Hyperbolic Discounting and the Sustainability of Rotational Savings Arrangements

By KARNA BASU*

People across the developing world join rotational savings and credit associations (roscas) to fund repeated purchases of nondivisible goods. When the scope for punishment is weak, there is a natural question about why agents not defect from roscas. This paper models roscas as commitment savings devices and derives conditions under which hyperbolic discounters will never defect, even in the absence of formal contracting, social punishment, and reputation. I show why, unlike with standard commitment devices, a hyperbolic discounter will not postpone entry into a rosca. Finally, this paper makes predictions about the relative survival of random and fixed roscas.
JEL: D03, D91, O12
Keywords: Hyperbolic discounting, Commitment devices, Roscas, Saving, Informal banking

In settings where informational and contractual problems limit the scope of formal banking, informal financial arrangements may continue to thrive.[1] Roscas (rotating savings and credit associations) are a prominent form of saving in several countries.[2] A rosca consists of a group of individuals who meet at regular intervals and contribute a fixed amount to a collective "pot", which is then allocated to one of the members. A rosca "cycle" consists of exactly as many meetings as there are members. Within a cycle, each member gets to take the pot home exactly once. The order in which the pot is allocated can be determined in several ways. I focus on "fixed" and "random" roscas. In a fixed rosca, the order is randomly determined at the first meeting of the rosca, and then repeated indefinitely through future cycles.[3] In a random rosca, the order is randomly determined at the start of each new cycle.

In this paper, I model roscas as commitment savings devices for sophisticated hyper-

* Department of Economics, Hunter College, City University of New York, 695 Park Ave, New York, NY 10065. kbasu@hunter.cuny.edu. I am deeply indebted to Abhijit Banerjee, Xavier Gabaix and Sendhil Mullainathan for comments and support. I thank Jean-Marie Baland, Sylvain Chassang, Antara Dutta, Dean Karlan, Karthik Muralidharan, Roger Myerson, and seminar participants at MIT, Michigan State University, Syracuse University, and NEUDC for discussions and comments. This version of the paper has benefited greatly from suggestions provided by three anonymous referees.

[1] See Jonathan Conning and Christopher Udry (2005) and Abhijit V. Banerjee and Esther Duflo (2010).

[2] In Senegal, members of the Tidiane community use them to fund pilgrimages to Mecca. In Philadelphia, women from the Ivory Coast join roscas to pay for childbirths and funerals. Roscas can be found in several parts of Kenya, where the money is put to various uses, from home repairs to the purchase of food and clothing. Alec R. Levenson and Timothy Besley (1996) point out that, in any given year, one-fifth of all households in Taiwan participate in a rosca. F. J. A. Bouman (1995) cites several studies of African roscas where participation rates among the population are even higher - for example, 50 percent in the Republic of the Congo and 80 percent in Cameroon.

[3] At times, other considerations (such as age) may come into play in determining the initial ordering. See, for example, Siwan Anderson and Jean-Marie Baland (2002).

bolic discounters. The model allows us to solve some puzzles about the high observed survival of roscas, and to derive implications for existence and sustainability under varying underlying conditions. The existing literature on roscas stresses the need for social sanctions to ensure that participants do not defect.[4] One might expect that these sanctions need to be stronger when individuals have present-biased preferences. I show that this is not the case. In fact, roscas with hyperbolic discounters may survive indefinitely even without formal contracting or social sanctions. Hyperbolic discounters create their own punishments for defection – knowing that they under-save in autarky can make them more likely to stay on in roscas. The model also demonstrates that credit histories, or reputations, are not essential for rosca survival. Finally, I show how the depth of reputation determines whether fixed or random roscas are more likely to survive.

Following Besley, Coate, and Loury (1993), I assume agents would like to save for a nondivisible good. If they were to save alone, each would have to wait a certain number of periods before she could acquire the good. A rosca allows agents to pool their savings so that some members get the nondivisible good sooner. The likelihood of an early nondivisible gives agents an incentive to join a rosca. However, as Anderson, Baland, and Moene (2009) show, members face ex-post incentives to defect. Consider an agent in a fixed rosca who has just received the pot. If she now leaves the rosca, she can *at least* replicate the rosca outcome by saving alone (there is no longer a positive probability of getting the good sooner through the rosca). In a random rosca, agents have an even greater incentive to defect. Consider the first-ranked agent in a cycle. There is no guarantee that she will again be ranked first in the next cycle. If she leaves the rosca after receiving the pot, she can ensure that she continues to be effectively "first-ranked" by independently saving just as she would in a rosca. By staying on in the rosca, the agent is continuing to save at a negative interest rate. Therefore, under standard exponential time preferences, the threat of punishment must be severe enough that an agent who would otherwise choose to leave a rosca will now choose to continue participating.

Within this framework, I seek to answer the following questions. First, there is evidence that roscas survive even when formal contracting is impossible and the threat of social punishment is minimal.[5] Why do agents not leave after receiving the nondivisible? Second, for any given outside option, agents in random roscas have greater incentives to leave than do agents in fixed roscas. Why do random roscas exist, given that they provide the same ex-ante utility as fixed roscas? Third, even if the threat of social sanctions can be used to ensure participation within a cycle, members are free to leave at the end of a cycle since their debt to the group is repaid**.** Then, why would the last-ranked member in a fixed rosca stay on?[6] If she were an exponential discounter, she should leave after a cycle if there is any probability that she will find a rosca that ranks her higher (since, even if she does not find a rosca, she can save alone and do no worse than in her original

[4]The theoretical case for this is made by Besley, Stephen Coate, and Glenn Loury (1993) and Anderson, Baland, and Karl O. Moene (2009).

[5]Mary K. Gugerty (2007) finds that roscas last several years in settings where sanctions are difficult to impose. In her dataset, the average rosca is 6.5 years long (around 5 cycles old).

[6]In addition to Gugerty (2007), Anderson and Baland (2002) provide evidence that fixed roscas survive over multiple cycles. The median rosca in their sample lasts for 3.2 cycles.

rosca).

The model below is built on two key assumptions. First, agents have time-inconsistent preferences and are aware of it. There is empirical evidence that this is indeed the case – Gugerty (2007) and Olivier Dagnelie and Philippe LeMay-Boucher (2010) find that rosca members most often cite self-control problems as the reason for joining.[7] Second, I take seriously the fact that roscas are informal institutions that may exist even in environments where contracting and social networks are weak.[8] I assume that participation cannot be contracted upon and that social punishment is infeasible.

This paper is organized as follows. Section I provides a literature review. Section II characterizes the autarky equilibrium, and shows how saving slows as agents get more present-biased. Section III articulates the central logic of rosca sustainability. I assume there is a single provider of roscas that cannot make credible threats of punishment. The only enforceable tool is expulsion. In this setting, roscas generate two complementary benefits for hyperbolic discounters – high expected rank and commitment. The high expected rank, including the possibility of being an instant winner, ensures that the agent will not postpone entry. Within the rosca, though there is no positive punishment for defection, illiquidity and the threat of expulsion create commitment. Illiquidity ensures that, if an agent participates when she is furthest from her next nondivisible, her future selves will continue to participate. The threat of expulsion ensures that non-payment locks an agent in autarky forever. I show that there is a parameter region in which the agent will always participate and repay the "debt" after receiving the pot. This happens when she is intermediately hyperbolic – she must be sufficiently time-inconsistent that her future selves perform better in a rosca than in autarky, but not so time-inconsistent that she is unwilling to save in the present.

A sophisticated hyperbolic discounter has two objectives: to indulge today, and to discourage her future selves from doing the same. A rosca, by appealing to her second objective, gives her a reason to forego instant gratification.

In Section IV, I model roscas as decentralized equilibria by lifting two assumptions of the benchmark case. Now, agents' past rosca behavior might not be observable, and new roscas can be formed over time (in any period, there is an exogenous probability with which people get matched into new roscas). Agents play a dynamic game and strategies can be conditioned on reputation, to the extent that it is available. The objective is to find equilibria that prevent frivolous defection (which would involve leaving one rosca in search of a higher rank in another). This is a serious consideration, particularly in developing economies, when information cannot be shared across financial institutions.

This setup provides two further insights. First, it is not necessary for defectors to be banned from future roscas. If the rate at which new roscas form is sufficiently slow,

[7] In Dagnelie and LeMay-Boucher (2010), 89 percent of rosca participants cite self-discipline as the motivation. In Gugerty (2007), 36 percent of rosca participants say self-control is the primary reason for participation. Some quotes: "You can't save alone–it is easy to misuse money;" "Saving money at home can make you extravagant in using it." (page 268)

[8] Even in studies that allude to social sanctions as an enforcement mechanism, there is often limited information about the precise nature and potency of the sanctions. In light of this, I think it is useful to show that even when sanctions exist, they may not be necessary. For discussions on sanctions, see Ivan Light (1972), Besley and Levenson (1996), Rogier van den Brink and Jean-Paul Chavas (1997), and Anderson and Baland (2002).

roscas can survive under complete anonymity. The longer a hyperbolic discounter expects to remain in autarky while searching for a new rosca, the less likely she is to defect. Second, I find conditions under which random roscas can be preferred to fixed roscas. To do so, I model what is perhaps the most realistic reputation environment – "partial reputation". This is the case where only agents who leave without completing a cycle can be identified and barred from future roscas. This allows us to restrict our focus to agents' desires to leave *between* cycles. The last-ranked member in a fixed rosca is permanently last ranked, so she might have an incentive to leave after a cycle in anticipation of a better rank in a new rosca. Now, random roscas have a particular advantage over fixed roscas – by re-randomizing at the start of every cycle, they internalize the attractiveness of the outside option. There is no incentive for a member of a random rosca to leave between cycles. Under certain conditions, random roscas survive longer and raise welfare relative to fixed roscas.

Section V discusses empirical implications of the model and Section VI concludes.

I. Related Literature

Standard explanations of roscas focus on the individual's desire to save for a nondivisible good. In addition to the papers described above, there is a wide range of descriptive papers on roscas in several parts of the world.[9]

Anderson and Baland (2002) find evidence that roscas are used by women to restrict their husbands' access to their savings. In their model, women have a greater preference for the nondivisible than men, but have limited power over expenditures within the household. If women were to save at home, their husbands would direct too much of their savings towards immediate consumption. If women save in a rosca, husbands have no access to their savings until the pot is received. At this stage, assuming the woman has sufficient bargaining power to purchase the nondivisible good, it is in fact purchased. In this setting, a rosca can be viewed as another kind of commitment savings device. A woman would like to save for the nondivisible, but she knows that saving at home is suboptimal. A rosca, by locking in savings, allows her to prevent overconsumption by her household in future periods.

In Section V, I pay particular attention to the analyses in Gugerty (2007) and Anderson, Baland, and Moene (2009). Gugerty's study is set in a rural community in Kenya where banks, if available, are far away, and there appears to be very limited scope for credible social sanctions. She finds that only 6 percent of members left a rosca is the last cycle studied. 37 percent of the roscas are fixed, 58 percent are random, and the rest use other forms of negotiation/randomization. The largest proportions of rosca funds are spent on cooking items, school fees, and food. Anderson, Baland, and Moene (2009) study a poor urban neighborhood near Nairobi, where 71 percent of the roscas are fixed and 29 percent are random. They find that funds generated through roscas are more often spent on nondivisible goods with immediate benefits than on durables.

[9]See Bouman (1994, 1995), Sudhanshu Handa and Claremont Kirton (1999), and Peter K. Kimuyu (1999).

Quasi-hyperbolic discounting has been used to describe time-inconsistent preferences in several papers.[10] The fact that these preferences result in a need for commitment has also been widely studied.[11] In the context of this literature, the point of this paper is to show that roscas are effective commitment devices even without "commitment" in the standard sense (in settings where agents cannot pre-commit to join and cannot be forced, through contracts or social punishment, to continue participating).

Finally, there are two aspects of roscas that are worth mentioning even though my results do not directly relate to them. First, the allocation of pots can also be determined by an auction. Though these "bidding" roscas are not common in the empirical papers that motivate this model, they nevertheless feature prominently in many developing countries. It is possible to conceive of them as risk-sharing arrangements under limited commitment (see Ethan Ligon, Jonathan P. Thomas, and Tim Worrall 2002). However, this explanation is less plausible with fixed and random roscas, where the allocation order is generally inflexible.[12]

Second, there is the question of efficiency. Besley, Coate, and Loury (1994) argue that random (and, by extension, fixed) roscas do not result in efficient allocations in general. Stefan Ambec and Nicolas Treich (2005) show how roscas can be the best possible institutions, ex ante, when agents value commitment (their paper assumes contracts are binding and that agents can commit to joining a rosca at a future date). This leaves open the question of whether, in an environment with hyperbolic discounting and limited contracting, an institution more efficient or sustainable than roscas exists. In the conclusion, I discuss further whether privately provided services, such as lotteries, can act as substitutes for roscas. However, since roscas do enjoy widespread success, I hope this paper takes a useful step towards understanding when and why they survive.

II. Autarky Equilibrium

In this section, I assume the individual does not have access to a rosca, and study her behavior as the equilibrium of a game played by her per-period selves. The agent would like to save for a nondivisible good. In any period, her saving decision depends on her discount factor and her expectations of future actions. There are complementarities to saving – the faster her future selves add to the savings pool, the more she is willing to save today.

Below, I solve for the equilibrium strategy at different levels of hyperbolic discounting. Hyperbolic discounters would like to save less in the present but continue to save rapidly in the future. As they get more present-biased, the gap between ideal future saving and actual future saving grows. This further reduces current incentives to save. As a result,

[10] Originally proposed by Edmund S. Phelps and Robert A. Pollack (1968), it has been developed in several papers including David Laibson (1997), Christopher Harris and Laibson (2001), and Per Krusell and Anthony A. Smith (2008).

[11] For examples, see Nava Ashraf, Nathalie Gons, Dean S. Karlan and Wesley Yin (2003), Ashraf, Karlan and Yin (2006), Philip Bond and Gustav Sigurdsson (2009), Karna Basu (2010), and Gharad Bryan, Karlan and Scott Nelson (2010).

[12] Some recent papers on bidding roscas include Charles W. Calomiris and Indira Rajaraman (1998), Stefan Klonner (2003), and Klonner and Ashok Rai (2007).

equilibrium saving is suboptimal not only from an overall welfare perspective but also from the perspective of the agent at any point in time. This immediately highlights how a rosca, by inducing faster future savings, encourages the current self to save more as well.

A. Setup

The agent is an infinitely lived, sophisticated quasi-hyperbolic discounter. She has a per-period non-stochastic income y, and no initial endowment. Borrowing is not possible, and no interest is earned on savings. There are two types of goods: a consumption good (denoted c; price 1) and a nondivisible good (denoted d; price ky, where k is a positive integer). Saving decisions are lump-sum, in multiples of y.[13] The agent's per-period utility function is $u(c + bd)$ (where u is strictly concave and defined over the domain $[0, \infty)$, with $u(0) = 0$; b is a positive constant). Discounted utility at time t is:

$$U_t = u(c_t + bd_t) + \beta \sum_{i=1}^{\infty} \delta^i u(c_{t+i} + bd_{t+i}), \text{ where } \beta \in (0,1) \text{ and } \delta \in (0,1)$$

Finally, I assume that the nondivisible good is "desirable":

$$\delta^{k-1} u(b) > \sum_{i=0}^{k-1} \delta^i u(y) \tag{1}$$

This ensures that, if the agent were an exponential discounter, she would repeatedly save all her wealth for the nondivisible good (any other saving rule would violate either time consistency or the condition above).

The individual can be viewed as a series of time-indexed selves with utility functions $\{U_t\}$ who play a Markov Perfect Equilibrium.[14] In any period, the agent observes her total wealth, w_t, and picks a level of gross saving, s_t. Wealth and savings are related in the following way: $w_t = s_{t-1} + y$. Since saving is lump-sum and initial wealth is 0, w_t must be a multiple of y. In any state w the action set, $\{0, y, 2y, ..., w\}$, includes all feasible levels of saving. A strategy, $\boldsymbol{\pi}$, associates every state w with a sequence of positive probabilities, $\{\pi_w(0), \pi_w(y), \pi_w(2y), ..., \pi_w(w)\}$, that sums to 1 and denotes a probability distribution over all feasible actions.

We can restrict our attention to the following set of states : $\{y, 2y, 3y, ..., (k-1)y\}$. Since the nondivisible provides the only incentive to save, and since initial wealth is 0, there will never be an equilibrium where the agent encounters wealth higher than ky. The agent with wealth ky will always save 0, regardless of future behavior.

A strategy is an equilibrium if and only if, for any state $w \in \{y, 2y, 3y, ..., (k-1)y\}$,

[13] This assumption allows us to model autarky equilibria using mixed strategies over finite choices.

[14] See Phelps and Pollack (1968) and Krusell and Smith (2003, 2008) for applications of this equilibrium concept to savings problems under time-inconsistency

every action s that is played with positive probability satisfies:

$$s \in \max_{s' \in \{0, y, 2y, \ldots, w\}} \left[u\left(w - s'\right) + \beta\delta V\left(s' + y; \boldsymbol{\pi}\right)\right]$$

Here, V is the continuation utility, defined recursively:

$$V\left(w; \boldsymbol{\pi}\right) = \sum_{j=0}^{w/y} \pi_{w}\left(jy\right)\left[v\left(w - jy\right) + \delta V\left(\left(j+1\right)y; \boldsymbol{\pi}\right)\right], \text{ where } v\left(x\right) = \begin{cases} u\left(x\right), \text{ if } x < ky \\ u\left(b\right), \text{ if } x = ky \end{cases}$$

B. Properties of Autarky Equilibrium

Multiple-self models with quasi-hyperbolic discounting can lead to a multiplicity of equilibria. This section presents some results that allow us to restrict the set of strategies that are candidates for equilibrium, and predict which equilibrium will be chosen in the case of multiplicity. At any level of β, there will be an equilibrium that is optimal in a Pareto sense – it weakly dominates all other equilibria in every state. The optimal equilibria involve faster saving at higher levels of β, and the equilibrium strategy and discounted utility are continuous across β. The proofs of the following lemmas and propositions are in the appendix.

It is helpful to define a few terms. A "saving" strategy (or equilibrium) is one in which the nondivisible is purchased with positive probability. A "full-saving" strategy is one in which the agent always saves her accumulated wealth until the nondivisible is purchased. A "partial-saving" strategy is a saving strategy that is not a full-saving strategy. A "non-saving" strategy is one in which the nondivisible is never purchased.

Consider any saving strategy. For this to be an equilibrium, the agent must always weakly prefer to save her accumulated wealth over any smaller amount. It follows directly from concavity that if an agent with low wealth chooses to save a certain amount, an agent with higher wealth cannot possibly wish to save any less. The stock must always weakly rise until ky is reached.

LEMMA 1: *In any saving equilibrium, an agent with wealth w will either save w or mix between w and $w - y$.*

For simplicity, describe any saving equilibrium as $\boldsymbol{\pi} = \left\{\pi_{y}, \pi_{2y}, \ldots, \pi_{(k-1)y}\right\}$, where π_{s} denotes the probability of saving s. At each w, the following will be true:

$$\beta\delta V\left(w + y; \boldsymbol{\pi}\right) \geq u\left(y\right) + \beta\delta V\left(w; \boldsymbol{\pi}\right) \tag{2}$$

If $\pi_{w} < 1$, then:

$$\beta\delta V\left(w + y; \boldsymbol{\pi}\right) = u\left(y\right) + \beta\delta V\left(w; \boldsymbol{\pi}\right) \tag{3}$$

It is mechanically true that, for any saving equilibrium:

$$\begin{aligned} V(w;\boldsymbol{\pi}) &= \pi_w \delta V(w+y;\boldsymbol{\pi}) + (1-\pi_w)(u(w)+\delta V(w;\boldsymbol{\pi})) \\ \Rightarrow & \quad V(w;\boldsymbol{\pi}) = \frac{\pi_w}{1-(1-\pi_w)\delta}\delta V(w+y;\boldsymbol{\pi}) + \frac{(1-\pi_w)}{1-(1-\pi_w)\delta}u(y) \end{aligned} \tag{4}$$

For an intuitive understanding of how a saving equilibrium is constructed, consider an arbitrary saving strategy, $\{\pi_y, \pi_{2y}, ..., \pi_{(k-1)y}\}$. In any state w, the agent decides whether to save w ("save up") or $w-y$ ("save down") by comparing the continuation utilities associated with each action. A rise in any π_{-w} strengthens the incentive to save up at w. However, a rise in π_w weakens the same incentive. This is because the punishment associated with saving down gets weakened – if I do not save up today, a high π_w ensures that my future selves will nevertheless save rapidly starting in the next period. Therefore, for a saving strategy to constitute an equilibrium, π_w must be low enough and π_{-w} high enough to create a weak incentive for state w to save up.

The next three lemmas follow from this. Lemma 2 establishes that, in any saving equilibrium, the probability of saving up rises as the agent gets closer to the nondivisible. Lemmas 3 and 4 show that if there are two distinct saving equilibria, one must involve higher probabilities of saving up in all states, and correspondingly, a higher continuation utility at all levels of wealth.

LEMMA 2: *If $\boldsymbol{\pi}$ is a saving equilibrium, then $0 < \pi_y \leq \pi_{2y} \leq ... \leq \pi_{(k-1)y} \leq 1$. Furthermore, the probability of saving up must strictly rise until it reaches 1.*

LEMMA 3: *Let $\boldsymbol{\pi}$ and $\boldsymbol{\mu}$ be two saving equilibria at some β. If, for some w, $\pi_w > \mu_w$, then for all w', $\pi_{w'} > \mu_{w'}$ or $\pi_{w'} = \mu_{w'} = 1$.*

LEMMA 4: *Let $\boldsymbol{\pi}$ and $\boldsymbol{\mu}$ be two equilibria at some β. Let $\boldsymbol{\pi}$ be a saving equilibrium. If $V(y;\boldsymbol{\pi}) > V(y;\boldsymbol{\mu})$, then for all w, $V(w;\boldsymbol{\pi}) > V(w;\boldsymbol{\mu})$.*

Proposition 1 consolidates the lemmas to show that an optimal equilibrium always exists. A saving equilibrium is always better than a non-saving equilibrium, and the optimal saving equilibrium is the one that involves the fastest saving.

PROPOSITION 1: *There is always an equilibrium that is "optimal" in the sense that, at any level of wealth that is reached in equilibrium, the agent does not strictly prefer to play any other equilibrium. If saving equilibria exist, the optimal equilibrium is unique and characterized by the highest value of π_y.*

Now, any saving equilibrium $\boldsymbol{\pi}$ can be characterized by some $i \in \{0, y, 2y, ..., (k-1)y\}$ such that $\pi_w = 1$ for $i < w < ky$, and $\pi_w < 1$ for $y \leq w \leq i$ (the agent saves up with certainty in all states above i).

To understand the results of Proposition 2, it is easiest to imagine how equilibrium changes as β gets smaller. For β values below 1, the agent behaves exactly like an exponential discounter (full-saving equilibrium) down to $\bar{\beta}$ (as defined in Equation A8).

Once β drops below $\bar{\beta}$, the full-saving strategy is no longer an equilibrium – the agent in state y is sufficiently present-biased that she prefers to save down today if she would save fully from tomorrow anyway. Below $\bar{\beta}$, to create incentives to save, the agent in state y must play $\pi_y < 1$. Then, she knows that if she does not save up today, there is a possibility that she will not even save tomorrow. By worsening the consequences of saving down today, she again becomes willing to save up. Extending this logic, as β drops further, there may be a region with equilibria where states y and $2y$ play a mixed strategy, followed by a region where states y, $2y$, and $3y$ play a mixed strategy, and so on. The optimal equilibrium will be characterized by higher levels of i as β drops. Ultimately, there will be some $\underline{\beta}$ such that there will be no saving at $\beta \leq \underline{\beta}$. At $\underline{\beta}$, the equilibrium has the property that $\pi_y = 0$ and in state y the agent is indifferent between saving up and down. An explicit lower bound on $\underline{\beta}$ is given by $\underline{\underline{\beta}}$ as defined in Equation A9.[15]

Let U_w refer to the discounted utility from the optimal equilibrium at wealth w. The proposition also shows that U_y drops continuously as β drops. Within any region that supports an equilibrium characterized by a particular i, U_y drops as β drops through two mechanisms: (a) the future is discounted more, and (b) future selves further slow down the rate of saving. When i gets bigger, the impact of mechanism (b) gets more pronounced. Therefore, as β drops (and i rises), U_y drops at increasing rates, down to $\underline{\beta}$. Below $\underline{\beta}$, mechanism (b) ceases to exist, so U_y no longer drops as sharply. This is formalized in statements 3 and 4 of the proposition, and depicted visually in Figure 2.

PROPOSITION 2: *Let* $\boldsymbol{\pi}(\beta)$ *denote the optimal equilibrium at any* β*. Then: (1)* $\pi_y(\beta)$ *is continuous and weakly increasing in* β*; (2) there are values* $0 < \underline{\beta} < \bar{\beta} < 1$ *such that* $\pi_y = 0$ *if* $\beta \leq \underline{\beta}$, $\pi_y \in (0,1)$ *if* $\beta \in \left(\underline{\beta}, \bar{\beta}\right)$*, and* $\pi_y = 1$ *if* $\beta \geq \bar{\beta}$*; (3)* U_y *is continuous and strictly increasing in* β*; (4)* U_y *is weakly concave for* $\beta \in \left(\underline{\beta}, 1\right)$*, and* $\frac{dU_y}{d\beta}$ *is strictly greater for* $\beta \in \left(\underline{\beta}, \bar{\beta}\right)$ *than for* $\beta \in \left(\bar{\beta}, 1\right)$.

While it might not be the case that the agent actually plays the optimal equilibrium, this serves as a reasonable benchmark, especially as it stacks the odds against rosca survival (the better the autarky option, the lower the agent's incentive to stay on in a rosca).

III. Roscas as Commitment Devices

A rosca is a group of k people (k as defined in Section II), with one cycle lasting k periods. The per-period contribution is y. This rosca can be either fixed or random. When agents are hyperbolic discounters, it is natural to think of roscas as effective commitment savings devices.

First, consider an agent in a fixed rosca, in the period after which she has received the nondivisible. Suppose she values the commitment provided by the rosca and chooses to

[15] $\underline{\underline{\beta}}$ is defined as the point below which the following is true: *even if* $\pi_w = 1$ for all $w \geq 2y$, there is no positive value of π_y such that the agent in state y will weakly prefer to save up.

stay. Then, she knows that in all future periods she will continue to stay. This is because, for every additional period that she participates in the rosca, more of her savings get locked in (they are consumed by someone else, so there is no way for her to access them). This argument can be similarly applied to the first-ranked agent in a random rosca. If she chooses to stay in the rosca in the second period, the lock-in property ensures that she will always choose to stay.

Second, even if roscas cannot punish members for defection, they can create an effective punishment mechanism by threatening to send defectors back to autarky. Since the hyperbolic agent views her future selves as saving too slowly in autarky, she might strictly prefer to stay in a rosca even when she is furthest away from her next nondivisible.

Third, a potential entrant into a rosca knows she may be an instant winner. This gives roscas a specific advantage over pure commitment savings devices and ensures that agents who value commitment will not postpone entry.

A. Sustainability

I first analyze a rosca that operates according to the following rule: any agent who leaves can never rejoin any rosca, but cannot be punished in any other way. This can be interpreted as a case where roscas can only form at some central location, which allows agents' past behavior to be monitored.[16] While this is admittedly a stylized environment, it is one in which roscas with exponential discounters cannot survive (and hence allows us to isolate the mechanism that generates different results for hyperbolic discounters).

FIXED ROSCAS. — For a rosca participant, the rosca is least attractive when she is $k-1$ periods from the next pot (for example, consider the last-ranked member in the first period of a cycle). The discounted utility from continued participation is $\frac{\beta\delta^{k-1}u(b)}{1-\delta^k}$, denoted U_F. If, in this period, she has a strict incentive to stay on in the rosca, it follows that she will always have a strict incentive to stay on. If the agent were an exponential discounter, she would lack this strict incentive since there are no social sanctions and she could replicate the rosca outcomes in autarky.[17]

Suppose the last-ranked agent is a hyperbolic discounter who is unable to achieve a full-saving equilibrium in autarky. She would ideally like to pause the rosca for one period (so she could consume her income today) and continue with rosca participation from tomorrow (full saving in the future maximizes her continuation utility). Since pausing the rosca is not an option, she faces a trade-off between autarky (high instantaneous utility, low future utility) and staying in the rosca (low instantaneous utility, high future utility). If she values the commitment provided by the rosca highly enough that she is willing to forego current consumption, she will strictly prefer to stay on.

[16] This assumption in relaxed in Section IV.
[17] Furthermore, if autarky involved a rising pattern of savings, she would strictly prefer to leave the rosca.

PROPOSITION 3: *Suppose a member of a fixed rosca knows that the other members will never defect. Then, she always strictly prefers to remain in the rosca iff* $\beta \in \left(\beta^*, \bar{\beta}\right)$*, where* $\bar{\beta}$ *is defined in Equation A8 and* β^* *lies in* $\left(0, \underline{\underline{\beta}}\right)$ *as defined in Equation 5 below.*

PROOF:
Let the optimal autarky equilibrium at any β be denoted $\boldsymbol{\pi}(\beta)$. Recall that the discounted utility from this equilibrium is denoted U_w. If the agent strictly prefers to stay in the rosca when she is furthest from the nondivisible, she will always strictly prefer to stay. For $\beta \geq \bar{\beta}$, $U_y = \frac{\beta\delta^{k-1}u(b)}{1-\delta^k} = U_F$, so a rosca is not strictly preferred. For $\beta \in [\underline{\beta}, \bar{\beta})$, $U_y = \beta\delta V(2y; \boldsymbol{\pi}(\beta))$ (since there is partial saving in autarky) and $\delta V(2y; \boldsymbol{\pi}(\beta)) < \frac{\delta^{k-1}u(b)}{1-\delta^k}$ (from Condition 1, since autarky does not involve full saving). This implies that $U_y < U_F$, so a rosca is strictly preferred. For $\beta < \underline{\beta}$, $U_y = u(y) + \beta\delta\frac{u(y)}{1-\delta}$, so a rosca will be strictly preferred iff $U_F > U_y$, or:

$$\beta > \beta^* \equiv \frac{u(y)}{\frac{\delta^{k-1}u(b)}{1-\delta^k} - \frac{\delta u(y)}{1-\delta}} \tag{5}$$

By Condition 1 and Equation A9, $\beta^* \in \left(0, \underline{\underline{\beta}}\right)$.

Note that roscas are sustainable even in regions with no autarky saving. In the appendix, I discuss how illiquidity and threat of expulsion contribute differentially to commitment across parameter regions.

RANDOM ROSCAS. — The only difference between a random and a fixed rosca is that, in a random rosca, the ordering is re-randomized at the start of each cycle. A random rosca is least attractive to the agent who receives the pot in period 1. In period 2, her expected discounted utility from continued participation is $\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k}$, denoted U_R. If, in this period, she stays in the rosca, she will always stay in since this is the furthest from the nondivisible she can ever be. Note that $U_R < U_F$ (because $\delta < 1$ and $k > 1$).

PROPOSITION 4: *Suppose a member of a random rosca knows that the other members will never defect. Then, if* δ *is sufficiently large, she always strictly prefers to remain in the rosca iff* $\beta \in \left(\beta^*_{ran}, \bar{\beta}_{ran}\right)$*, where* $\bar{\beta}_{ran} \in (0, \underline{\beta})$ *and* β^*_{ran} *is defined in Equation 6 below. Furthermore,* $\left(\beta^*_{ran}, \bar{\beta}_{ran}\right)$ *will be a strict subset of* $\left(\beta^*, \bar{\beta}\right)$*.*

PROOF:
Let the optimal autarky equilibrium at any β be denoted $\boldsymbol{\pi}(\beta)$. Note that U_R is linear over β, U_y is continuous over β, $U_y > U_R$ at $\beta = 0$ and $\beta \in [\bar{\beta}, 1)$, U_y is linear and increasing over $\beta \in \left(0, \underline{\beta}\right)$, and U_y is concave and increasing over $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$. Therefore, if $U_y < U_R$ at any β, then this inequality must be true for all $\beta \in \left(\beta^*_{ran}, \bar{\beta}_{ran}\right)$, where $\beta^*_{ran} \in (0, \underline{\beta})$ and $\bar{\beta}_{ran} \in (\underline{\beta}, \bar{\beta})$.

If such a β^*_{ran} exists, it must satisfy:

$$\frac{\beta^*_{ran}\delta^{k-1}u\left(b\right)}{\left(1-\delta\right)k}=u\left(y\right)+\frac{\beta^*_{ran}\delta u\left(y\right)}{1-\delta}\Leftrightarrow\beta^*_{ran}=\frac{\left(1-\delta\right)u\left(y\right)}{\delta^{k-1}u\left(b\right)-k\delta u\left(y\right)} \tag{6}$$

By Condition 1, $\beta^*_{ran}>0$ if δ is sufficiently high and $\lim_{\delta\to 1}\beta^*_{ran}=0$. Since $\underline{\underline{\beta}}\leq\underline{\beta}$ and $\lim_{\delta\to 1}\underline{\underline{\beta}}=\frac{u(y)}{u(b)-(k-1)u(y)}>0$, there must exist $\beta^*_{ran}\in(0,\underline{\beta})$ that satisfies Equation 6. This ensures that, for sufficiently high δ, there will be a region in which an agent always prefers to remain in a random rosca.

Finally, since $U_R<U_F$ for $\beta>0$, it must be true that $\left(\beta^*_{ran},\bar{\beta}_{ran}\right)\subset\left(\beta^*,\bar{\beta}\right)$.

The intuition for the above result is the following: as δ gets large, the agent does not mind the fact that her rank in the next cycle is uncertain (her expected utility converges to the expected utility of staying on in a fixed rosca). However, when δ is small, she cares about the fact that she has to wait particularly long for the next nondivisible. This limits the range of β values for which she will choose to remain in the rosca. Figures 1 and 2 provide a summary of the results.

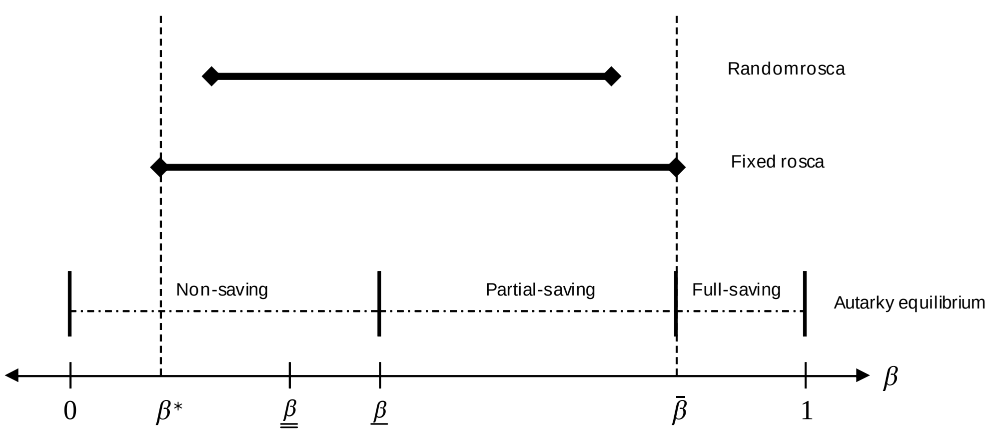

FIGURE 1. COMPARISON OF REGIONS THAT SUPPORT RANDOM AND FIXED ROSCAS

B. Entry and Welfare

A potential problem with commitment savings devices, especially when start dates cannot be contracted upon, is that the agent might have an incentive to postpone entry.[18]

[18]If the agent's welfare is measured as the discounted utility with $\beta=1$, suppose the welfare maximizing outcome is the one where she starts saving immediately.

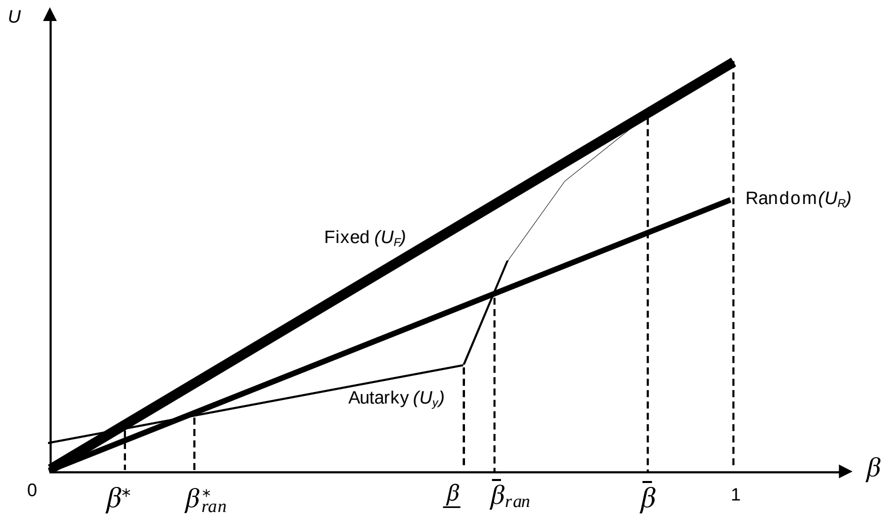

FIGURE 2. WORST-CASE DISCOUNTED UTILITIES FROM FIXED AND RANDOM ROSCAS, COMPARED TO AUTARKY

As O'Donoghue and Rabin (1999) and Basu (2009) show, when tasks are costly in the present and have delayed benefits, a hyperbolic discounter will procrastinate even though the welfare-maximizing outcome involves completing the task immediately. Since commitment saving typically entails saving today for future benefits, the agent might wait to join even if she values the commitment. This problem disappears with roscas because of the initial randomization in ranking. The possibility of getting the nondivisible in the current period ensures that the hyperbolic discounter will not want to postpone entry.[19]

Suppose there is no defection from a rosca once it forms. An agent faces identical expected values from joining a random or fixed rosca. In each case, she expects to get the nondivisible once every k periods. The expected value is $\frac{u(b)}{k}\left[1+\frac{\beta\delta}{1-\delta}\right]$.

PROPOSITION 5: *If an agent knows that she will always stay in a rosca once she joins, she will join in the first period of her life.*

PROOF:

Suppose an agent can join a rosca in any period, and knows she will remain in it forever once she enters. She will join in the first period of her life if she would rather not postpone by one period:

$$\frac{u(b)}{k}\left[1+\frac{\beta\delta}{1-\delta}\right] > u(y) + \frac{u(b)}{k}\left[\frac{\beta\delta}{1-\delta}\right] \Leftrightarrow \frac{u(b)}{u(y)} > k$$

[19]This result relies on the assumption that goods yield immediate benefits. However, even when goods are durable, agents will be less likely to postpone entry into roscas than into other commitment devices.

This condition does not depend on β and is always satisfied because of Condition 1. Therefore, hyperbolic discounters will not postpone entry into a rosca.

Since the individual has present-biased preferences, she is tempted by the possibility of an immediate reward in the same way that she is tempted to overconsume in the present. If the conditions for rosca sustainability are met, the expected value of the rosca is sufficiently high that she will choose to join immediately.

IV. Roscas as Decentralized Equilibria

Section III demonstrates that it is possible for a hyperbolic discounter to strictly prefer to remain in a rosca if reentry is banned. It follows that if all agents in a rosca satisfy the parameter conditions, then no member has an incentive to leave if others choose to stay. However, it is reasonable to ask whether permanent expulsion is actually feasible when social sanctions are not. Given the informational problems associated with enforcing expulsion, it is conceivable that new roscas are simply unable to learn about past actions of their members. This section loosens two assumptions. First, the available information about an agent's past rosca behavior is allowed to vary. This places a restriction on what rosca rules can be conditioned on. Second, new roscas can form in any period. The rate at which this happens is pinned down by an exogenous probability with which rosca aspirants get matched into groups of size k. In such settings, we might again expect rosca members to defect not just to play autarky but in search of higher ranks in new roscas.

The new assumptions generate two key results. First, we would like to know how roscas might survive if permanent expulsion is infeasible. Even under complete anonymity, roscas can survive if the exogenous probability of finding new roscas is sufficiently low. This follows directly from the commitment value for hyperbolic discounters. If all agents were exponential discounters, even an infinitesimal probability of finding a higher rank in a new rosca would create a strict incentive to leave.

Second, Section III does not provide a reason for random roscas to exist (the greatest incentive to defect from random roscas is stronger than for fixed roscas, while ex-ante utilities are the same). In this section, under limited information and fast matching, random roscas are more resilient than fixed roscas. Suppose we have "partial reputation" – agents who have left roscas in the middle of past cycles are marked as defectors, but agents who leave after completing a cycle are not (this is reasonable – we are most likely to remember those who owe us money). This limits the problem of defection within cycles, but leaves open the possibility that an agent might wish to leave after completing a cycle. Now, an advantage of random roscas becomes salient. Since the ordering is randomized at the start of each cycle, no agent has an incentive to leave a random rosca between cycles. On the other hand, the last-ranked member of a fixed rosca might still wish to leave at the end of a cycle, with her reputation intact and the chance to rejoin a new rosca with a higher expected rank. This gives us conditions under which random roscas are both more sustainable and more welfare-generating than fixed roscas.[20]

[20] This distinction between fixed and random roscas survives even if agents are exponential discounters. It should be

A. Assumptions

Consider three reputation environments. Under "anonymity", agents' past behavior is completely invisible. Under partial reputation, agent's histories become public to the extent that others know if they have ever defected from a rosca *during* a cycle. Finally, under "full reputation", an agent's entire past rosca behavior is publicly known (strategies can be conditioned on whether the agent has ever left a rosca). The better the reputation environment is, the easier it is for roscas to condition strategies on agents' past behavior, thus increasing the sustainability and benefits of roscas.

I assume a large, growing population of identical hyperbolic discounters. For analytical convenience, I restrict the agents to $\beta \in \left(\beta^*, \underline{\beta}\right)$. In this region, agents value a rosca and would not save in autarky (so their actions are limited to rosca-related decisions). Finally, I assume that in any period, an infinitesimal proportion of the population experiences a shock that leaves them unable to save from the current period onwards. For the purposes of this exercise, we can think of them as becoming fully myopic ($\beta = 0$).

TIMING AND STRATEGIES. — There are two rosca "pools" – the pool of agents looking for a new rosca (New), and the pool of agents who wish to fill an open slot in an existing rosca (Old). The timing of the game is as follows. Each period is divided into 5 subperiods:

a) Agents in existing roscas choose whether to stay (Y) or leave (N). Agents who are not in a rosca choose whether to move to Old (by default, they are in New).

b) Each existing rosca (defined as a rosca with at least one remaining member) with vacated slots makes rank-specific offers to agents in pool Old. In pool New, some proportion, p, of agents are randomly matched into groups of size k. (p is exogenously determined. This is an indicator of how easily people are able to form groups).

c) Agents accept (Y) or reject (N) offers of membership in roscas (in either pool).

d) New roscas randomly determine the ordering. Existing random roscas randomly determine the ordering if a new cycle is starting.

e) Agents in roscas decide whether to stay (Y) or leave (N).

If any agent leaves a rosca or rejects an offer, she can only re-enter the pool in the following period. Any rosca that is unable to fill its slots breaks up and agents re-enter the pool in the following period.

In any period, the following actions are available to agents: those in an existing rosca observe their state (profile of other members and distance to the next nondivisible) and

possible to extend the results of this section to settings without time-inconsistency (of course, we would then have to also introduce some exogenous punishment for defection).

must choose Y or N in sub-periods (a) and (e). Agents who are not in a rosca choose whether to move to Old in sub-period (a). If they receive an offer, they choose Y or N in sub-period (c). Finally, in sub-period (e), agents can again choose Y or N after learning their rank.

A rosca cycle starts in sub-period (d) of period 1 and continues for k periods. Then, for example, under partial reputation, if an agent leaves in sub-period (c) of cycle 1 (before the ordering is determined), she does not acquire a reputation as a defector.

A rosca strategy is a rule about how to choose members from pool Old for each possible opening in a rosca (all members are aware of their rosca strategy). An equilibrium is an action associated with each information set (for individuals) and a rosca strategy for each rosca configuration, such that no agent has an incentive to deviate from her strategy at any information set.

B. Outside Option

The problem of sustainability is directly affected by an agent's outside option, or the expected discounted utility from leaving a rosca in any period. When information is limited, an agent might know that she has a realistic chance of leaving a rosca in which she has a low rank, and re-entering one with a higher expected rank.

Assume all roscas survive forever. Consider an agent in a fixed rosca who is k periods away from the next nondivisible. Suppose she is free to leave the rosca and re-enter any other rosca starting in the next period. If she is certain to get a new rosca ($p = 1$), clearly she prefers to leave. We would like to find conditions under which she will not leave her rosca. The agent will have a strict incentive to stay if $p < p^*$, where p^* is defined by:

$$\frac{\beta\delta^{k-1}u(b)}{1-\delta^k} = u(y) + \beta\delta\left[\frac{p^*\left(\frac{u(b)}{k(1-\delta)}\right) + (1-p^*)\,u(y)}{1-(1-p^*)\,\delta}\right]$$

$$\Rightarrow \quad p^* = \frac{(1-\delta)\left(\frac{\beta\delta^{k-1}u(b)}{1-\delta^k}\right) - (1-\delta+\beta\delta)\,u(y)}{\frac{\beta\delta u(b)}{k(1-\delta)} - \frac{\beta\delta^k u(b)}{1-\delta^k} + (\delta-\beta\delta)\,u(y)} \tag{7}$$

Similarly, consider the first-ranked agent in a random rosca that survives forever. If, by leaving, she can find a new rosca with some probability p, she will only stay on in her current rosca for $p \le p^*_{ran}$, where p^*_{ran} is given by:

$$\beta\delta^{k-1}\left(\frac{u(b)}{k(1-\delta)}\right) = u(y) + \beta\delta\left[\frac{p^*_{ran}\left(\frac{u(b)}{k(1-\delta)}\right) + \left(1-p^*_{ran}\right)u(y)}{1-\left(1-p^*_{ran}\right)\delta}\right]$$

$$\Rightarrow \quad p^*_{ran} = \frac{(1-\delta)\left(\frac{\beta\delta^{k-1}u(b)}{k(1-\delta)}\right) - (1-\delta+\beta\delta)\,u(y)}{\frac{\beta\delta u(b)}{k(1-\delta)} - \frac{\beta\delta^k u(b)}{k(1-\delta)} + (\delta-\beta\delta)\,u(y)} \tag{8}$$

It follows that $p^*_{ran} < p^*$.

C. Fixed Roscas

Under anonymity, strategies cannot be conditioned on any aspect of an agent's past behavior. If there is an opening in a rosca, the rosca simply decides whether to make an arbitrary offer to a person in the pool. Similarly, an agent has no information about the other members' past rosca experience. If p is low enough, there is an equilibrium with roscas surviving over time. However, if p is high, such an equilibrium cannot exist. Every agent will have an incentive to leave her rosca immediately after receiving the nondivisible, as she can consume her income today and join a new rosca with a higher expected rank in the near future.

Under partial reputation, an agent is marked as a defector if she has left during any cycle, which is defined as starting after the ordering is randomized in period 1 and ending after period k (and continuing every k periods after that). Now, the problems associated with anonymity are alleviated to some extent. It is possible for agents to have strategies where they refuse to join roscas with defectors (believing that anyone who has done so is now a $\beta = 0$ type). However, such strategies cannot stop the last-ranked member of a fixed rosca from leaving at the end of a cycle. Since she cannot be distinguished from those who have never been in a rosca, she will have an incentive to join a new rosca if p is high enough.

Under full reputation, strategies can be conditioned on whether an agent has left a rosca in the past, and if so, at what stage of a cycle she left. Here, fixed roscas can be sustained even with $p = 1$ since it is possible to have equilibrium strategies that permanently expel defectors. These results are formalized in the proposition below.

PROPOSITION 6: *Consider p^* as defined in Equation 7. (1) Under anonymity, strategies in which agents always stay in fixed roscas constitute an equilibrium only when $p \leq p^*$. (2) Under partial reputation, strategies in which agents always stay in fixed roscas constitute an equilibrium only when $p \leq p^*$. (3) Under full reputation, strategies in which agents always stay in fixed roscas constitute an equilibrium at any p.*

PROOF:

(1) Suppose $p \leq p^*$. Consider the following strategy: Agents in roscas, or with rosca offers, always play Y; agents outside roscas always enter New; roscas with openings randomly make offers from Old. Then, by definition of p^*, the outside option from defection is sufficiently small that the agent who is $k - 1$ periods away from the next nondivisible prefers to remain in the rosca. Under this equilibrium, every group of k agents that forms a rosca will never separate.

Suppose $p > p^*$. By the definition of p^*, a rosca cannot survive forever in equilibrium: if all agents have a strategy of never leaving a rosca, then any individual who has just received a nondivisible has a strict incentive to deviate.

(2) Suppose $p \leq p^*$. Then, agents can play the same equilibrium strategy as under anonymity.

Suppose $p > p^*$. Roscas cannot last forever with the same membership. Since there is no strategy that can be conditioned on whether an agent left at the end of a cycle, the last ranked player has a strict incentive to leave after a cycle if she knows that, in the future, all agents will stay in a rosca forever.

(3) Consider the following beliefs: Any agent who has ever left a past rosca is now a $\beta = 0$ type. Consider the following strategies: a rosca with openings randomly makes offers to anyone who has never left a rosca before; agents outside roscas remain in New; agents in roscas or with rosca offers always play Y unless there is a $\beta = 0$ type in the group. Since any agent who leaves a rosca must play autarky forever, no agent will leave. The strategies described are an equilibrium, and the beliefs are justified.

D. Random Roscas

Following the arguments above, under full reputation and anonymity, the set of parameter values (β, p) at which there is no defection from random roscas is a subset of the values at which there is no defection from fixed roscas. Under full reputation, both random and fixed roscas will survive at any p, but this happens for a larger set of β-values in the case of fixed roscas. Under anonymity, since $p^*_{ran} < p^*$ and $\beta^*_{ran} > \beta^*$, fixed roscas will survive over both a larger set of β-values and a larger set of matching probabilities.

However, with partial reputation, when p is high, random roscas can survive forever even when fixed roscas cannot. The intuition for this result is the following. When $p > p^*$, there cannot be a fixed rosca in which all agents stay forever, because the last ranked member would have an incentive to leave. However, there is no such incentive to leave a random rosca between cycles. Therefore, if β is high enough so that agents *within* the rosca prefer to stay rather than play autarky, then random roscas will survive forever, and any agent's expected value from a rosca will be higher than it would be under fixed roscas (see Figure 3).

PROPOSITION 7: *Assume partial reputation. If $\beta > \beta^*_{ran}$ (as defined in Equation 6), then random roscas can survive forever in equilibrium. If, in addition, $p > p^*$, then random roscas are strictly welfare improving relative to fixed roscas.*

PROOF:

Consider the following beliefs: any agent who has left a rosca during a past cycle has $\beta = 0$. Consider the following strategies: a rosca with openings randomly makes offers to anyone who is not marked as a defector; individuals outside roscas enter New; agents in roscas or with rosca offers play Y unless any members of the group are marked as defectors. Since $\beta > \beta^*_{ran}$ (as assumed), the first-ranked member will always prefer to remain in the rosca rather than defect mid-cycle. Therefore, these beliefs and strategies will constitute an equilibrium in which roscas survive forever.

Now suppose $p > p^*$. Then, the expected value for any agent entering a random rosca is: $\frac{u(b)}{k}\left(1 + \frac{\beta\delta}{1-\delta}\right)$. Since there is no equilibrium in which fixed roscas last forever, the expected value from fixed roscas must be lower.

These results are discussed further in Section V.

V. Empirical Implications

I now study how the results in previous sections can be related to our empirical understanding of roscas. My focus is on implications that directly relate to quasi-hyperbolic discounting, including comparative statics generated by the model and predictions about the survival of random and fixed roscas

If hyperbolic discounting is indeed a primary explanation of rosca participation, then this model predicts that members of long-lasting roscas will exhibit intermediate levels of time-inconsistency and be aware of their preferences.[21] The actual size of the β region within which roscas survive depends on several parameter values. Consider the boundaries of this region as δ gets high: $\lim_{\delta \to 1} \bar{\beta} = k\frac{u(y)}{u(b)}$ and $\lim_{\delta \to 1} \beta^* = \lim_{\delta \to 1} \beta^*_{ran} = 0$. All else equal, roscas are more likely to survive as the nondivisible gets more expensive relative to income. Also, roscas are more likely to survive as the utility from small units of consumption increases relative to the utility from the nondivisible (up to the point where the nondivisible is no longer valued even by exponential discounters).

In the previous section, we have seen how it is possible for roscas to survive in decentralized settings. The defection incentive is created by the option value of a higher rank in a new rosca. In environments that are completely anonymous, fixed roscas can survive forever only if matching for new roscas is sufficiently low. When there is some reputation, fixed roscas again survive when matching is sufficiently slow, but random roscas can be welfare-improving when matching probabilities are high. When roscas can access more information about an agent's past rosca behavior, equilibria with repeating fixed roscas can exist even under perfect matching (see Figure 3).

This gives us some testable predictions. If we conjecture that the likelihood of matching is positively correlated with population density, then p rises as communities get urbanized. When reputation is informal, the availability of information is likely to be inversely correlated with urbanization (full reputation is a feature of very small rural communities, while urban areas are more anonymous). Then, the model predicts that fixed roscas are more likely than random roscas at fully rural and fully urban extremes. In semi-rural communities, we are more likely to encounter conditions suited to the survival of random roscas.[22]

The Anderson & Baland (2002) study, set in an urban neighborhood, finds that a majority of roscas are fixed. The Gugerty (2007) study, set in rural Kenya, finds that a majority of roscas are random. These patterns appear consistent with the above predictions, but we would require more information for a comprehensive analysis.

In this model, the nondivisible good yields a one-period benefit. This is not an unreasonable assumption, since there is a range of empirical evidence suggesting that rosca members do not spend the money on durable goods. However, it is useful to identify potential implications for rosca survival if agents save for durable goods instead. When

[21] Gugerty (2007) and Dagnelie & LeMay-Boucher (2010) provide some evidence in support of this. Tomomi Tanaka and Quang Nguyen (2010) find that participants in fixed roscas (relative to bidding roscas) are less present-biased but more sophisticated about their preferences.

[22] Anderson, Baland, and Moene (2009) make predictions about participation in random and fixed roscas under individual heterogeneity. An approach that integrates their predictions with mine is probably feasible and useful.

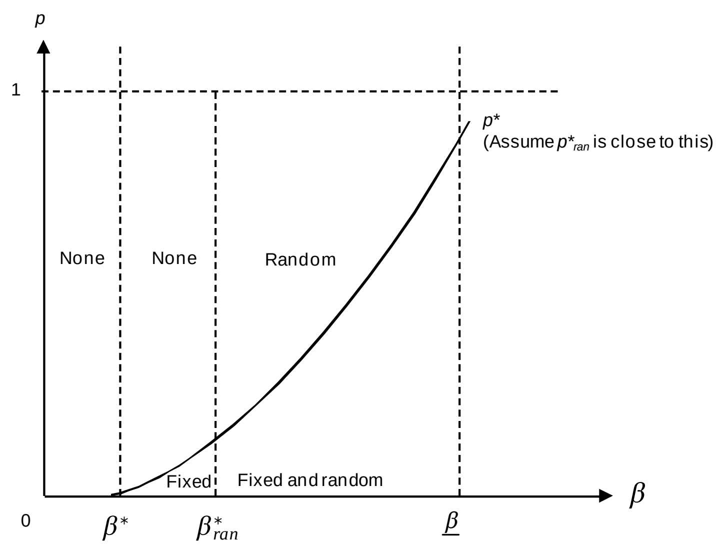

FIGURE 3. PARTIAL REPUTATION: TYPES OF ROSCAS (IF ANY) THAT SURVIVE INDEFINITELY IN DIFFERENT PARAMETER REGIONS

benefits are spread across multiple periods, agents might place less value on immediate consumption. If this is the case, agents may choose to delay entry into roscas[23] but also be less tempted to defect from one in search of higher ranks elsewhere.

VI. Conclusion

This paper shows that roscas can be effective commitment savings devices even without the threat of social sanctions. Agents with self-control problems derive benefits from staying in a rosca and improving their savings behavior. The randomization of rank and subsequent commitment play complementary roles – the first draws an agent into a rosca, and the second gives her a reason to stay. I also highlight the relative advantages of fixed and random roscas. Within a cycle, agents in random roscas have a weaker incentive to stay than they would in fixed roscas. However, between cycles, agents in random roscas never have an incentive to leave, while late ranked agents in fixed roscas might prefer to leave if they can join a new rosca. Empirically, both random and fixed roscas exist in large numbers. This paper makes predictions about the survival of each, based on the depth of reputation and the speed of matching.

There are several directions for further research. Models that allow for heterogenous populations (with varying time preferences and saving objectives) are likely to alter the

[23]O'Donoghue & Rabin (1999) provide one explanation for delayed entry.

size and structure of roscas and conditions under which they survive in the absence of contracting. Additional insights might also be gained from endogenizing the matching speed, p. Also, there is scope for testing the predictions of the model, especially if convincing measures of time-inconsistency, reputation, and matching speed can be constructed.

It is instructive to conclude with a brief discussion of other institutions that could replicate the wealth-pooling and commitment properties of roscas. Conceptually, the most natural substitute would be a simple lottery that replicates the deposit pattern and winning odds of a rosca. Indeed, such an institution has its advantages – by making each player a potential winner in every period, the question of enforcement is made irrelevant. *Numbers gambling*, a long-standing practice in American cities, is one such example.[24] In this game, individuals place small bets on three-digit numbers, after which the winning number is selected at random. Light (1977), despite concluding that roscas are more welfare-enhancing than numbers gambling, shows that a common justification of gambling echoes the rationalization of roscas as commitment devices.[25]

Nevertheless, there are reasons to believe that the expected value of a rosca is higher than that of gambling. First, it could be argued that agents are better informed about their rosca partners than about a gambling organizer. Hyperbolic discounters who know that their rosca partners share their preferences will be more confident about being repaid than if they were to sink equivalent sums into anonymous gambles. Second, organizational costs associated with gambling operations imply that participants make losses in expectation (according to Light 1977, middlemen charge a commission of 10 percent). Third, agents might value the fact that a rosca never delivers two pots too close together or too far apart. This is beyond the scope of my model, but consistent with the idea that participants save for recurring expenses like school uniforms and household repairs.

It should be noted that the question of lotteries does not apply only to roscas as described in this paper. On the contrary, the relative advantages of a lottery become more salient when roscas consist of exponential discounters, since these roscas require stronger punishment mechanisms to survive.

Given the resilience of roscas across the world, the idea of viewing them as commitment devices in environments with poor contracting is potentially useful and informative.

REFERENCES

Ambec, Stefan, and Nicolas Treich. 2007. "Roscas as Financial Agreements to Cope with Self-Control Problems." *Journal of Development Economics*, 82(1): 120-137.

Anderson, Siwan, and Jean-Marie Baland. 2002. "The Economics of Roscas and Intrahousehold Resource Allocation." *Quarterly Journal of Economics*, 117(3): 963-995.

[24] Lawrence J. Kaplan and James M. Maher (1970) and Light (1977) describe the institution in detail.

[25] "Once a numbers collector has a man's quarter, . . . there is no getting it back in a moment of weakness. If, on the other hand, the quarter were stashed at home, a saver would have to live with the continuing clamor of unmet needs. In a moment of weakness, he might spend the quarter." (page 896)

Anderson, Siwan, Jean-Marie Baland, and Karl O. Moene. 2009. "Enforcement in Informal Saving Groups." *Journal of Development Economics*, 90(1): 14-23.

Ashraf, Nava, Nathalie Gons, Dean S. Karlan, and Wesley Yin. 2003. "A Review of Commitment Savings Products in Developing Countries." Asian Development Bank Economics and Research Department Working Paper #45.

Ashraf, Nava, Dean S. Karlan, and Wesley Yin. 2006. "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines." *Quarterly Journal of Economics*, 121(2): 635-672.

Banerjee, Abhijit V., and Esther Duflo. 2010. "Giving Credit Where it is Due." *Journal of Economic Perspectives*, 24(3): 61-80.

Basu, Karna. 2009. "Renegotiating with One's Self." http://urban.hunter.cuny.edu/~basu/reconsider.pdf.

Basu, Karna. 2010. "The Provision of Commitment in Informal Banking Markets: Implications for Takeup and Welfare." http://urban.hunter.cuny.edu/~basu/informalbanking.pdf.

Bauer, Michal, Julie Chytilova, and Jonathan Morduch. 2009. "Behavioral Foundations of Microcredit: Experimental and Survey Evidence From Rural India." http://www.nyu.edu/projects/morduch/documents/2009-12-Behavioral-Foundations-of-Microcredit.pdf.

Bertrand, Marianne, Dean S. Karlan, Sendhil Mullainathan, Eldar Shafir, and Jonathan Zinman. 2009. "What's Advertising Content Worth? Evidence from a Consumer Credit Marketing Field Experiment." Yale University Economic Growth Center Discussion Paper 968.

Bertrand, Marianne, Sendhil Mullainathan, and Eldar Shafir. 2004. "A Behavioral Economics View of Poverty." *American Economic Review Papers and Proceedings*, 94(2): 419-423.

Besley, Timothy, and Stephen Coate. 1995. "Group Lending, Repayment Incentives, and Social Collateral." *Journal of Development Economics*, 46(1): 1-18.

Besley, Timothy, Stephen Coate, and Glenn Loury. 1993. "The Economics of Rotating Savings and Credit Associations." *American Economic Review*, 83(4): 792-810.

Besley, Timothy, Stephen Coate, and Glenn Loury. 1994. "Rotating Savings and Credit Associations, Credit Markets and Efficiency." *Review of Economic Studies*, 61(4): 701-719.

Besley, Timothy, and Alec R. Levenson. 1996. "The Role of Informal Finance in Household Capital Accumulation: Evidence from Taiwan." *The Economic Journal*, 106(434): 39-59.

Bond, Philip, and Gustav Sigurdsson. 2009. "Commitment Contracts." http://finance.wharton.upenn.edu/~pbond/research/CommitmentContracts20091208.pdf.

Bond, Philip, and Ashok S. Rai. 2009. "Borrower Runs." *Journal of Development Economics*, 88: 185-191.

Bouman, F. J. A. 1994. "ROSCA and ASCRA: Beyond the Financial Landscape." In

Financial Landscapes Reconstructed: The Fine Art of Mapping Development, ed. F.J.A. Bouman and Otto Hospes, 375-394. Boulder, CO: Westview Press.

Bouman, F. J. A. 1995. "Rotating and Accumulating Savings and Credit Associations: A Development Perspective." *World Development*, 23(3): 371-384.

Brink, Rogier van den, and Jean-Paul Chavas. 1997. "The Microeconomics of an Indigenous African Institution: The Rotating Savings and Credit Association." *Economic Development and Cultural Change*, 45(4): 745-772.

Bryan, Gharad, Dean Karlan, and Scott Nelson. 2010. "Commitment Devices." *Annual Review of Economics*, 2: 671-698.

Calomiris, Charles W., and Indira Rajaraman. 1998. "The Role of ROSCAS: Lumpy Durables or Event Insurance?" *Journal of Development Economics*, 56(1): 207-216.

Conning, Jonathan, and Christopher Udry. 2005. "Rural Financial Markets in Developing Countries." Yale University Economic Growth Center Discussion Paper 914.

Dagnelie, Olivier, and Philippe LeMay-Boucher. 2010. "Rosca Participation in Benin: a Commitment Issue." Forthcoming. *Oxford Bulletin of Economics and Statistics*.

Dagnelie, Olivier. 2009. "Life and Death of Roscas: Leadership, Election and Screening." http://dagnelie.iae-csic.org/papers/life_death_roscas_dagnelie_apr09.pdf.

Frederick, Shane, George Loewenstein, and Ted O'Donoghue. 2002. "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature*, 40(2): 351-401.

Fudenberg, Drew, and Jean Tirole. 1991. *Game Theory*. Cambridge, MA: MIT Press.

Ghatak, Maitreesh. 1999. "Group lending, local information and peer selection." *Journal of Development Economics*, 60: 27-50.

Gugerty, Mary K. 2007. "You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya." *Economic Development and Cultural Change*, 55(2): 251-282.

Handa, Sudhanshu, and Claremont Kirton. 1999. "The Economics of Rotating Savings and Credit Associations: Evidence from the Jamaican 'Partner'." *Journal of Development Economics*, 60(1): 173-194.

Harris, Christopher, and David Laibson. 2001. "Dynamic Choices of Hyperbolic Consumers." *Econometrica*, 69(4): 935-957.

Kaplan, Lawrence J., and James M. Maher. 1970. "The Economics of the Numbers Game." *American Journal of Economics and Sociology*, 29(4): 391-408.

Karlan, Dean S. 2007. "Social Connections and Group Banking." *Economic Journal*, 117(517): 52-84.

Kimuyu, Peter K. 1999. "Rotating Saving and Credit Associations in Rural East Africa." *World Development*, 27(7): 1299-1308.

Kirby, Kris N., and Nino N. Marakovic. 1995. "Modeling Myopic Decisions: Evidence for Hyperbolic Delay-Discounting within Subjects and Amounts." *Organiza-*

tional Behavior and Human Decision Processes, 64(1): 22-30.

Klonner, Stefan. 2003. "Rotating Savings and Credit Associations when Participants are Risk Averse." *International Economic Review*, 44(3): 979-1005.

Klonner, Stefan, and Ashok Rai. 2007. "Adverse Selection in Credit Markets: Evidence from a Policy Experiment." http://www.williams.edu/Economics/rai/adverse-klonner-rai.pdf.

Krusell, Per, and Anthony A. Smith Jr. 2003. "Consumption-Savings Decisions with Quasi-Geometric Discounting." *Econometrica*, 71(1): 365-375.

Krusell, Per, and Anthony A. Smith Jr. 2008. "Consumption-Savings Decisions with Quasi-Geometric Discounting: The Case with a Discrete Domain." http://www.econ.yale.edu/smith/discretedomain.pdf.

Laibson, David. 1997. "Golden Eggs and Hyperbolic Discounting." *Quarterly Journal of Economics*, 112(2): 443-477.

Levenson, Alec R., and Timothy Besley. 1996. "The Anatomy of an Informal Financial Market: Rosca Participation in Taiwan." *Journal of Development Economics*, 51(1): 45-68.

Light, Ivan. 1972. *Ethnic Enterprise in America*. Berkeley, CA: University of California Press.

Light, Ivan. 1977. "Numbers Gambling Among Blacks: A Financial Institution." *American Sociological Review*, 42(6): 892-904.

Ligon, Ethan, Jonathan P. Thomas, and Tim Worrall. 2002. "Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies." *Review of Economic Studies*, 69(1): 209-244.

Mullainathan, Sendhil. 2004. "Development Economics through the Lens of Psychology." http://www.yjs.fi/tiedotefiles/Mullainathan.pdf.

O'Donoghue, Ted, and Matthew Rabin. 1999. "Doing it Now or Later." *American Economic Review*, 89(1): 103-124.

Phelps, Edmund S., and Robert A. Pollack. 1968. "On Second-Best National Saving and Game-Equilibrium Growth." *Review of Economic Studies*, 35(2): 185-199.

Tanaka, Tomomi, Colin F. Camerer and Quang Nguyen. 2010. "Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam." *American Economic Review*, 100(1): 557-571.

Tanaka, Tomomi, and Quang Nguyen. 2010. "Committing the Uncommitted: Rosca as a Saving Commitment Device for Sophisticated Hyperbolic Discounters." http://www.public.asu.edu/~ttanaka1/ROSCA.pdf.

APPENDIX

A1. Proofs for Section II

Equations 3 and 4 tell us that $V(w;\boldsymbol{\pi}) \leq \delta V(w+y;\boldsymbol{\pi})$. Combining Equations 2 and 4, we get the following inequalities:

$$V(w;\boldsymbol{\pi}) \geq \frac{u(y)}{1-\delta}\left[\pi_w\left(\frac{1}{\beta}-1\right)+1\right] \tag{A1}$$

$$V(w+y;\boldsymbol{\pi}) \geq \frac{u(y)}{1-\delta}\left[\pi_w\left(\frac{1}{\beta}-1\right)+1\right]+\frac{u(y)}{\beta\delta} \tag{A2}$$

Furthermore, if $\pi_w < 1$, we can combine Equations 3 and 4 to explicitly solve for $V(w;\boldsymbol{\pi})$ and $V(w+y;\boldsymbol{\pi})$ in terms of π_w:

$$V(w;\boldsymbol{\pi}) = \frac{u(y)}{1-\delta}\left[\pi_w\left(\frac{1}{\beta}-1\right)+1\right] \tag{A3}$$

$$V(w+y;\boldsymbol{\pi}) = \frac{u(y)}{1-\delta}\left[\pi_w\left(\frac{1}{\beta}-1\right)+1\right]+\frac{u(y)}{\beta\delta} \tag{A4}$$

PROOF OF LEMMA 1:

In a saving equilibrium, $\pi_y(y) > 0$. I show that, if $\pi_w(s) > 0$, then for all $w' > w$ and for all $s' < s$, $\pi_{w'}(s') = 0$. $\pi_w(s) > 0 \Rightarrow u(w-s)+\beta\delta V(s+y;\boldsymbol{\pi}) \geq u(w-s') + \beta\delta V(s'+y;\boldsymbol{\pi})$. Assume there exist $w' > w$ and $s' < s$ such that $\pi_{w'}(s') > 0$. Then: $u(w'-s')+\beta\delta V(s'+y;\boldsymbol{\pi}) \geq u(w'-s)+\beta\delta V(s+y;\boldsymbol{\pi})$. Combining these two inequalities, we get: $u(w'-s')-u(w'-s) \geq u(w-s')-u(w-s)$. This violates strict concavity of u. Therefore, the assumption must be false.

PROOF OF LEMMA 2:

Suppose there is some $\pi_w < 1$. From Equation A3, $V(w;\boldsymbol{\pi}) = \frac{u(y)}{1-\delta}\left[\pi_w\left(\frac{1}{\beta}-1\right)+1\right]$. From Equation A2, $V(w;\boldsymbol{\pi}) \geq \frac{u(y)}{1-\delta}\left[\pi_{w-y}\left(\frac{1}{\beta}-1\right)+1\right]+\frac{u(y)}{\beta\delta}$. Therefore, $\pi_{w-y} < \pi_w$.

PROOF OF LEMMA 3:

At some w, $\pi_w > \mu_w$. By applying Equation A2 to π_w and Equation A4 to μ_w, we know $V(w+y;\boldsymbol{\pi}) > V(w+y;\boldsymbol{\mu})$ and $V(w;\boldsymbol{\pi}) > V(w;\boldsymbol{\mu})$. Now assume $\pi_{w+y} \leq \mu_{w+y}$ and $\pi_{w+y} < 1$. By applying Equation A3 to π_{w+y} and Equation A1 to μ_{w+y}, we get $V(w+y;\boldsymbol{\pi}) \leq V(w+y;\boldsymbol{\mu})$. Contradiction. Also, since $\pi_w > \mu_w$, by applying Equation A1 to π_w and Equation A3 to μ_w, we know $V(w;\boldsymbol{\pi}) > V(w;\boldsymbol{\mu})$. Now assume $\pi_{w-y} \leq \mu_{w-y}$. By Lemma 2, $\mu_{w-y} < 1$. Applying Equation A2 to μ_{w-y} and Equation A4 to π_{w-y}, we get $V(w;\boldsymbol{\pi}) \leq V(w;\boldsymbol{\mu})$. Contradiction. Therefore, if $\pi_w > \mu_w$, then for all w', $\pi_{w'} > \mu_{w'}$ or $\pi_{w'} = \mu_{w'} = 1$.

PROOF OF LEMMA 4:

This can be proved recursively. Suppose $\boldsymbol{\pi}$ is a saving equilibrium and for all $w < \bar{w}$, $V(w,\boldsymbol{\pi}) > V(w;\boldsymbol{\mu})$. Then, by Lemma 1 and Equation 2, for all $w < \bar{w}$, $V(\bar{w};\boldsymbol{\pi}) \geq \frac{u(\bar{w}-w)}{\beta\delta}+V(w;\boldsymbol{\pi}) > \frac{u(\bar{w}-w)}{\beta\delta}+V(w;\boldsymbol{\mu})$. Now assume $V(\bar{w};\boldsymbol{\mu}) \geq V(\bar{w};\boldsymbol{\pi})$. Then, for all $w < \bar{w}$, $V(\bar{w};\boldsymbol{\mu}) > \frac{u(\bar{w}-w)}{\beta\delta}+V(w;\boldsymbol{\mu})$. Then, $\mu_{\bar{w}-y}(\bar{w}-y) = 1$, so $V(\bar{w}-y;\boldsymbol{\mu}) =$

$\delta V(\bar{w};\boldsymbol{\mu}) \geq \delta V(\bar{w};\boldsymbol{\pi}) \geq V(\bar{w}-y;\boldsymbol{\mu})$. This contradicts the initial assumption that $V(\bar{w}-y;\boldsymbol{\pi}) > V(\bar{w}-y;\boldsymbol{\mu})$. Therefore, it must also be the case that $V(\bar{w};\boldsymbol{\pi}) > V(\bar{w};\boldsymbol{\mu})$.

PROOF OF PROPOSITION 1:

At least one Markov Perfect Equilibrium exists (Fudenberg and Tirole, 1991). Next, I show that there is a complete and transitive ordering across all equilibria. Consider any two equilibria, $\boldsymbol{\pi}$ and $\boldsymbol{\mu}$. If $\pi_y = \mu_y = 0$, then there is no saving in either equilibrium, so the agent is indifferent between the two. If $\pi_y = \mu_y > 0$, then it follows from Lemma 3 that they are identical equilibria.

If $\pi_y > \mu_y$, then from Equation A1, we know $V(y;\boldsymbol{\pi}) \geq \frac{u(y)}{1-\delta}\left[\pi_y\left(\frac{1}{\beta}-1\right)+1\right]$. Since $\mu_y < 1$, we know $\beta\delta V(2y;\boldsymbol{\mu}) \geq u(y)+\beta\delta V(y;\boldsymbol{\mu})$. Combining this with Equation 4, we get $V(y;\boldsymbol{\mu}) \leq \frac{u(y)}{1-\delta}\left[\mu_y\left(\frac{1}{\beta}-1\right)+1\right]$. So, $V(y;\boldsymbol{\pi}) > V(y;\boldsymbol{\mu})$. By Lemma 4, $V(w;\boldsymbol{\pi}) > V(w;\boldsymbol{\mu})$ for all w. Therefore, equilibrium $\boldsymbol{\pi}$ will be strictly preferred in all states.

Using Equation 4 (and since $V(ky;\boldsymbol{\pi}) = u(b)+\delta V(y;\boldsymbol{\pi})$), we know that for $w > i$:

$$V(w;\boldsymbol{\pi}) = \delta^{k-\frac{w}{y}}u(b) + \delta^{k-\frac{w}{y}+1}V(y;\boldsymbol{\pi}) \tag{A5}$$

Applying Equation 3 recursively to each $w \leq i$, and combining with Equation A5, we get the following for each $w \leq i$:

$$V(w;\boldsymbol{\pi}) = \frac{\delta^{k-\frac{i}{y}-1}}{1-\delta^{k-\frac{i}{y}}}u(b) - \left(\frac{\frac{i}{y}}{1-\delta^{k-\frac{i}{y}}}+1-\frac{w}{y}\right)\left(\frac{u(y)}{\beta\delta}\right) \tag{A6}$$

Combining Equations A3 and A6, we get an explicit solution for an equilibrium characterized by i:

$$\pi_w = \begin{cases} \frac{\beta(1-\delta)}{1-\beta}\left[\frac{\delta^{k-\frac{i}{y}-1}}{1-\delta^{k-\frac{i}{y}}}\left(\frac{u(b)}{u(y)}\right) - \left(\frac{\frac{i}{y}}{1-\delta^{k-\frac{i}{y}}}+1-\frac{w}{y}\right)\left(\frac{1}{\beta\delta}\right) - \frac{1}{1-\delta}\right], & \text{if } w \leq i \\ 1, & \text{if } w > i \end{cases} \tag{A7}$$

Note that, if there is a β interval in which an equilibrium with i exists everywhere, then π_w and $V(w;\boldsymbol{\pi})$ are continuous and increasing in β. This is because $w \leq i$, so:

$$\frac{\frac{i}{y}}{1-\delta^{k-i}}+1-\frac{w}{y} > 0$$

Also, note the following: for $w > i$, $V(w;\boldsymbol{\pi})$ is strictly convex in w. Therefore, in any equilibrium with i, since $\beta\delta V(i+2y;\boldsymbol{\pi}) \geq \beta\delta V(i+y;\boldsymbol{\pi})+u(y)$, for $w > i+y$ it must be true that $\beta\delta V(w+y;\boldsymbol{\pi}) > \beta\delta V(w;\boldsymbol{\pi})+u(y)$. In other words, for all $w \leq i$, the agent is indifferent about saving up; for $w = i$, the agent is either indifferent or strictly prefers to save up; and for $w > i$, the agent strictly prefers to save up.

PROOF OF PROPOSITION 2:

(1)(a) Consider some $\hat{\beta}$ at which there is an optimal saving equilibrium $\boldsymbol{\pi}\left(\hat{\beta}\right)$. Then, at any $\tilde{\beta} > \hat{\beta}$, it must be true that at each w, $\tilde{\beta}\delta V\left(w + y; \boldsymbol{\pi}\left(\hat{\beta}\right)\right) > u(y) + \tilde{\beta}\delta V\left(w; \boldsymbol{\pi}\left(\hat{\beta}\right)\right)$. Since (by construction) each $V(w; \boldsymbol{\pi})$ is continuous in $\boldsymbol{\pi}$ and each π_x is bounded above by 1, there must be an equilibrium $\boldsymbol{\pi}\left(\tilde{\beta}\right) \geq \boldsymbol{\pi}\left(\hat{\beta}\right)$. By Proposition 1, this will be strictly preferred to any equilibrium with $\pi_y < \tilde{\pi}_y$. Therefore, whenever a saving equilibrium exists, the optimal equilibrium strategy is weakly increasing in β.

(b) Consider some $\tilde{\beta}$ such that $\pi_y\left(\tilde{\beta}\right) > 0$. Then, the equilibrium is associated with some $\tilde{\imath}$. I prove left-continuity of $\boldsymbol{\pi}$ at $\tilde{\beta}$. Suppose, under this equilibrium, $\beta\delta V(\tilde{\imath} + 2y; \boldsymbol{\pi}) > \beta\delta V(\tilde{\imath} + y; \boldsymbol{\pi}) + u(y)$. In other words, whenever there is full-saving, this is strictly preferred. From (1)(a), we know that for $\beta < \tilde{\beta}$, the optimal equilibrium must satisfy $\boldsymbol{\pi}(\beta) \leq \boldsymbol{\pi}\left(\tilde{\beta}\right)$. Then, the best possible equilibrium will be characterized by i. Consider a strategy where $\boldsymbol{\pi}(\beta)$ is determined by Equation A7. Since $V(w; \boldsymbol{\pi})$ is always continuous in $\boldsymbol{\pi}$ and each π_w is continuous in β, there must be an interval $\left[\hat{\beta}, \tilde{\beta}\right]$ in which this strategy is an equilibrium (in this interval, at $w > \tilde{\imath}$ the individual strictly prefers to save, and at $w \leq \tilde{\imath}$, Equation A7 yields each $\pi_w > 0$).

Suppose, under this equilibrium, $\beta\delta V(\tilde{\imath} + 2y; \boldsymbol{\pi}) = \beta\delta V(\tilde{\imath} + y; \boldsymbol{\pi}) + u(y)$. Then, this strategy can identically be characterized by $j = \tilde{\imath} + 1$, such that $\beta\delta V(j + 2y; \boldsymbol{\pi}) > \beta\delta V(j + y; \boldsymbol{\pi}) + u(y)$. Repeating the argument above, there must be an interval $\left[\hat{\beta}, \tilde{\beta}\right]$ in which a strategy satisfying Equation A7 for j exists and is an equilibrium.

(c) Consider some $\tilde{\beta}$ such that $\pi_y\left(\tilde{\beta}\right) > 0$. Then, the equilibrium is associated with some $\tilde{\imath}$. I prove right-continuity of $\boldsymbol{\pi}$ at $\tilde{\beta}$. Let j be the highest value of i that characterizes the optimal equilibria at $\beta > \tilde{\beta}$. Assume $j < \tilde{\imath}$. Consider some $\hat{\beta} > \tilde{\beta}$ at which a j-equilibrium is optimal. Then, a j-equilibrium exists for all β in $(\tilde{\beta}, \hat{\beta}]$. Then $\lim_{\beta \to \tilde{\beta}^+} \boldsymbol{\pi}(\beta) = \boldsymbol{\alpha} > 0$. Then, at $\tilde{\beta}$, there must exist a j-equilibrium with $\boldsymbol{\pi}\left(\tilde{\beta}\right) = \boldsymbol{\alpha}$. Then, the i-equilibrium could not have been optimal. Contradiction. Therefore, there must be an interval $\left[\tilde{\beta}, \beta'\right]$ in which the optimal equilibrium is characterized by $\tilde{\imath}$. By Equation A7, $\boldsymbol{\pi}(\beta)$ is continuous in β in this interval.

(d) Consider some $\tilde{\beta}$ such that $\pi_y\left(\tilde{\beta}\right) = 0$ (the optimal equilibrium is non-saving). I prove continuity of $\boldsymbol{\pi}$ at $\tilde{\beta}$.

By part (a), left-continuity of π_y is automatically satisfied. If there is an interval to the right of $\tilde{\beta}$ where the optimal equilibrium is non-saving, then right-continuity is satisfied. If not let j be the highest value of i that characterizes the optimal equilibria at $\beta > \tilde{\beta}$. Consider some $\hat{\beta} > \tilde{\beta}$ at which a j-equilibrium is optimal. Then, a j-equilibrium exists for all β in $(\tilde{\beta}, \hat{\beta}]$. Let $\lim_{\beta \to \tilde{\beta}^+} \pi_y(\beta) = \alpha$. Assume $\alpha > 0$. Then, at $\tilde{\beta}$, there must exist a j-equilibrium with $\pi_y\left(\tilde{\beta}\right) = \alpha$. Then, the non-saving equilibrium is not optimal. Contradiction. Therefore, our assumption is wrong, and $\alpha = 0$. This proves continuity of $\pi_y(\beta)$ at any $\tilde{\beta}$ where the optimal equilibrium is non-saving.

(2) (a) Define the full-saving strategy as $\boldsymbol{\mu} \equiv (1, 1, 1, ..., 1)$. By construction, $V(w; \boldsymbol{\mu}) = \delta^{k-w} u(b) + \delta^{k-w+1} V(y; \boldsymbol{\mu})$ is strictly convex in w. So, $\boldsymbol{\mu}$ is an equilibrium iff:

$$\text{(A8)} \qquad \beta\delta V(2y; \boldsymbol{\mu}) \geq u(y) + \beta\delta V(y; \boldsymbol{\mu}) \Leftrightarrow \beta \geq \bar{\beta} \equiv \frac{1-\delta^k}{\delta^{k-1}[u(b)]} \cdot \frac{u(y)}{1-\delta}$$

By Condition 1, $0 < \bar{\beta} < 1$. Therefore, a full-saving ($i = 0$) equilibrium is optimal at $\beta \in [\bar{\beta}, 1)$.

(b) From 1(d), above, we know the optimal equilibrium will be no-saving for $\beta \in (0, \underline{\beta}]$, where $\underline{\beta} < \bar{\beta}$. Consider the strategy $\boldsymbol{\mu} \equiv (0, 1, 1, ..., 1)$. Define $\underline{\underline{\beta}}$ as the point at which the agent at wealth y is indifferent between saving and not saving at wealth y:

$$\text{(A9)} \qquad \underline{\underline{\beta}}\delta V(2y; \boldsymbol{\mu}) = u(y) + \underline{\underline{\beta}}\delta V(y; \boldsymbol{\mu}) \Rightarrow \underline{\underline{\beta}} = \frac{u(y)}{\delta^{k-1}u(b) - \frac{(\delta-\delta^k)u(y)}{1-\delta}}$$

$\underline{\underline{\beta}} > 0$ and is a lower bound on $\underline{\beta}$. Therefore, $0 < \underline{\beta} < \bar{\beta}$.

(3) Consider any $\tilde{\beta}$ such that $\pi_y\left(\tilde{\beta}\right) > 0$. Since $\boldsymbol{\pi}$ is continuous at $\tilde{\beta}$, and since by construction $V(w; \boldsymbol{\pi})$ is continuous in $\boldsymbol{\pi}$, and since $U_y = \beta\delta V(2y; \boldsymbol{\pi})$, U_y is continuous in β.

Consider any $\tilde{\beta}$ such that $\pi_y\left(\tilde{\beta}\right) = 0$. Since, for all $\beta < \tilde{\beta}$, $U_y = u(y) + \beta\delta V(y; \boldsymbol{\pi}) = \left(1 + \frac{\beta\delta}{1-\delta}\right) u(y)$, U_y is left-continuous in β. For $\beta > \tilde{\beta}$, either non-saving equilibrium continue to be optimal (in which case U_y is continuous) or there is a saving equilibrium with $\lim_{\beta \to \tilde{\beta}^+} \pi_y(\beta) = 0$. If the latter is true, then $\lim_{\beta \to \beta^+} U_y = \tilde{\beta}\delta V(2y; \boldsymbol{\pi}) = u(y) + \tilde{\beta}\delta V(y; \boldsymbol{\pi})$. This proves continuity of U_y.

(4) Observe the following: If $\beta \in (0, \underline{\beta}]$, $U_y = u(y) + \beta\frac{\delta u(y)}{1-\delta}$; if $\beta \in \left(\underline{\beta}, \bar{\beta}\right)$, $U_y = \beta\delta V(2y; \boldsymbol{\pi}) = \beta\frac{\delta^{k-\frac{i}{y}}}{1-\delta^{k-\frac{i}{y}}} u(b) - \left(\frac{\frac{i}{y}}{1-\delta^{k-\frac{i}{y}}} - 1\right) u(y)$; if $\beta \in [\bar{\beta}, 1)$, $U_y = \beta\frac{\delta^{k-1}u(b)}{1-\delta^k}$. Since i is weakly dropping in β, and since $\frac{\delta^{k-\frac{i}{y}}}{1-\delta^{k-\frac{i}{y}}}$ is strictly rising in i, U_y is strictly increasing and weakly concave for $\beta \in \left(\underline{\beta}, 1\right)$. Furthermore, for $\beta \in \left(\underline{\beta}, \bar{\beta}\right)$, $\frac{dU_y}{d\beta}$ is greater than for $\beta \in [\bar{\beta}, 1)$.

A2. Source of Commitment

A natural question in Section III is: what exactly about the rosca provides commitment to the hyperbolic discounter? A rosca comes with (1) the threat of being barred for non-payment and (2) illiquidity of savings. We can consider two alternative commitment savings devices that separately perform these functions: a friend who promises to monitor your saving and credibly threatens to stop helping if you under-save, and a fixed-deposit that locks up your savings until you reach a target amount (in this case, ky).

In the $\left(\underline{\beta}, \bar{\beta}\right)$ range, illiquidity plays no role, since even in autarky the agent never dips into her savings. As long as we are in a region where some saving occurs in equilibrium, access to a fixed deposit cannot improve savings behavior. Here, a fixed rosca is very similar to a friend who offers to "help". The fact that the rosca can offer a credible threat to deny access to defectors creates a large enough utility gap between the rosca and autarky equilibria to ensure participation.[26]

At lower levels of β, the illiquidity provided by the rosca can play a role. Consider the region in which an agent stays in a rosca but would not save in autarky ($\beta < \underline{\beta}$):

$$\frac{\beta\delta^{k-1}u\left(b\right)}{1-\delta^{k}} \geq u\left(y\right) + \frac{\beta\delta u\left(y\right)}{1-\delta}$$

Now, suppose the agent had the helpful friend instead of the rosca. The above condition might no longer be sufficient to ensure cooperation. She would also need to ensure that at wealth $2y$ she did not have an incentive to consume everything:

$$\frac{\beta\delta^{k-2}u\left(b\right)}{1-\delta^{k}} \geq u\left(2y\right) + \frac{\beta\delta u\left(y\right)}{1-\delta}$$

If u is not very concave, and δ is high, then the second condition can fail even if the first is satisfied. With δ high, the agent does not benefit as much from being one period closer to the nondivisible. If u is almost linear, the benefit of consuming $2y$ can be enough to outweigh the fact that she will no longer save in the future.[27] However, even when the illiquidity plays a role, this does not make the contracting aspect of a rosca irrelevant. With a fixed-deposit instead of a rosca, the agent might simply not deposit any money in subsequent periods.

[26] An individual's ability to play history-dependent equilibria with her future selves is limited by the fact that, if she were to deviate from her equilibrium path, she could easily renegotiate with herself to not play the punishment strategy.

[27] One could expand the region in which illiquidity plays a role if there is either an intermediate "temptation" good or if income is stochastic. In each of these cases, the agent will have a greater incentive to dip into her savings in autarky, and she might therefore further value the fact that a rosca will prevent her from doing so.