no}; 1 + s_2^{no})
\end{array} \right. \]

**Lemma 2**

(a) At any \( \beta \), \( s_1^{min} \) exists. (b) At any \( \beta \), \( s_2^{max} \geq p - 1 \) iff \( s_1 \geq s_1^{min} (\beta) \). (c) \( s_1^{min} (\beta) \) is continuous, and is strictly decreasing in \( \beta \) except when \( s_1^{min} (\beta) = p - 2 \). (d) \( s_1^{min} (1) \in [p - 2, \frac{p - 1}{2}] \) and \( \lim_{\beta \to 0^+} s_1^{min} (\beta) = \infty \).

This gives us, for any \( \beta \), the minimum that period 1 would have to save to ensure that the nondivisible is purchased. As \( \beta \) drops, period 2 gets more interested in instant gratification; hence, \( s_1 \) must rise to motivate him to save up for the nondivisible. However, we know that as \( \beta \) drops, period 1’s willingness to comply must also drop. We can define \( s_1^{max} \) as the maximum that period 1 is willing to save (under the assumption that, if feasible, period 2 will continue to save for the nondivisible):

\[
s_1^{max} (\beta) \equiv \max \{ s_1 \in [p - 2, 1] : U_1 (\beta; 1 - s_1, 2 + s_1 - p, b) \geq U_1 (\beta; 1, 1, 1) \}
\]

If such a term does not exist, let \( s_1^{max} (\beta) = 0 \).

**Lemma 3**

(a) Given a choice between saving \( s_1 > s_1^{max} \) (for the nondivisible) and not saving, the period 1 agent strictly prefers not saving. (b) For \( \beta < \beta^{min} \), \( s_1^{max} (\beta) = 0 \). (c) \( s_1^{max} (\beta^{min}) = s_1^{opt} (\beta^{min}) \) and for all \( \beta > \beta^{min} \), \( s_1^{max} (\beta) > s_1^{opt} (\beta) \). (d) For \( \beta \geq \beta^{min} \), \( s_1^{max} (\beta) \) is continuous and strictly increasing.

Now, we have a decreasing function, \( s_1^{min} \) (period 2’s minimum requirement from period 1) and an increasing function \( s_1^{max} \) (period 1’s maximum willingness). This gives us some \( \beta_{mid} \) below which the nondivisible cannot be bought in equilibrium:

\[
\beta_{mid} \equiv \min \{ \beta : s_1^{max} \geq s_1^{min} \}
\]

Finally, let us define \( \beta_{max} \) as the lowest value of \( \beta \) at which period 2 would agree to
save for the nondivisible even if period 1 saved only his optimal amount:

\[ \beta_{\text{max}} \equiv \min \{ \beta : s_1^{\text{opt}} \geq s_1^{\text{min}} \} \]  

(12)

We can now describe the equilibrium outcome at any level of \( \beta \).

**Proposition 1** The autarky equilibrium outcome is:

\[
\begin{align*}
    s_1^{\text{aut}} &= \begin{cases} 
    s_1^{\text{opt}}, & \text{if } \beta \in [\beta_{\text{max}}, 1] \\
    s_1^{\text{min}}, & \text{if } \beta \in [\beta_{\text{mid}}, \beta_{\text{max}}) \\
    0, & \text{if } \beta \in (0, \beta_{\text{mid}})
    \end{cases} \\
    s_2^{\text{aut}} &= \begin{cases} 
    p - 1, & \text{if } \beta \in [\beta_{\text{mid}}, 1] \\
    0, & \text{if } \beta \in (0, \beta_{\text{mid}})
    \end{cases}
\end{align*}
\]

Note that, as \( \beta \) drops, \( s_1^{\text{aut}} \) drops, then rises, and then drops again. The next section provides further intuition for the results above.

### 4.3 Graphical Analysis of Equilibrium

Let us first recap the functions defined so far. Refer to Figure 1 (it is easiest to start at \( \beta = 1 \) and move left). \( s_1^{\text{opt}} \) is period 1’s optimal level of saving. Clearly, when \( \beta = 1 \), \( s_1^{\text{opt}} = \frac{p-1}{2} \). This evenly balances the savings burden between periods 1 and 2, and is the welfare-maximizing outcome. As \( \beta \) drops, period 1 would ideally like to buy the nondivisible, but with less saving in the present and more in the next period. Ultimately, below \( \beta_{\text{min}} \), period 1’s optimal outcome involves not saving at all.

The function \( s_1^{\text{max}} \) indicates the maximum period 1 would be willing to save if he were assured that period 2 would make the additional contributions necessary for the nondivisible to be purchased. As we would expect, \( s_1^{\text{max}} \) lies above \( s_1^{\text{opt}} \) and drops as \( \beta \) drops. The function \( s_1^{\text{min}} \) indicates the minimum level of period 1 savings that would make period 2 willing to contribute towards the nondivisible. Again, this constraint becomes harder to satisfy as \( \beta \) drops.

Now we can see that saving will occur in autarky as long as \( s_1^{\text{max}} \geq s_1^{\text{min}} \), or \( \beta \geq \beta_{\text{mid}} \). In this region, there is always a way to purchase the nondivisible so that periods 1 and 2 are left better off than under no saving. However, \( s_1 \) will initially drop, and then rise again. The reasoning for this is the following: At high values of \( \beta \) (above \( \beta_{\text{max}} \)), period 2 is sufficiently forward looking to know that he is willing to save even if period 1 forces a disproportionate burden on him. Thus, period 1 is able to save according to \( s_1^{\text{opt}} \). For lower levels of \( \beta \), period 1 can no longer achieve his optimal. So he saves as little as possible (\( s_1^{\text{min}} \)) as long as this gives him greater utility than not saving at all.\(^9\)

\(^9\)To see when each of these regions will be non-trivial, note that one of the following must be
Figure 1: Autarky equilibrium: When $\beta \geq \beta_{\text{max}}$, period 1’s optimal is achievable. When $\beta \in [\beta_{\text{mid}}, \beta_{\text{max}})$, period 1 saves more than his optimal. When $\beta < \beta_{\text{mid}}$, there is no saving in equilibrium.

This setup creates a natural need for commitment. When period 1 is unable to achieve his optimal outcome, he might wish to change period 2’s incentives. In the next section, I describe commitment savings as a minimal restriction on period 2’s choices, and study how this affects equilibrium outcomes.

5 The Role of Commitment

A commitment savings product is defined as a savings account in which period 1’s deposits remain illiquid until period 3. While one can easily conceive of more effective commitment contracts, it is useful to understand how even minimal commitment can change equilibrium outcomes.\(^\text{10}\)

\(\beta_{\text{min}} = \beta_{\text{mid}} = \beta_{\text{max}}\) (intersection at the discontinuity) or $\beta_{\text{min}} < \beta_{\text{mid}} < \beta_{\text{max}}$. At $\beta_{\text{min}}$, $U_1(\beta_{\text{min}}; 1-s_1^{\text{yes}}(\beta_{\text{min}}), 2+s_1^{\text{yes}}(\beta_{\text{min}})-p, b) = U_1(\beta_{\text{min}}; 1, 1, 1)$. For the regions to be non-trivial, we need $U_2(\beta_{\text{min}}; 1-s_1^{\text{yes}}(\beta_{\text{min}}), 2+s_1^{\text{yes}}(\beta_{\text{min}})-p, b) < U_2(\beta_{\text{min}}; 1, 1, 1)$. Combining these two conditions, we get

$$\frac{u(1)-u(2+s_1^{\text{yes}}(\beta_{\text{min}}))}{u(1)-u(2)} < 1 - \beta_{\text{min}}.$$  

When the RHS is low, the LHS is high (and vice versa). So this condition is true if $\beta_{\text{min}}$ is low enough, which will be the case if the nondivisible good is sufficiently attractive.\(^\text{10}\) This conception of commitment is also natural when the service provider has limited ability to enforce more complex contracts.
To see how commitment might help, let us define a function $s_{minlock}^1(\beta)$, which is the minimum that period 1 needs to save to give period 2 the incentive to top up for the nondivisible (conditional on period 1 savings being inaccessible to period 2). Formally:

$$s_{minlock}^1(\beta) = \min \left\{ s_1 \geq p - 2 : U_2(\beta; 1 - s_1, 1 + s_1 - s_{yes}^2, b + 1 + s_{yes}^2 - p) \right\}$$

(13)

As defined in Equation 7, $s_{yes}^2$ is period 2’s optimal savings decision conditional on the nondivisible being purchased in period 3. The construction of $s_{minlock}^1$ differs from the construction of $s_{min}^1$ in one respect—not saving is now relatively less attractive to period 2, since he cannot consume period 1’s deposits. This makes it easier for period 1 to incentivize period 2 to save for the nondivisible.

**Lemma 4** (a) $s_{minlock}^1$ exists. (b) When period 1 savings are locked, period 2 will save for the nondivisible iff $s_1 \geq s_{minlock}^1(\beta)$. (c) $s_{minlock}^1(\beta)$ is continuous, and is weakly decreasing in $\beta$ unless $s_{min}^1(\beta) = p - 2$. (d) $s_{minlock}^1(1) \in [p - 2, \frac{p - 1}{2}]$, $\lim_{\beta \to 0^+} s_{minlock}^1(\beta) = p - 1$, and $s_{minlock}^1(\beta) < s_{min}^1(\beta)$ except when $s_{min}^1(\beta) = p - 2$.

Since $s_{minlock}^1$ is lower than $s_{min}^1$, it will intersect $s_{opt}^1$ and $s_{max}^1$ at lower values of $\beta$. 

Figure 2: Commitment lowers the minimum period 1 savings required by period 2, from $s_{min}^1$ to $s_{minlock}^1$. 

0  $\beta_{min}$  $\beta_{midlock}$  $\beta_{mid}$  $\beta_{maxlock}$  $\beta_{max}$  1

$p - 1$

$s_{min}$  $s_{opt}^1$  $s_{max}$  $s_{minlock}^1$
than $\beta_{\text{max}}$ and $\beta_{\text{mid}}$, respectively. As before, we can define:

\[
\beta_{\text{midlock}} \equiv \min \left\{ \beta : s_{1}^{\text{max}} \geq s_{1}^{\text{minlock}} \right\} \tag{14}
\]

\[
\beta_{\text{maxlock}} \equiv \min \left\{ \beta : s_{1}^{\text{opt}} \geq s_{1}^{\text{minlock}} \right\} \tag{15}
\]

As shown in Figure 2, commitment savings expands the set of $\beta$-values at which the agent is able to save for the nondivisible. For $\beta \in [\beta_{\text{midlock}}, \beta_{\text{mid}})$, there would have been no saving in the absence of commitment. For $\beta \in [\beta_{\text{mid}}, \beta_{\text{max}})$, commitment savings allows period 1 to save less than before and still purchase the nondivisible.

We can immediately see how, for $\beta$ just below $\beta_{\text{max}}$, commitment lowers welfare. In this region, in autarky, period 1 saves more than he would like. But this is good from a welfare perspective, since period 2 is forcing him to save an amount closer to $\frac{p_{0}}{2}$. Commitment allows him to once again save $s_{1}^{\text{opt}}$, skewing the savings burden more heavily towards period 2 and thus lowering welfare.

6 Profit-Maximizing Bank

Consider a setting where the only financial services are provided by a monopolist bank whose objective is to maximize the sum of profits over three periods. The bank is aware of the agent’s preferences and may offer a contract in period 1. The agent will accept such a contract if it leaves him no worse off than in autarky.

The following two cases are analyzed separately: (1) lending contracts cannot be enforced; (2) lending contracts are feasible (i.e. repayment can be enforced). In case 1, the bank will offer commitment savings for a range of $\beta$-values. This has some interesting welfare implications: for low $\beta$, commitment savings will raise welfare, but for high $\beta$, welfare will drop. This shows how constraining the period 2 agent could end up hurting the period 0 agent.

In case 2, the bank will no longer offer commitment savings and will instead give period 1 a loan. Welfare will be lower than in autarky, even if the loan enables the agent to purchase the nondivisible when earlier he could not. This is because period 1 is willing to incur large future repayments for greater immediate consumption.

6.1 Commitment Savings

Consider the case where the bank does not have the capacity to enforce loan repayments. Then, the only possible service is commitment savings. Access to illiquidity can change period 2 incentives, thus raising surplus for the period 1 agent. This surplus can then be extracted by the bank through up-front fees. I first describe the bank’s optimal contract,
and then look at welfare implications.

The contract takes the following form: the monopolist sets a fee \( f^{\text{mon}} \) for commitment savings. Period 1 observes \( f^{\text{mon}} \) and chooses how much to save \( s^{\text{mon}}_1 \). Technically, this is equivalent to monopolist choosing \( s^{\text{mon}}_1 \) in exchange for providing the nondivisible in period 3. The higher period 1’s discounted utility from commitment savings (relative to autarky), the greater the fee the monopolist can charge. Since the monopolist cannot enforce loan contracts, the fee is charged in period 1. Through savings, the agent can decide how much of this fee burden to transfer to period 2).

If the bank does find it profitable to offer such a contract, it will be the solution to the following problem:

\[
\begin{align*}
\max_{f,s_1} & \quad f \\
\text{s.t.} & \quad U_1 (\beta; 1 - f - s_1, 2 + s_1 - p, b) \geq U_1 (\beta; c_1^{\text{aut}}, c_2^{\text{aut}}, c_3^{\text{aut}}) \quad (16) \\
& \quad U_2 (\beta; 1 - f - s_1, 2 + s_1 - p, b) \geq U_2 (\beta; 1 - f - s_1, 1, 1 + s_1) \quad (17)
\end{align*}
\]

Constraint 16 is a participation constraint—period 1’s discounted utility must be at least as high as in autarky. Constraint 17 (equivalently, \( s^{\text{mon}}_1 \geq s^{\text{minlock}}_1 \)) can be interpreted as an incentive compatibility constraint—period 2 must actually be willing to save the remaining amount towards the nondivisible (if the agent in period 2 is not willing to save \( p-1 \), he is best off saving nothing at all). The reason such a contract can improve outcomes relative to autarky is that, by making the period 1 savings illiquid, the temptation for period 2 to consume rather than save is dampened (period 2’s decision is determined by \( s^{\text{minlock}}_1 \) rather than \( s^{\text{min}}_1 \)). Note that the agent is never being forced to make a payment. The incentive to do so comes directly from the modified tradeoffs that emerge under illiquidity.

The monopolist will offer commitment savings only if the agent adopts it at some \( f^{\text{mon}} \geq 0 \).\(^{11}\) The actual contract depends on which constraints bind. Constraint 16 will always bind—if it didn’t, the monopolist could raise profits by raising \( f^{\text{mon}} \). When Constraint 17 doesn’t bind, the bank maximizes profits by setting \( u' (1 - f^{\text{mon}} - s^{\text{mon}}_1) = \beta u' (2 + s^{\text{mon}}_1 - p) \). By allowing period 1 to equalize marginal utility today with the discounted marginal utility tomorrow, period 1’s surplus is maximized (and by extension, this maximizes the amount the bank can charge for the service).

As \( \beta \) drops, Constraint 16 leads to consumption more skewed in period 1’s favor. But simultaneously, period 2 is more willing to not save anything at all. When Constraint 17 binds, it is no longer possible to equalize discounted marginal utilities, so \( u' (1 - f^{\text{mon}} - s^{\text{mon}}_1) > \beta u' (2 + s^{\text{mon}}_1 - p) \). This reduces the bank’s ability to extract sur-

\(^{11}\)If indifferent between commitment savings and autarky, assume the agent will adopt commitment savings as long as it changes his equilibrium actions.
Proposition 2 A monopolist bank will offer a commitment savings contract with $f_{\text{mon}} \geq 0$ only for $\beta \in [\beta_{\text{midlock}}, \beta_{\text{max}}]$. In any contract, Constraint 16 will bind.

Regardless of whether Constraint 17 binds, period 1’s discounted utility will be the same as in autarky. Welfare, however, might be lower. Consider the case where there was no saving in autarky. Here, any commitment savings contract involves periods 1 and 2 voluntarily reducing their consumption, while raising period 3 consumption. Since the welfare function weighs future periods relatively more than period 1 does, this reallocation of consumption into the future would raise welfare. However, if the agent were already saving in autarky, commitment savings would hurt welfare. Here, the bank is able to help period 1 by making period 2 save more. Since period 1 has a high willingness to transfer the savings burden to period 2, welfare goes down.\footnote{In part of this region, period 2 was already saving too much in autarky, so welfare would drop even if commitment savings were free.}

Proposition 3 For $\beta \in (\beta_{\text{midlock}}, \beta_{\text{mid}})$, monopolist commitment savings strictly raises welfare relative to autarky. For $\beta \in (\beta_{\text{mid}}, \beta_{\text{max}})$, monopolist commitment savings strictly lowers welfare relative to autarky.

We see in the next section that, even if commitment savings lowers welfare relative to autarky, lending further lowers it. Under commitment savings, the agent making the banking decision is giving up current consumption to fund future consumption. Since he has present-biased preferences, he is reluctant to make this trade-off, which constrains the amount of surplus than can be extracted by the bank.

6.2 Loans

Under lending, the agent is offered a contract denoted by the vector $L_{\text{mon}} = (l_{\text{mon}}, t_{\text{mon}}^2, t_{\text{mon}}^3)$. This constitutes a loan $l_{\text{mon}} \geq 0$ in period 1, with repayment $t_{\text{mon}}^2 \geq 0$ and $t_{\text{mon}}^3 \geq 0$ in periods 2 and 3, respectively. I assume that repayment is fully enforceable and that the bank can credibly commit to not renegotiate terms.

We can easily see that the bank will always do better with a loan than with a savings contract. Consider the profit-maximizing savings contract and re-frame it as a loan. With a loan of $p - 1$ and subsequent repayments of $f_{\text{mon}} + s_{\text{mon}}^1$ and $p - 1 - s_{\text{mon}}^1$, period 1’s discounted utility would be strictly higher. Therefore, the bank could extract additional fees in periods 2 and 3.
Given a loan contract $L$, let the agent’s consumption choice in period $i$ be denoted $c_i(L)$. This is determined by backward induction as before. The monopolist’s maximization problem is:  

$$
\begin{align*}
\max_{L} & \quad -l + t_2 + t_3 \\
\text{s.t.} & \quad U_1(\beta; c_1(L), c_2(L), c_3(L)) \geq U_1(\beta; c_1^{\text{aut}}, c_2^{\text{aut}}, c_3^{\text{aut}})
\end{align*}
$$

(18)

The next proposition describes the profit-maximizing contract. In any contract, it must be true that $t_2 = t_3 = t$. Since the agent would like to equalize marginal utilities across all future periods, a balanced repayment plan will maximize the amount the bank can charge for the loan. Furthermore, the agent will no longer save. He will either consume the nondivisible in period 1 or not at all.

**Proposition 4**

(a) When lending contracts can be enforced, any agent with $\beta < 1$ will receive a loan from a monopolist bank. (b) An agent who receives a loan will not save. (c) In any contract, $t_{2}^{\text{mon}} = t_{3}^{\text{mon}} = t^{\text{mon}}$. (d) Under the loan contract $L^{\text{mon}}$, either $u'(c_1(L^{\text{mon}})) = \beta u'(c_2(L^{\text{mon}}))$ or $l^{\text{mon}} = p - 1$.

Figures 3 and 4 illustrate two possible equilibrium contracts. In each case, the bank’s objective is to locate itself on the highest isoprofit line subject to the participation constraint. Figure 3 depicts an agent with $\beta$ close to 1. In this case, the period managed to implement his optimal savings path in autarky. Up to $l = p - 1$, the participation constraint is linear, which means he is willing to accept any loan as long as total repayment adds up to the same amount—he will simply pass down the borrowed amount to his future selves and replicate the autarky outcome. The participation constraint faces a jump at $l = p - 1$ since he prefers to consume the nondivisible immediately than in period 3. Subsequently, the constraint will be concave and relatively flat—any loan beyond $p - 1$ is relatively less attractive as he is unable to ensure balanced consumption in periods 2 and 3. In this case, the profit-maximizing loan is $p - 1$.

Figure 4 depicts an agent with $\beta$ close to 0. This agent was not saving in autarky. Since he is highly time-inconsistent, he is willing to accept large repayments for small loans. His participation constraint still has a discontinuity at $l = p - 1$, but the bank’s profit-maximizing contract involves a smaller loan. When the bank can extract a large portion of future income as repayment for a small loan, it cannot raise profits by extending a larger loan that would enable a purchase of the nondivisible.

The agent in period 1 is willing to pay more (in terms of future consumption) for a loan than he would if he were time-consistent. Since the monopolist’s loan contract satisfies

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13 Assume, if there is more than one profit-maximizing contract, the firm chooses the one with the lowest loan size.
Figure 3: The monopolist lender’s profit maximization problem when the agent’s $\beta$ is close to 1.

Figure 4: The monopolist lender’s profit maximization problem when the agent’s $\beta$ is close to 0.
Proposition 5  When a monopolist bank offers loan contracts, welfare for any agent with \( \beta < 1 \) is strictly lower than in autarky and strictly lower than under monopolist commitment savings.

7  Welfare-Maximizing NGO

For the purposes of this model, an NGO is defined as a bank that seeks to maximize the welfare of the agent, subject to a break-even constraint. I again consider separately the cases where it cannot enforce loan contracts and where it can. Unlike the monopolist bank, the NGO will restrict the agent’s access to services to ensure that welfare does not drop below the autarky level. When lending is possible, it will choose to lend because lending functions as a more effective commitment device than commitment savings (marginal utilities are equalized across those periods in which the nondivisible is not consumed).

7.1  Commitment Savings

The NGO must decide which values of \( \beta \) to offer commitment savings to, and at what fee \( (f_{ngo} \geq 0) \). An agent who accepts such an offer must choose \( s_{1}^{ngo} \) to maximize \( U_{1} \) subject to an incentive compatibility constraint similar to Condition 17.

First, note that if commitment savings is offered, \( f_{ngo} = 0 \). Any higher fee reduces consumption strictly in period 1 and weakly in period 2. Second, commitment savings will not be offered for those values of \( \beta \) that lie just below \( \beta_{max} \), since commitment reduces welfare in that region. For lower values of \( \beta \), commitment will be offered either when there is no saving in autarky, or when \( s_{1}^{min} \) is sufficiently high that the NGO can generate a more equitable savings path by raising period 2’s incentive to save. This raises both welfare and period 1’s discounted utility.

Proposition 6  The NGO will always choose \( f_{ngo} = 0 \). It will offer commitment savings for a strict subset of \( [\beta_{midlock}, \beta_{max}] \).

---

\footnote{14}{Here, the nondivisible good might function as a "temptation" good that further reduces welfare. The following example illustrates this. Suppose the agent had no option of purchasing the nondivisible good. Then, he would receive some loan contract \((l, \bar{l}, \bar{f})\). Now, suppose the possibility of purchasing a nondivisible good is introduced, and continue to assume he does not do so in autarky. It is possible that the bank will now change the contract to allow the purchase of the nondivisible, to \((p - 1, \bar{l}, \bar{f})\). Since the indifference constraint is satisfied in both cases, \( U_{1} (\beta; 1 + \bar{l}, 1 - \bar{l}, 1 - \bar{f}) = U_{1} (\beta; b, 1 - \bar{l}, 1 - \bar{f}) \). Since period 1 is indifferent between the two outcomes, but \( b > 1 + \bar{l} \) and \( \beta < 1 \), welfare must be lower. Since the nondivisible raises instantaneous utility in period 1, it simply allows the bank to charge more for the loan.}

\footnote{15}{Denying commitment savings is equivalent to charging an impossibly high fee.}
The NGO offers commitment savings only when it raises welfare. Unlike the monopolist bank, the it denies service to some who want it.\textsuperscript{16}

7.2 Loans

An advantage of loan contracts relative to commitment savings is that they do not require the NGO to know the agent’s type. The NGO’s optimal lending contract is straightforward: to every agent, it will offer a loan of \( p - 1 \) with repayments of \( \frac{p-1}{2} \) in periods 2 and 3.\textsuperscript{17} By pulling forward nondivisible consumption, period 1 gets a higher discounted utility than in autarky. The agent attains the first-best welfare since the nondivisible is purchased and marginal utilities of consumption are equalized across periods 2 and 3.

8 Coexistence of Bank and NGO

This is a setting of particular interest in developing economies that experience an expansion of financial services. Suppose an NGO enters a region served by a monopolist bank. We would like to know how this affects equilibrium contracts, and whether the NGO can eliminate the welfare costs imposed by the bank. I consider three cases below: (a) both offer commitment savings, (b) the bank offers loans while the NGO offers commitment savings, and (c) both offer loans. In each of the cases, we can solve for the Nash equilibrium of a game played in period 1: the bank offers a contract that maximize profits given the NGO’s contract, the NGO offers a contract that maximizes agent welfare given the bank’s contract, and the agent chooses a contract (if any) that maximizes period 1’s discounted utility.

8.1 Bank and NGO Commitment Savings

For any \( \beta \), the bank and NGO must each choose whether to offer commitment savings, and at what fee. The monopolist’s objective is to maximize the fee while the NGO’s objective is to maximize \( U_0 \). The agent chooses between the two offers and autarky, with the goal of maximizing \( U_1 \).

Clearly, an agent who desires commitment will choose the contract with the lowest fee. For any \( \beta \) where the NGO initially offered commitment savings, the same contract will continue to be offered, and will be adopted. The NGO will also be forced to expand commitment savings to the remaining \( \beta \) values in \([\beta_{\text{midlock}}, \beta_{\text{max}}]\) that it did not initially

\textsuperscript{16}The NGO could expand commitment savings by setting a minimum deposit size (it would try to get \( s_1 \) as close to \( \frac{p-1}{2} \) as possible, subject to the constraints imposed by by autarky utility and \( s_{\text{max}} \)). Nevertheless, it would not offer commitment for \( \beta \) values immediately below \( \beta_{\text{max}} \).

\textsuperscript{17}It is necessary to limit the loan size as agents with low \( \beta \) will prefer loans larger than \( p - 1 \).
serve. In these cases, the resulting welfare will be lower than in autarky. However, by undercutting the bank’s fees, the NGO will ensure a welfare improvement relative to the monopoly case.

**Proposition 7** When bank and NGO commitment savings coexist, any equilibrium will have the following property: 

(a) The agent will adopt a contract with \( f = 0 \) for any \( \beta \in [\beta_{\text{midlock}}; \beta_{\text{max}}] \). 

(b) Within this range, for any \( \beta \) that received commitment savings under an NGO alone, welfare will be the same as under the NGO. For any \( \beta \) that did not receive commitment savings under an NGO alone, welfare will be strictly lower than under the NGO and strictly higher than under the bank.

The starker results emerge below, when the monopolist is able to lend. Despite the NGO’s willingness to return surplus to the agent, a monopolist moneylender can always lure hyperbolic discounters away from NGO commitment savings.

### 8.2 Monopolist Moneylender and NGO Commitment Savings

Given that commitment savings is attractive to the period 1 agent, and given the evidence that there is some demand for it, the low real-world availability and takeup of commitment savings continues to pose a puzzle. In this section, I show how the presence of a monopolist moneylender (or bank) can, for two reasons, drive agents away from commitment saving.

Consider an agent who has adopted NGO commitment savings. The only advantage (over autarky) that this product offers is illiquidity, which leads period 2 to behave differently than he otherwise would. But if period 2 has access to loans, he is willing to pay the moneylender to make his savings from the previous period liquid. The moneylender can, in effect, make all illiquid savings liquid, rendering commitment savings worthless. It might be possible for the NGO to overcome this problem, either by making final payments in terms of a good that cannot be resold, or by making it difficult for the moneylender to verify that the agent has illiquid assets.

However, even if the above problem is solved, the moneylender can always offer a contract that period 1 strictly prefers to commitment savings. As in Section 6.2, the NGO’s commitment savings contract can always be re-framed as a loan. This improves period 1’s discounted utility, thus enabling the moneylender to charge for this service.

Therefore, the NGO will never be able to attract agents in competition with a moneylender. However, it does have the power to alter the agent’s outside option, which changes the participation constraint the moneylender must satisfy. For those with \( \beta \in [\beta_{\text{midlock}}; \beta_{\text{max}}] \), an offer of commitment savings raises the minimum \( U_1 \) that the agent must be left with. Therefore, even though commitment savings will not be adopted, in equilibrium period 1’s discounted utility will be higher than if the moneylender were operating alone.
Proposition 8 (a) In equilibrium with a moneylender and NGO commitment savings, the agent will adopt a loan contract. For $\beta \notin (\beta_{\text{midlock}}, \beta_{\text{max}})$, the moneylender will offer the same contract as in Proposition 4 ($L^{\text{mon}}$). For $\beta \in (\beta_{\text{midlock}}, \beta_{\text{max}})$, the moneylender’s contract will be constrained by the NGO’s contract, which will have $f = 0$. (b) Relative to NGO commitment savings alone, welfare will be strictly lower. Relative to monopolist lending: if the agent originally received $l^{\text{mon}} \geq p - 1$, welfare will be strictly higher, and if the agent originally received $l^{\text{mon}} < p - 1$, welfare changes are ambiguous.

This demonstrates that zero take-up of commitment savings does not imply that it had no effect. When the NGO can alter the agent’s outside option, the moneylender is forced to modify its loan contract. Clearly, welfare cannot be as high as in a setting with only NGO commitment savings. Furthermore, even relative to monopolist lending, the welfare impacts are ambiguous.

First, there are cases where welfare must rise relative to monopolist lending. Consider Figure 5. Suppose, in the absence of an NGO, the moneylender’s contract is given by $L^a$ and lies at the point of tangency with the participation constraint. Then, it must be true that $u'(c_1(L^a)) = \beta u'(c_2(L^a))$. Now, imagine the NGO’s offer raises the agent’s reservation utility. Since the moneylender can no longer remain on the original isoprofit line, it must offer a new loan that satisfies the new participation constraint (indicated by the thick curve). One possibility (indeed, the only possibility if there were no nondivisible), would be to relocate at the point of tangency between an isoprofit line and the new participation constraint. The new contract, $L^b$, must offer a larger loan and smaller repayment than before (in order to keep the ratio between $c_1$ and $c_2$ the same, consumption in both periods must rise). Here, the entry of an NGO must lead to a rise in welfare. (Similarly, if the original monopoly loan size was at least $p - 1$, which would be true at high values of $\beta$, the entry of NGO commitment savings would again raise welfare.)

On the other hand, because of the discontinuity in the participation constraint, it is also possible that the improved outside option would result in a more dramatic move: from $L^a$ (a small loan) to $L^B$ (a larger loan with larger repayment). To see this more precisely, suppose at the original participation constraint the moneylender was offering the agent $L^a$, but was virtually indifferent between that and a larger loan of $p - 1$ at $L^A$. The proof of Proposition 9 shows that such a point must exist. It can then be shown that, as the participation constraint tightens, the moneylender strictly prefers to offer $p - 1$.

Here, as a result of the improved outside option, the agent gets a larger loan along with a larger repayment. If the NGO’s offer generates a sufficiently small improvement in the outside option, the agent will end up with strictly lower welfare (period 1’s discounted utility under $L^a$ and $L^B$ is nearly the same, but under $L^B$ it is achieved through a larger future payment burden). In such a case, the NGO’s refusal to offer commitment savings could raise welfare. However, that would not constitute an equilibrium: the moneylender’s
contract in the absence of the NGO would depress the agent’s discounted utility so much that the NGO would have an incentive to offer a commitment savings contract.

8.3 Bank and NGO Lending

Finally, consider the coexistence of a moneylender and a lending NGO. The NGO would prefer to limit the loan size to $p - 1$ in period 1, with repayments of $\frac{p-1}{2}$ in the next two periods. If $\beta$ is sufficiently high, this will be the equilibrium outcome. However, if $\beta$ is low, the moneylender could generate profits by lending more than the welfare optimizing amount. In such cases, the equilibrium loan size will be $l > p-1$, with repayments of $\frac{l}{2}$ (the NGO always has an incentive to drive repayment down to meet the zero-profit condition).

Even in cases where the moneylender is forced to lend more than the optimal amount, there will be a strict welfare improvement relative to the outcome with a moneylender alone since the NGO can raise welfare by driving profits down.

**Proposition 9** If a moneylender and lending NGO coexist: (a) The equilibrium contract will always satisfy a zero-profit condition and will enable immediate purchase of the non-divisible ($l \geq p - 1$ and $t = \frac{l}{2}$). There is some $\tilde{\beta}$ such that, if $\beta \in [\tilde{\beta}, 1)$, the agent receives $l = p - 1$ and $t = \frac{l}{2}$, and if $\beta \in (0, \tilde{\beta})$, the agent receives $l > p - 1$ and $t = \frac{l}{2}$. (b) For $\beta \in [\tilde{\beta}, 1)$, welfare will be the same as under a lending NGO, and for $\beta \in (0, \tilde{\beta})$, welfare will be strictly lower than under a lending NGO. For all $\beta < 1$, welfare will be strictly higher than under a monopolist lender.
9 Discussion

9.1 Implications for Policy and Experiments

These results put structure on the welfare losses that can arise even in the absence of consumer mistakes or exploitative behavior by banks. The model has some implications for the design of commitment savings and suggests directions for further experimental study. While the mechanisms described above are stylized, they provide a link between equilibrium contracts and welfare, and suggest heterogeneous treatment effects that can be measured in new and existing datasets.

A testable prediction is that, in autarky, agents who display moderate time-inconsistency will save in a more balanced manner than those who are less time-inconsistent. This is because moderately time-inconsistent agents are more constrained by the next period’s reluctance to save than mildly time-inconsistent agents are. In addition to the growing literature on commitment savings and microcredit, there have been several attempts to document time preferences and the degree of time-inconsistency.\textsuperscript{18} Similar studies in unbanked settings, combined with data on savings behavior, could be used to test the autarky predictions of this model. To the extent that welfare, as defined in this and previous papers, is measurable, it is in principle possible to also test the prediction that welfare does not always drop in the degree of time-inconsistency.

This also helps us to think further about the design and takeup of simple financial products. While data on lending has been examined for potential welfare losses (which could happen through a number of channels), less attention has been paid to the possibility that commitment savings too could lower welfare. This need not happen merely through the ex post realization of shocks or misallocation across accounts. In particular, it would be useful to identify cases where commitment savings helps individuals to adjust their savings patterns to excessively disadvantage future selves, and study if this has impacts for broader outcomes.

For intermediate values of $\beta$, commitment, rather than allowing greater or more balanced savings, helps period 1 further indulge his taste for instant gratification. This could happen in two possible settings: with a profit-maximizing monopolist whose goal is to maximize period 1’s discounted utility and therefore meets all demand, or with an NGO that does not limit access to commitment savings (either because it is unaware of potential welfare losses, or because it must compete with a profit-maximizing bank). These results on commitment savings lend themselves to further empirical investigation. Ashraf, Karlan, and Yin (2006) and Dupas and Robinson (2013), for example, provide evidence that access to commitment savings raises average savings and welfare. Since access was

\textsuperscript{18}See Ameriks, Caplin, Leahy, and Tyler (2007); Andersen, Harrison, Lau, and Rutstrom (2008); and Wang, Rieger, and Hens (2011)
randomly assigned, it should be possible to examine such data for possible non-monotonic relationships between time preferences and welfare.

Furthermore, the model’s examination of the interaction between commitment savings and lending can provide partial explanations for some existing puzzles in the literature. Ashraf et al. (2006) ask: "A natural question arises concerning why, if commitment products appear to be demanded by consumers, the market does not already provide them." (pg. 638) One possible answer is the following: since profit-maximizing banks can earn higher profits by offering hyperbolic discounters credit instead of savings, an apparent demand for commitment will be met through credit (which itself embeds commitment through repayment requirements) rather than by explicit commitment savings. Brune, Gine, Goldberg, and Yang (2013) find that offers of commitment savings improve household outcomes along several dimensions, even though there is no apparent rise in the use of the commitment savings account. While their paper itself provides some compelling explanations, the model here suggests that the puzzle could be further resolved by recognizing that an offer of commitment savings, even if it is not adopted, could have a welfare impact. The impacts of commitment emerge not just through participation, but through changes in reservation utilities that must be met by lenders.

By abstracting away from questions of default, the model of lending presented in this paper is able to generate predictions for credit markets that are independent of contract enforceability. Lending can help hyperbolic discounters in two ways. First, it can improve welfare by providing commitment through a balanced repayment path. Second, it allows them to buy nondivisible goods that they might not have been able to save up for. However, when a monopolist provides loans, welfare drops relative to autarky since the lender is able to feed period 1’s desire for instant gratification while extracting repayment from future selves.

Loan contracts vary based on time preferences. As agents get more hyperbolic, their demand for loans rises. A welfare-minded NGO should be unresponsive to this, as its goal is to enable the individual to take advantage of non-convexities in consumption, not instant gratification. A profit-maximizing bank, however, is sensitive to time preferences. When operating in isolation, it provides small loans to highly time-inconsistent agents. When competing with a welfare-minded NGO, it offers the same agents loans that are larger than optimal (since it can no longer extract large repayments from small loans, it seeks to expand its profits by offering larger loans than the NGO would like).

There remain open questions about the welfare effects of microfinance, with mixed evidence (see Morduch, 1998; Pitt and Khandker, 1998; and Duflo, Banerjee, Glennerster, and Kinnan, 2013). Duflo et al. (2013), in particular, find that some heterogeneous treatment effects can explain seemingly ambiguous effects of microfinance. This paper makes the complementary, and not quite novel, point that outcomes might also vary by time
preference. If loan sizes are not fixed (or fixed sufficiently high), hyperbolic discounters can find themselves worse off than exponential discounters through over-borrowing.

The model also sheds some light on an important question, posed by Banerjee and Duflo (2011) and Karlan and Appel (2011): why is microcredit less popular than initially expected? Banerjee and Duflo (2011) describe how, in Hyderabad, India, despite having access to multiple sources of microcredit, more than half of their sample continued to borrow from moneylenders. As they demonstrate, the rigidity of microfinance can explain some of this. In the context of this paper, rigidity matters in a specific way: if an MFI restricts loan sizes based on an independent welfare calculation, individuals with low $\beta$ will continue to turn to moneylenders who can offer larger loans, even if those loans are offered at higher rates. Again, however, the presence of microcredit will affect moneylender rates, so a positive impact of microcredit might be discernable even on those who don’t adopt it.

Finally, it is important to observe that the model does not necessitate paternalistic restrictions on contracts. A number of the results above emerge from what I argue is a realistic and necessary assumption—that in the period when an agent adopts contracts to alter future behavior, he is time-inconsistent himself. It should be possible for some regulation to be enacted by "period 0" agents, before temptation plays a role. Just as parents restrict the set of actions available to their children, we can make the case for contracts where individuals voluntarily restrict the contracting ability of their future selves. This is most effectively done when the current self has no immediate stake in the decision.

9.2 Hidden Types

While this paper has focused on highlighting some nuances associated with commitment savings and lending contracts under time-inconsistency, and shown how autarky equilibrium, banking equilibrium, and welfare can sometimes diverge from intuitive predictions, a practical application of the results might depend on whether consumer preferences are observable by the bank or NGO, as is assumed in the model. It is arguable that, in sufficiently close-knit environments where the service provider is a member of the community, the assumptions are not entirely far-fetched. Nevertheless, it is worth discussing how equilibrium contracts might change if consumer time preferences are private information.

An NGO faces a particular advantage over a bank in that its optimal contracts are less reliant on the agent’s type. With commitment savings, the NGO faces a simple tradeoff. Since any contract it offers must satisfy $f = 0$, it will offer commitment as long as the average welfare loss to those who would be hurt by commitment is at least balanced by the average gain to others. With loans, since the welfare-maximizing contract does not
depend on $\beta$, the NGO does not need to know the agent’s type at all.

A bank, on the other hand, clearly relies on its knowledge of $\beta$ to extract the maximum surplus. For commitment savings, there is no possibility of screening—the bank must choose a single $f$ to maximize profits. To see how this choice depends on the distribution of preferences, we can compare derivatives of period 1’s discounted utility with respect to $\beta$ (refer to Figure 2). The following are easily derived. For $\beta \in [\beta_{\text{mid}}, \beta_{\text{max}})$, when savings occur but period 1 is constrained by period 2:

$$\frac{dU^{\text{aut-constrained}}_1}{d\beta} = -\frac{\delta s^{\text{min}}_1}{\partial \beta} (u'(c_1) - \beta u'(c_2)) + u(c_2) + u(b) > 0$$ (19)

For $\beta \in [\beta_{\text{midlock}}, \beta_{\text{mid}})$, when savings do not occur but would under commitment:

$$\frac{dU^{\text{aut-nosave}}_1}{d\beta} = 2u(1) > 0$$ (20)

Under commitment savings at fee $f$, if period 2’s incentive compatibility constraint (see Condition 17) is not binding ($s_1$ denotes period 1’s optimal savings at fee $f$):

$$\frac{dU^{\text{comm-unconstrained}}_1}{d\beta} = u(c_2) + u(b) > 0$$ (21)

Under commitment savings at fee $f$, if period 2’s incentive compatibility constraint is binding:

$$\frac{dU^{\text{comm-constrained}}_1}{d\beta} = -\frac{\delta s^{\text{minlock}}_1}{\partial \beta} (u'(c_1) - \beta u'(c_2)) + u(c_2) + u(b) > 0$$ (22)

We know that, if $f > 0$, the agent prefers autarky over commitment at $\beta_{\text{midlock}}$ and $\beta_{\text{max}}$. Observe the following:

$$\frac{dU^{\text{aut-constrained}}_1}{d\beta} > \frac{dU^{\text{comm-unconstrained}}_1}{d\beta} > \frac{dU^{\text{aut-nosave}}_1}{d\beta}$$

This tells us that an agent benefits increasingly from commitment savings as $\beta$ drops from $\beta_{\text{max}}$ to $\beta_{\text{mid}}$, and then decreasingly as $\beta$ continues to drop from $\beta_{\text{mid}}$ to $\beta_{\text{midlock}}$. Therefore, as the bank raises fees for commitment savings, its client base drops to a narrower window around $\beta_{\text{mid}}$. This observation, combined with an actual distribution of types, allows the bank to set its profit-maximizing fee.

Under lending, the bank’s optimal decision under private types is subject to more complex considerations. To illustrate this, it is convenient to assume that the agent is,
with equal probability, one of two possible types, $\beta_L$ or $\beta_H$, with $\beta_L < \beta_H$. First, consider the bank’s lending problem in the absence of a nondivisible good. If the bank decides to serve only one type, it will choose $\beta_L$ and select the profit-maximizing contract that satisfies the participation constraint (the more time-inconsistent the agent is, the higher are the bank’s profits).

If the bank decides to serve both types, it will always engage in screening. Let $L^H$ denote the bank’s profit-maximizing contract at $\beta_H$. The first-order condition requires that the slope of the participation constraint, $\frac{u'(1+t^H)}{2\beta_H u'(1-t^H)}$, be equal to the slope of an isoprofit line, $\frac{1}{2}$. But at this contract, $\frac{u'(1+t^H)}{2\beta_L u'(1-t^H)} > \frac{1}{2}$ (the participation constraint for $\beta_L$ is steeper). This means that the bank can offer a second contract with a larger loan and larger repayment that yields higher profits than $L^H$ while being acceptable to $\beta_L$. So, in the absence of a nondivisible, the bank will select the highest type ($\beta^*$) it chooses to serve, and will then offer a menu of contracts that allow full screening across all types below $\beta^*$.

Now, we can see how the outcome might change when the nondivisible is introduced. The reasoning above applies everywhere except at the discontinuity of the participation constraint. Suppose, at $\beta_H$, the agent receives a contract with $l^H = p - 1$. At this contract, it is possible that $\frac{u'(1+t^H)}{2\beta_H u'(1-t^H)} < \frac{u'(1+t^H)}{2\beta_L u'(1-t^H)} < \frac{1}{2}$. Therefore, there might exist no other contract that raises the lender’s profits while being acceptable to $\beta_L$. In this case, there will be no separation of types.

Finally, consider the coexistence of a bank and NGO. When both offer commitment savings, it is possible for a positive fee to survive in equilibrium. Unlike in the case where types were publicly known, the NGO no longer has an automatic incentive to undercut the bank’s fees. Now, undercutting would have two effects: it would make existing customers of commitment savings better off, but it would also attract new customers, some of whom might experience a welfare loss from adoption. If the bank offers loans and the NGO offers commitment savings, the outcome will be subject to similar considerations as when the bank operates alone, except that a higher reservation utility would have to be met. When both the bank and NGO offer loans, the equilibrium contracts will be identical to the case with public information. The NGO’s incentive to drive any contract down to zero profits remains the same as before. Since each agent is offered the contract that maximizes period 1’s discounted utility, he will continue to reveal his type through his choice.\(^{20}\)

\(^{19}\)The bank will not necessarily choose to locate at the profit-maximizing contract for $\beta_H$. If it offers $\beta_H$ a smaller loan and repayment, this lowers available profits from $\beta_H$ but loosens the participation constraint that must be satisfied for $\beta_L$, generating higher profits from that type.

\(^{20}\)The preceding discussion has assumed that $p$ and $b$ remain constant across individuals. In cases where there is also variation in the nondivisible good being purchased, screening might occur across additional dimensions, even under commitment savings.
10 Conclusion

This paper attempts to characterize equilibrium commitment contracts for hyperbolic dis- 
counters under different banking environments. This suggests several areas for continued 
research.

There is room for analyzing in greater detail the role of external interest rates. As 
interest rates rise, lending clearly becomes less attractive since there are non-commitment 
motives to save. However, the relative appeal of lending over saving will still be greater 
for more time-inconsistent agents. There also remains a potentially interesting question 
of how these results would change under an infinite horizon. Such an analysis would 
introduce the possibility of multiple equilibria. Here, a bank’s contract will depend not 
just on the agent’s type but on his choice of autarky equilibrium. Finally, this paper 
makes the assumption that contracts, once signed, are exclusive and cannot be renegoti-
ated. While this is plausible with a monopolist or an NGO, it is harder to justify under 
competition, when banks could offer agents secondary loans that undermine the bene-
fits of commitment. This is the subject of continuing work.

While the model presented above is a stylized representation of markets for commit-
ment savings and loans, the goal of the paper has been to articulate the sometimes subtle 
mechanics at work in the interaction between hyperbolic discounters and informal banks.

11 Appendix

Proof of Lemma 1. (a) Period 1 will either save for nondivisible consumption or not 
at all. \( s_{1}^{yes}(\beta) \) is differentiable, and strictly increasing in \( \beta \) (since \( u'(0) = \infty \)). Note that 
\[ U_{1}(1; 1 - s_{1}^{yes}(1), 2 + s_{1}^{yes}(1) - p, b) > U_{1}(1; 1, 1, 1) \] (Assumption 1) and
\[ \lim_{\beta \to 0^{+}} U_{1}(\beta; 1 - s_{1}^{yes}(\beta), 2 + s_{1}^{yes}(\beta) - p, b) < \lim_{\beta \to 0^{+}} U_{1}(\beta; 1, 1, 1) \] (since \( \lim_{\beta \to 0^{+}} s_{1}^{yes}(\beta) \geq p - 2 > 0 \)). Also note that \( \frac{dU_{1}(\beta; 1, 1, 1)}{d\beta} \) is constant and \( \frac{dU_{1}(\beta; 1 - s_{1}^{yes}(\beta), 2 + s_{1}^{yes}(\beta) - p, b)}{d\beta} = u(2 + s_{1}^{yes}(\beta) - p) + u(b) \), which is increasing in \( \beta \) (both terms are differentiable every-
where). \( \beta_{min} \) can therefore be uniquely determined.

(b) By construction of \( \beta_{min} \), \( s_{1}^{opt}(\beta) \) determines the optimal savings decision.

(c) For \( \beta \geq \beta_{min} \), \( s_{1}^{opt}(\beta) = s_{1}^{yes}(\beta) \), which is continuous and strictly increasing.
\[ s_{1}^{yes}(1) = \frac{p - 1}{2} \]. At \( \beta_{min} \), \( c_{2} = 1 + s_{1}^{yes}(\beta_{min}) - (p - 1) > 0 \) (because \( u'(0) = \infty \)), so
\[ s_{1}^{opt}(\beta_{min}) = s_{1}^{yes}(\beta_{min}) > p - 2 \].

Proof of Lemma 2. (a) The following are true: for sufficiently large \( s_{1} \),
\[ U_{2}(\beta; 1 - s_{1}, 1 + s_{1} - s_{2}^{yes}, 0 + 1 + s_{2}^{yes} - p) > U_{2}(\beta; 1 - s_{1}, 1 + s_{1} - s_{2}^{no}, 1 + s_{2}^{no}) \]; the LHS 
and RHS in the previous inequality are continuous in \( s_{1} \); and by definition, \( s_{1}^{min} \) is 
bounded below at \( p - 2 \). Therefore, \( s_{1}^{min} \) exists.
(b) Suppose there is an \( \hat{s}_1 \) s.t. \( U_2 (\beta; 1 - \hat{s}_1, 1 + \hat{s}_1 - s_2^{yes} (\hat{s}_1), b + 1 + s_2^{yes} (\hat{s}_1) - p) \leq U_2 (\beta; 1 - \hat{s}_1, 1 + \hat{s}_1 - s_2^{no} (\hat{s}_1), 1 + s_2^{no} (\hat{s}_1)) \). Then, it must be true that \( s_2^{yes} \) attains a corner solution \((p-1)\), and \( s_2^{yes} > s_2^{no} \). So, by concavity of \( u \), at all \( s < \hat{s}_1 \), \( U_2 (\beta; 1 - s, 1 + s - s_2^{yes} (s), b + 1 + s_2^{yes} (s) - p) \leq U_2 (\beta; 1 - s, 1 + s - s_2^{no} (s), 1 + s_2^{no} (s)) \), which means the period 2 agent will strictly prefer to not save for the nondivisible.

(c) Since \( U_2 (\beta; 1 - s_1, 1 + s_1 - s_2^{yes}, b + 1 + s_2^{yes} - p) \) is continuous in \( s_1, s_2^{yes} \), and \( \beta \); \( U_2 (\beta; 1 - s_1, 1 + s_1 - s_2^{no}, 1 + s_2^{no}) \) is continuous in \( s_1, s_2^{no} \), and \( \beta \); and \( s_2^{yes} \) and \( s_2^{no} \) are continuous in \( \beta \); it must be true that \( s_1^{min} \) is continuous in \( \beta \).

Consider any \( \hat{s}_1 \) and \( \hat{\beta} \) s.t. \( U_2 (\beta; 1 - \hat{s}_1, 1 + \hat{s}_1 - s_2^{yes} (\hat{\beta}), b + 1 + s_2^{yes} (\hat{\beta}) - p) \leq U_2 (\beta; 1 - \hat{s}_1, 1 + \hat{s}_1 - s_2^{no} (\hat{\beta}), 1 + s_2^{no} (\hat{\beta})) \). Then, \( s_2^{yes} \) attains a corner solution \((p-1)\) and \( u(1 + s_2^{no}) < u(b) \). So, at all \( \beta < \hat{\beta} \), \( U_2 (\beta; 1 - \hat{s}_1, 1 + \hat{s}_1 - s_2^{yes} (\beta), b + 1 + s_2^{yes} (\beta) - p) < U_2 (\beta; 1 - \hat{s}_1, 1 + \hat{s}_1 - s_2^{no} (\beta), 1 + s_2^{no} (\beta)) \), which means that \( s_1^{min} (\beta) \) is strictly decreasing in \( \beta \) except when \( s_1^{min} (\beta) = p - 2 \).

(d) By assumption, \( s_1^{min} (1) \in [p - 2, \frac{p-1}{2}] \). For any \( s_1 \), however large, there exists \( \beta \) s.t. \( U_2 (\beta; 1 - s_1, 1 + s_1 - s_2^{yes}, b + 1 + s_2^{yes} - p) < U_2 (\beta; 1 - s_1, 1 + s_1 - s_2^{no}, 1 + s_2^{no}) \). So, \( \lim_{\beta \to 0^+} s_1^{min} (\beta) = \infty \).

Proof of Lemma 3. (a) This is true by definition of \( s_1^{max} \).

(b) For \( \beta < \beta_{min} \), there is no \( s_1 \geq p-1 \) s.t. \( U_1 (\beta; 1 - s_1, 2 + s_1 - p, b) \geq U_1 (\beta; 1 + s_1, 1) \).

So, \( s_1^{max} (\beta) = 0 \).

(c) Since, by definition of \( \beta_{min} \), \( U_1 (\beta; 1 - s_1, 2 + s_1 - p, b) < U_1 (\beta; 1 + s_1, 1) \) for all \( s_1 \neq s_1^{yes} (\beta_{min}) \), \( s_1^{max} (\beta_{min}) = s_1^{opt} (\beta_{min}) \). For \( \beta > \beta_{min} \), \( U_1 (\beta; 1 - s_1^{opt} (\beta), 2 + s_1^{opt} (\beta) - p, b) > U_1 (\beta; 1 + s_1, 1) \). Since \( U_1 (\beta; 1 - s_1, 2 + s_1 - p, b) \) is continuous in \( s_1 \), \( s_1^{max} > s_1^{opt} \) when \( \beta > \beta_{min} \).

(d) Since \( U_1 (\beta; 1 - s_1, 2 + s_1 - p, b) \) is continuous in \( \beta \) and \( s_1 \), and since \( U_1 (\beta; 1 + s_1, 1) \) is continuous in \( \beta \), \( s_1^{max} (\beta) \) is continuous for \( \beta > \beta_{min} \). Consider any \( \tilde{\beta} \geq \beta_{min} \). For any \( \beta > \tilde{\beta} \), \( U_1 (\beta; 1 - s_1^{max} (\tilde{\beta}), 2 + s_1^{max} (\tilde{\beta}) - p, b) > U_1 (\beta; 1 + s_1, 1) \). Therefore \( s_1^{max} (\beta) \) is strictly increasing in this region.

Proof of Proposition 1. By Lemmas 1-3, \( \beta_{max} \) and \( \beta_{mid} \) exist, and \( \beta_{max} \geq \beta_{mid} \geq \beta_{min} \).

For \( \beta \in [\beta_{max}, 1] \), \( s_1^{opt} \geq s_1^{min} \). So, \( s_2^{out} (s_1^{opt}) = p-1 \). Therefore, \( s_1^{out} = s_1^{opt} \).

For \( \beta \in [\beta_{mid}, \beta_{max}] \), \( s_1^{opt} < s_1^{min} \leq s_1^{max} \). So, \( s_2^{out} (s_1) = p-1 \) iff \( s_1 \geq s_1^{min} \). Since, \( s_1^{min} \leq s_1^{max} \) and since \( U_1 (\beta; 1 - s_1, p-1, b) \) is strictly decreasing in \( s_1 \) for \( s_1 \in [s_1^{opt}, s_1^{max}] \), \( s_2^{out} = s_1^{min} \) and \( s_2^{out} (s_1^{min}) = p-1 \).

For \( \beta \in (0, \beta_{mid}) \), \( s_1^{min} > s_1^{max} \). Therefore, in this region \( s_2^{out} = s_2^{out} = 0 \).

Proof of Lemma 4. (a) Same argument as in Lemma 2. Also note that, for any \( \beta \), \( s_1^{spinlock} < p-1 \).
Proof of Proposition 3. To show that the bank makes positive profits, we must prove that \( s_2 \) is feasible only for \( \hat{s}_1 \), that Constraints 16 and 17 continue to be satisfied, and that \( s_2 \) is continuous in \( \hat{s}_1 \).

Suppose there is a \( \hat{s}_1 \) s.t. \( U_2 (\hat{s}_1, 1 + \hat{s}_1 - s_2^{yes}(\hat{s}_1), b) \leq U_2 (\hat{s}_1, 1 + \hat{s}_1, 1 + \hat{s}_1) \). Then, it must be true that \( s_2^{yes} \) attains a corner solution \( (p - 1), \) and \( 1 + s_2^{yes} - p < 1 + \hat{s}_1 \). So, by concavity of \( u \), at all \( s_1 < \hat{s}_1 \),

\[
U_2 (\hat{s}_1, 1 + s_1 - s_2^{yes}(s_1), b + 1 + s_2^{yes}(s_1) - p) < U_2 (\hat{s}_1, 1 + s_1, 1 + s_1),
\]

which means period 2 agent will strictly prefer to not save for the nondivisible.

(c) Since \( U_2 (\hat{s}_1, 1 + s_1 - s_2^{yes}, b + 1 + s_2^{yes} - p) \) is continuous in \( s_1, s_2^{yes} \), and \( \hat{s}_1 \) is continuous in \( s_1 \) and \( \hat{s}_1 \); \( s_2^{yes} \) is continuous in \( \hat{s}_1 \); it must be true that \( s_1^{minlock} \) is continuous in \( \hat{s}_1 \).

Proof of Proposition 2. The monopolist can charge a fee only if there is some \( s \) such that \( s \geq s_1^{minlock} \) and \( s \neq s_1^{aut} \), and \( U_1 (\hat{s}_1, 1 + s, 2 + s - p, b) \geq U_1 (\hat{s}_1, 1 + s, 2 + s - p, b) \). This is feasible only for \( \hat{s}_1 \in [\hat{s}_1^{midlock}, \hat{s}_1^{max}] \).

Suppose Constraint 16 doesn’t bind. Then, there must be a higher value of \( f^{mon} \) such that Constraints 16 and 17 continue to be satisfied.

Proof of Proposition 3. For \( \hat{s}_1 \in (\hat{s}_1^{midlock}, \hat{s}_1^{max}) \), there is no saving in autarky. Since Constraint 16 binds, \( U_1 (\hat{s}_1, 1 - f^{mon} - s_1^{mon}, 2 + s_1^{mon} - p, b) = U_1 (\hat{s}_1, 1, 1, 1) \). Since \( 1 - f^{mon} - s_1^{mon} < 1 \) and \( \hat{s}_1 < 1 \), this equality implies that \( U_0 (\hat{s}_1, 1 - f^{mon} - s_1^{mon}, 2 + s_1^{mon} - p, b) > U_0 (\hat{s}_1, 1, 1, 1) \).

For \( \hat{s}_1 \in (\hat{s}_1^{mid}, \hat{s}_1^{max}) \), \( U_1 (\hat{s}_1, 1 - f^{mon} - s_1^{mon}, 2 + s_1^{mon} - p, b) = U_1 (\hat{s}_1, 1 - s_1^{min}, 2 + s_1^{min} - p, b) \). Since \( u (1 - f^{mon} - s_1^{mon}) > u (1 - s_1^{min}) \) and \( \hat{s}_1 < 1 \), it must be true that \( U_0 (\hat{s}_1, 1 - f^{mon} - s_1^{mon}, 2 + s_1^{mon} - p, b) < U_0 (\hat{s}_1, 1 - s_1^{min}, 2 + s_1^{min} - p, b) \).

Proof of Proposition 4. (a) Consider \( \hat{L} \equiv (p - 1, \frac{p - 1}{2}, \frac{p - 1}{2}) \). If \( \hat{s}_1 < 1 \), it follows that \( U_1 (\hat{s}_1, c_1 (\hat{L}), c_2 (\hat{L}), c_3 (\hat{L})) > U_1 (\hat{s}_1, c_1^{aut}, c_2^{aut}, c_3^{aut}) \). Then, there exists \( \varepsilon > 0 \) such that for \( \hat{L} \equiv (p - 1, \frac{p - 1}{2} - \varepsilon, \frac{p - 1}{2} - \varepsilon) \),

\[
U_1 (\beta, c_1 (\hat{L}), c_2 (\hat{L}), c_3 (\hat{L})) \geq U_1 (\beta, c_1^{aut}, c_2^{aut}, c_3^{aut}),
\]

Therefore, for any agent with \( \beta > 0 \), there is at least one loan contract \( \hat{L} \) under which the bank makes positive profits.
(b) If the agent saves, he must save a positive amount in period 1. Suppose, under $L^{mon}$, the agent saves $(s_1 > 0, s_2 \geq 0)$ but does not purchase the nondivisible. Consider the alternative loan contract $\tilde{L} = (L^{mon} - s_1, t_2^{mon} - s_1 + s_2, t_3 - s_2)$. The agent’s consumption will be identical under $L^{mon}$ and $\tilde{L}$, so he is indifferent between the two. Both contracts yield the same profit. But $\tilde{L}$ has a strictly lower loan size, so the firm should have selected it.

Suppose, under $L^{mon}$, the agent saves and purchases the nondivisible in period 3 (the same argument can be made if it is purchased in period 2). Then, it must be true that $c_3 (L^{mon}) = b$ and $c_1 (L^{mon}) + c_2 (L^{mon}) = 2 - (p - 1) + L^{mon} - t_2^{mon} - t_3^{mon}$. Consider the contract $\tilde{L} = (p - 1, \frac{2 - c_1 (L^{mon}) - c_2 (L^{mon})}{2}, \frac{2 - c_3 (L^{mon}) - c_2 (L^{mon})}{2})$. This yields the same profit to the bank as $L^{mon}$. Since $U_1 (\beta; c_1 (L), c_2 (\tilde{L}), c_3 (\tilde{L})) > U_1 (\beta; c_1 (L^{mon}), c_2 (L^{mon}), c_3 (L^{mon}))$, $L^{mon}$ cannot be a profit-maximizing contract.

(c) This follows directly from the strict concavity of $u$. If $t_2 \neq t_3$, the bank can raise its profits by offering $\tilde{t}_2 = t_3 = t_3 + t_3 \frac{\varepsilon}{2}$ for some $\varepsilon > 0$.

(d) Consider any loan contract with $l > 0$, $l \neq p - 1$, such that the agent’s consumption choices do not involve saving. Suppose $u' (1 + l) > \beta u' (1 - t)$. Then, there are some $\varepsilon_1$, $\varepsilon_2$, such that $\varepsilon_1 < \varepsilon_2$ and such that the agent will accept a contract $(l + \varepsilon_1, t + \frac{s_2}{2}, t + \frac{s_3}{2})$. Since the modified contract raises profits, the original contract was not profit-maximizing. Instead, suppose $u' (1 + l) < \beta u' (1 - t)$. Then, there are some $\varepsilon_1$, $\varepsilon_2$, such that $\varepsilon_1 > \varepsilon_2$ and such that the agent will accept a contract $(l - \varepsilon_1, t - \frac{s_2}{2}, t - \frac{s_3}{2})$. Since the modified contract raises profits, the original contract was not profit-maximizing.

Proof of Proposition 5. At any $\beta < 1$, $U_1 (\beta; c_1 (L^{mon}), c_2 (L^{mon}), c_3 (L^{mon})) = U_1 (\beta; c_1^{aut}, c_2^{aut}, c_3^{aut})$. It must also be true that $c_1 (L^{mon}) > c_1^{aut}$. Since $\beta < 1$, it follows that $U_0 (\beta; c_1 (L^{mon}), c_2 (L^{mon}), c_3 (L^{mon})) < U_0 (\beta; c_1^{aut}, c_2^{aut}, c_3^{aut})$. The same argument can be used to prove that welfare is lower than under commitment savings.

Proof of Proposition 6. (a) For any fee $f$, let $s_1^{n go} (f)$ be period 1’s best possible savings choice under commitment savings (subject to $s_1^{n go} (f) \geq s_1^{min lock}$). Then, $s_1^{n go} (0) \in [s_1^{n go} (f), s_1^{n go} (f) + f]$. So, consumption in each period rises weakly when $f$ drops to 0. Welfare must rise. (b) NGO commitment saving will be offered if two conditions are satisfied: (i) Period 1 values commitment (which is the case for $\beta \in (\beta_{mid lock}, \beta_{mid})$), and (ii) Commitment weakly raises welfare. By construction, $s_{min} (\beta_{max}) < \frac{p - 1}{2}$. Since $s_{min}^{n go}$ is continuous and falling in $\beta$, there must be some $\beta \geq \beta_{mid}$, such that for $\beta \in (\beta, \beta_{max})$, $s_{min}^{n go} (\beta) \leq \frac{p - 1}{2}$. In this region, commitment will further lower $s_{min}^{n go} (\beta)$, thus further lowering welfare. Therefore, condition (ii) is not satisfied in this region.

Proof of Proposition 7. (a) First consider $\beta \notin [\beta_{mid lock}, \beta_{max}]$. In this region, there is no $f \geq 0$ at which the agent will adopt commitment. Next, consider $\beta = \beta_{mid lock}$. Here,
the agent can be induced to adopt commitment only at \( f = 0 \). Finally, consider any \( \beta \in (\beta_{\text{midlock}}, \beta_{\text{max}}) \). The NGO must offer commitment savings with \( f = 0 \). (Consider any alternate NGO strategy. Then the monopolist’s best response involves a positive fee, which, as shown in Proposition 6, will result in lower welfare than if the NGO offered commitment for free. Therefore the alternative NGO strategy cannot constitute a best response to the bank’s best response to it).

(b) If an agent received commitment savings under an NGO alone, he receives the same contract under coexistence. Therefore, his welfare remains the same as under an NGO. If an agent did not receive commitment savings under an NGO alone, his welfare must be lower than under an NGO (by definition of the NGO’s objective) and higher than under a bank (since in this region the fee has dropped from \( f_{\text{mon}} > 0 \) to 0).

Proof of Proposition 8. (a) The agent cannot accept a commitment savings contract in equilibrium: for any commitment savings contract, there exists a loan contract that will satisfy his participation constraint and yield positive profits for the bank.

For \( \beta \notin (\beta_{\text{midlock}}, \beta_{\text{max}}) \), commitment savings cannot change \( \bar{U}_1 \), so the monopolist moneylender will offer the same contract as in Proposition 4. Consider \( \beta \in (\beta_{\text{midlock}}, \beta_{\text{max}}) \). Let \( U_{1, \text{aut}} \) and \( U_{0, \text{aut}} \) be, respectively, period 1 discounted utility and welfare in autarky. Let \( U_{1}(f) \) and \( U_{0}(f) \) be, respectively, period 1 discounted utility and welfare under commitment savings with fee \( f \). And let \( U_{1}(L) \) and \( U_{0}(L) \) be, respectively, period 1 discounted utility and welfare under a loan contract \( L \). Given \( f \), the monopolist’s offers \( L(f) \) given by:

\[
\max_{L} -l + t_2 + t_3
\]

\[
\text{s.t. } U_{1}(L) \geq \max \{U_{1, \text{aut}}, U_{1}(f)\}
\]

Suppose \( f = \bar{f} > 0 \). Then, there is some \( f' < \bar{f} \) such that \( U_{1}(f') > U_{1}(L(\bar{f})) \). Since \( \beta < 1 \) and period 1 consumption is lower under commitment savings than under lending, this also means that \( U_{0}(f') > U_{0}(L(\bar{f})) \). So \( f > 0 \) cannot be a best response to the moneylender’s best response to it. In equilibrium, the NGO must offer \( f = 0 \). Since \( U_{1}(0) > U_{1, \text{aut}} \), the moneylender’s contract changes relative to Proposition 4. The agent is indifferent between the NGO and moneylender’s contract, and accepts the moneylender’s contract.

(b) Let the equilibrium contract be denoted \( \hat{L} \). Let the reservation utility under coexistence be denoted \( \hat{U}_1 \) and reservation utility under autarky be denoted \( U_{1, \text{aut}} \). First, notice that welfare must be strictly lower than under NGO commitment savings: Since, by the participation constraint, \( U_{1} \) is identical under \( \hat{L} \) and and free commitment savings, and since \( c_1 \) is higher under \( \hat{L} \) than under commitment savings, \( U_{0} \) must be strictly lower under \( \hat{L} \). Next, compare welfare under coexistence to welfare under monopolist lending.
Suppose, under the moneylender, the equilibrium contract was $l^{mon} > p - 1$. This means that $\frac{u(c_1(l^{mon}))}{\beta u(c_2(l^{mon}))} = \frac{1}{2}$. Then, under coexistence, it must be true that, on the new (tighter) participation constraint, there is some $\hat{l} > l^{mon}$ and $\hat{l} < t^{mon}$ such that $\frac{u(c_1(\hat{l}))}{\beta u(c_2(\hat{l}))} = \frac{1}{2}$. At any other point on the new participation constraint, $\frac{u(c_1)}{\beta u(c_2)} > \frac{1}{2}$, so it cannot be profit-maximizing for the bank. Therefore, if $l^{mon} > p - 1$, welfare rises under coexistence.

Suppose $l^{mon} = p - 1$. Now suppose $\hat{l} < p - 1$. Then, $u(1 + \hat{l}) + 2\beta u(1 - \hat{l}) = X$, where $X$ denotes the discounted utility of the outside option. Let $\hat{\pi}(X)$ denote the lender's profits. As $X$ changes, suppose the firm were to continue to satisfy the participation constraint with $\hat{l}$ constant. Then, implicitly differentiating the participation constraint, we get $\hat{\pi}'(X) = \frac{-1}{\beta u'(1 - \hat{l})} < 0$. Consider an alternate contract, $\bar{L}$, which satisfies $\bar{l} = p - 1$ and $u(b) + 2\beta u(1 - \bar{l}) = X$. Let $\bar{\pi}(X)$ denote the lender's profits from such a contract. Again, keeping $\bar{l}$ constant, we get $\bar{\pi}'(X) = \frac{-1}{\beta u'(1 - \bar{l})} < 0$. Since $\hat{\bar{l}} > \bar{l}$, $\hat{\bar{\pi}}'(X) < \bar{\pi}'(X)$. Since by assumption $\hat{\bar{\pi}}(\hat{\bar{U}}_1) > \bar{\pi}(\hat{\bar{U}}_1)$, and $\hat{\bar{U}}_1^{out} < \hat{\bar{U}}_1$, it must be true that $\hat{\pi}(\hat{\bar{U}}_1^{out}) > \bar{\pi}(\hat{\bar{U}}_1^{out})$. But this is not possible since $l^{mon}$ was profit-maximizing. Therefore, if $l^{mon} = p - 1$, it must be true that $\hat{l} > l^{mon}$. Then, the new profit-maximizing point is either at tangency, in which case $\hat{l} < t^{mon}$, or along $l = p - 1$, in which case also $\hat{l} < t^{mon}$.

Therefore, from the two previous paragraphs, if $l^{mon} \geq p - 1$, welfare under coexistence must be higher than under monopolist lending.

Now consider $l^{mon} < p - 1$. If $\hat{l} < p - 1$, then it must be true that $\hat{l} > l^{mon}$ and $\hat{l} < t^{mon}$, so welfare rises relative to monopolist lending.

Finally, it is possible to construct a case where welfare will drop under coexistence. Suppose $l^{mon} < p - 1$. Also consider the alternate contract $\bar{L}$ with $\bar{l} = p - 1$ and $\bar{l}$ such that $u(b) + 2\beta u(1 - \bar{l}) = \bar{U}_1^{out}$. Suppose the moneylender is nearly indifferent between $L^{mon}$ and $\bar{L}$. There is some value of $\beta$ at which such indifference exists: Let $\pi_L(\beta)$ be the monopolist’s maximized profit from any loan conditional on the nondiscrete not being purchased, and let $\pi_H(\beta)$ be the monopolist’s maximized profit from any loan large enough to allow the purchase of the nondiscrete. At sufficiently high $\beta$, when the agent is uninterested in small loans, $\pi_H(\beta) > \pi_L(\beta)$. At sufficiently low $\beta$, when the agent is willing to pay nearly all future income for a small loan, $\pi_L(\beta) > \pi_H(\beta)$. Since $\pi_H(\beta)$ and $\pi_L(\beta)$ are continuous in $\beta$, there must be some $\beta$ at which the bank is indifferent between a small loan and a large loan.

If the moneylender is nearly indifferent between $l^{mon} < p - 1$ and $\bar{l} = p - 1$, it can be shown that as the participation constraint tightens, the moneylender will strictly prefer to offer $p - 1$. Let period 1’s reservation utility be denoted $X$. As $X$ rises, suppose the moneylender were to choose to remain at a point of tangency. The tangency condition is: $u(1 + l(t)) + 2\beta u(1 - t) \equiv X$, where $l(t)$ is given by the first-order condition from...
profit-maximization. Differentiating implicitly by \(X\), we get \(\frac{d\theta}{dX} = \frac{-1}{(\frac{d}{\theta} - 2)\beta u'(1-t)}\). So, as \(X\) rises, the change in the moneylender’s profits is given by: \(\frac{d\sigma}{dX} = -\frac{\partial \theta}{\partial t} \frac{d\theta}{dX} + 2 \frac{d\theta}{dX} = \frac{-1}{\beta u'(1-t)}\).
Alternatively, as \(X\) rises, suppose the moneylender were to maintain \(\tilde{t} = p - 1\). Then, the change in profits is given by: \(\frac{d\sigma}{dX} = \frac{-1}{\beta u'(1-t)}\). Profits drop faster at the point of tangency than at \(\tilde{t} = p - 1\). Therefore, if the reservation utility rises under coexistence, the lender will switch to a contract with \(\tilde{t} = p - 1\). Since welfare is strictly lower at \(\tilde{t}\) than at \(L_{mon}\), if \(U_1^*\) is sufficiently close to \(U_1^{aut}\), welfare will drop relative to monopolist lending. ■

**Proof of Proposition 9.** (a) Any equilibrium contract, \(L^*\), must satisfy \(t^* = \frac{t^*}{2}\) and:

\[
l^* = \arg \max_l U_1 \left( \beta; b + (l + 1 - p), 1 - \frac{l}{2}, 1 - \frac{l}{2} \right) \tag{24}
\]

If it did not satisfy the zero-profit condition \((t = \frac{l}{2})\), the NGO could raise welfare by lowering \(t\). If, conditional on satisfying the zero-profit condition, \(t^*\) did not satisfy Condition 24, the bank could raise profits by offering a different contract.

By condition 1, \(l^*\) must be at least \(p - 1\). Consider the first-order condition of the maximization problem: \(u'(b + (l^* + 1 - p)) = \beta u'(1 - \frac{l^*}{2})\). Implicitly differentiating with respect to \(\beta\), we get \(l^* = u'(1 - \frac{L^*}{2}) = \frac{u'(1 - \frac{l^*}{2})}{u'(b + (l^* + 1 - p) + \frac{1}{2} \beta u'(1 - \frac{l^*}{2})} < 0\). Since \(l^*\) is continuous in \(\beta\), is bounded below at \(p - 1\), and for sufficiently low \(\beta\) is greater than \(p - 1\), there must be some \(\tilde{\beta} < 1\) as defined in the statement of the proposition.

(b) Welfare is maximized at \(l = p - 1\) and \(t = \frac{p - 1}{2}\). Therefore, for \(\beta \in [\tilde{\beta}, 1)\), welfare will be the same as under a lending NGO and strictly higher than under monopolist lending.

Consider \(\beta \in (0, \tilde{\beta})\). Welfare will be strictly lower than under a lending NGO. Now we can show that it will be strictly higher than under a monopolist lender. Under the monopolist lender’s contract, \(\frac{u'(c_1(L_{mon}))}{2\beta u'(c_2(L_{mon}))} \leq \frac{1}{2}\). (Strict inequality can only occur at \(L_{mon} = p - 1\). Any other contract satisfies tangency between an isoprofit line and the participation constraint.)

Under coexistence, suppose \(t^* \geq t_{mon}\). Since the individual’s reservation utility is higher than under a monopolist lender, it must be true that \(t^* > t_{mon}\). Then, \(\frac{u'(c_1(L_{mon}))}{2\beta u'(c_2(L_{mon}))} \leq \frac{1}{2}\). But this means that \(l^*\) does not satisfy Condition 24, since \(U_1^*\) could be raised by lowering \(t\). Therefore, it must be true that \(t^* < t_{mon}\).

Under coexistence, suppose \(l^* < t_{mon}\). If \(l_{mon} \leq p - 1\), this is impossible by the definition of \(l^*\). If \(l_{mon} > p - 1\), since we have seen above that \(t^* < t_{mon}\), this means that \(\frac{u'(c_1(L_{mon}))}{2\beta u'(c_2(L_{mon}))} > \frac{u'(c_1(L_{mon}))}{2\beta u'(c_2(L_{mon}))} = \frac{1}{2}\). But then \(l^*\) does not satisfy Condition 24, since \(U_1\) could be raised by raising \(t\). Therefore, it must be true that \(l^* \geq t_{mon}\).

Since \(t^* < t_{mon}\) and \(l^* \geq l_{mon}\), welfare is strictly higher than under a monopolist lender. ■
References


